Association Analysis Short Review

cse634

Data Mining

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The Apriori Algorithm: Basics

The Apriori Algorithm

It is an influential algorithm for mining
frequent itemsets
and using them for creating
association rules

Key Concepts:

- Frequent Itemsets
- Apriori Property

The Apriori Algorithm: Basics

Key Concepts:

Frequent Itemsets

- The sets of item which has minimum support
 (denoted by L_i for i^{th}-Itemset)

- Apriori Property
- Any subset of frequent itemset must be frequent

- Join Operation
- To find L_k , a set of candidate k-itemsets is generated by **joining** L_{k-1} with itself.

The Apriori Algorithm in a Nutshell

- Apriori Algorithm **finds the** frequent itemsets

  i.e. the sets of items that have minimum support

  It follows the Apriori Principle:

  a subset of a frequent itemset must also be a frequent itemset

  i.e., if {*A, B*} is a frequent itemset, both {*A*} and {*B*} should be a frequent itemset

The Apriori Algorithm in a Nutshell

- Apriori Algorithm

  The algorithm Iteratively **finds** frequent itemsets with cardinality from 1 to *k* (k-itemset)

- As the next step in the Apriori Process
  we use the frequent itemsets to **generate**
  association rules

The Apriori Algorithm : Pseudo code

- Join Step: C_k is generated by **joining** L_{k-1} with itself
- Prune Step: Any (k-1)-itemset that is not frequent cannot be a subset of a frequent k-itemset

- Pseudo-code:

  C_k: Candidate itemset of size k
  L_k : frequent itemset of size k

```
L_1 = {frequent items};
for (k = 1; L_k !=∅; k++) do begin
    C_k+1 = candidates generated from L_k;
   for each transaction t in database do
          increment the count of all candidates in C_k+1
       that are contained in t
   L_k+1  = candidates in C_k+1 with min_support
   end
return ∪_k L_k;
```

The Apriori Algorithm: Example

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2 ,I3, I5
T100	I1, I2, I3

- Consider a database, D , consisting of 9 transactions.
- Suppose min. support count required is 2 (i.e. min_sup = 2/9 = 22 %)
- Let minimum confidence required is 70%.
- We have to first find out the **frequent itemset** using Apriori algorithm.
- Then, **Association rules** will be generated using min. support & min. confidence.

Step 1: Generating 1-itemset Frequent Pattern

Scan D for count of each candidate

Itemset	Sup.Count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

C_1

Compare candidate support count with minimum support count

Itemset	Sup.Count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

L_1

- The set of frequent 1-itemsets, L_1 , consists of the candidate
- 1- itemsets satisfying minimum support.
- In the first iteration of the algorithm, each item is a member of the set of candidates.

Step 2: Generating 2-itemset Frequent Pattern

- To **discover** the set of frequent 2-itemsets, L_2 , the algorithm uses **L_1 *Join* L_1** to generate a candidate set of 2-itemsets, C_2

- **Nex**t, the transactions in D are scanned and the support count for each candidate itemset in C_2 is accumulated (as shown in the middle table)

Step 2: Generating 2-itemset Frequent Pattern

- 2-itemsets, L_2 , is then **determined**, consisting of those candidate 2-itemsets in C_2 having minimum support

- **Note:** We haven't used Apriori Property yet

Step 2: Generating 2-itemset Frequent Pattern

Generate C_2 candidates from L_1

Itemset
{I1, I2}
{I1, I3}
{I1, I4}
{I1, I5}
{I2, I3}
{I2, I4}
{I2, I5}
{I3, I4}
{I3, I5}
{I4, I5}

C_2

Scan D for count of each candidate

Itemset	Sup. Count
{I1, I2}	4
{I1, I3}	4
{I1, I4}	1
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2
{I3, I4}	0
{I3, I5}	1
{I4, I5}	0

C_2

Compare candidate support count with minimum support count

Itemset	Sup Count
{I1, I2}	4
{I1, I3}	4
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2

L_2

Step 3: Generating 3-itemset Frequent Pattern

L_2 *Join* L_2 and prune **generated** C_3 candidates

Itemset
{I1, I2, I3}
{I1, I2, I5}

C_3

Scan D for count of each candidate

Itemset	Sup. Count
{I1, I2, I3}	2
{I1, I2, I5}	2

C_3

Compare candidate support count with min support count

Itemset	Sup Count
{I1, I2, I3}	2
{I1, I2, I5}	2

L_3

- In order to find C_3, we first compute L_2 *Join* L_2
- C_3 = L2 *Join* L2 = {{I1, I2, I3}, {I1, I2, I5}, {I1, I3, I5}, {I2, I3, I4}, {I2, I3, I5}, {I2, I4, I5}}.
- Now, Join step is **complete** and Prune step will be used to **reduce** the size of C_3
- Prune step helps to avoid heavy computation due to large C_k.

Step 3: Generating 3-itemset Frequent Pattern

- Apriori property says that all subsets of a frequent itemset must also be frequent

- C_3 = L2 *Join* L2 = {{I1, I2, I3}, {I1, I2, I5}, {I1, I3, I5}, {I2, I3, I4}, {I2, I3, I5}, {I2, I4, I5}}

- We determine now which of candidates in C_3 **can** and which **can not** possibly be frequent
- Take {I1, I2, I3}
- The 2-item subsets of it are {I1, I2}, {I1, I3}, {I2, I3}

  All of them are members of L_2

  We **keep** {I1, I2, I3} in C_3

Step 3: Generating 3-itemset Frequent Pattern

- Lets take {I2, I3, I5}
- The 2-item subsets are {I2, I3}, {I2, I5}, {I3,I5}
- But {I3, I5} is not a member of L_2 and hence it is not frequent violating Apriori Property
- Thus we **remove** {I2, I3, I5} from C_3

All 2-item subsets of {I1, I2, I5} members of L_2

Therefore C_3 = {{I1, I2, I3}, {I1, I2, I5}}

- Now, the transactions in D are scanned in order to determine L_3, consisting of those candidates 3-itemsets in C_3 having minimum support and we get that
- L_3 = {{I1, I2, I3}, {I1, I2, I5}}

Step 4: Generating 4-itemset Frequent Pattern

- The algorithm uses L_3 *Join* L_3 to generate a candidate set of 4-itemsets, C_4
- C_4 = L3 *Join* L3 = {{I1, I2, I3, I5}}
- This itemset {{I1, I2, I3, I5}} is pruned since its subset {{I2, I3, I5}} is **not frequent.**
- Thus, $C_4 = \varphi$ and algorithm **terminates**
- What's Next ?

  Obtained frequent itemsets are to be used to generate strong association rules

(where strong association rules are rules that satisfy both minimum support and minimum confidence)

Step 5: Generating Association Rules from Frequent Itemsets

- Procedure:
  - For each frequent itemset **I**, generate the set of all nonempty subsets of **I**
  - For every nonempty subset ***S*** of **I**,
  - output the rule **S → I - S**
  - if support_count(**I**) **/** support_count(**S**) **>=** min_conf
  - where min_conf is minimum confidence threshold.

- Example

  We obtained the set od all frequent itemsets

L = {{I1}, {I2}, {I3}, {I4}, {I5}, {I1,I2}, {I1,I3}, {I1,I5}, {I2,I3}, {I2,I4}, {I2,I5}, {I1,I2,I3}, {I1,I2,I5}}

- Lets take for example **I** = {I1,I2,I5}

Step 5: Generating Association Rules from Frequent Itemsets

- Lets take **I** = {I1,I2,I5}
  - Its all nonempty subsets are {I1,I2}, {I1,I5}, {I2,I5}, {I1}, {I2}, {I5}

  Let minimum confidence threshold be, say 70%
- The resulting association rules are shown below, each listed with its confidence.
  - R1: I1 ^ I2 → I5
    - Confidence = sc{I1,I2,I5}/sc{I1,I2} = 2/4 = 50%
    - R1 is Rejected.
  - R2: I1 ^ I5 → I2
    - Confidence = sc{I1,I2,I5}/sc{I1,I5} = 2/2 = 100%
    - R2 is Selected.
  - R3: I2 ^ I5 → I1
    - Confidence = sc{I1,I2,I5}/sc{I2,I5} = 2/2 = 100%
    - R3 is Selected.

Step 5: Generating Association Rules from Frequent Itemsets

- R4: I1 → I2 ^ I5
  - Confidence = sc{I1,I2,I5}/sc{I1} = 2/6 = 33%
  - R4 is rejected.
- R5: I2 → I1 ^ I5
  - Confidence = sc{I1,I2,I5}/{I2} = 2/7 = 29%
  - R5 is rejected.
- R6: I5 → I1 ^ I2
  - Confidence = sc{I1,I2,I5}/ {I5} = 2/2 = 100%
  - R6 is Selected

- We have found three strong association rules

Methods to Improve Apriori’s Efficiency

- Hash-based itemset counting:
- A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent.

- Transaction reduction:
- A transaction that does not contain any frequent k-itemset is useless in subsequent scans

- Partitioning:
- Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB.

Methods to Improve Apriori's Efficiency

- Sampling:
- mining on a subset of given data,
- lower support threshold
- add a method to determine the completeness

- Dynamic itemset counting:
- add new candidate itemsets **only** when all of their subsets are estimated to be frequent

Mining Frequent Patterns Without Candidate Generation

- **Compress** a large database into a compact,
- Frequent-Pattern tree (FP-tree) structure
  - highly condensed, but complete for frequent pattern mining
  - avoid costly database scans
- **Develop** an efficient, FP-tree-based frequent pattern mining method
  - A divide-and-conquer methodology: decompose mining tasks into smaller ones
  - **Avoid** candidate generation:

    sub-database test only!

FP-Growth Method : An Example

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2 ,I3, I5
T100	I1, I2, I3

- Consider the same previous example of a database, D , consisting of 9 transactions.
- Suppose min. support count required is 2 (i.e. min_sup = 2/9 = 22 %)
- The **first** scan of database is same as Apriori, which derives the set of 1-itemsets & their support counts.
- The set of frequent items is **sorted** in the order of descending support count.
- The resulting set is denoted as L = {I2:7, I1:6, I3:6, I4:2, I5:2}

FP-Growth Method: Construction of FP-Tree

- First, **create** the root of the tree, labeled with "null".

- **Scan** the database D a second time
- First time we scanned it to create 1-itemset and then
- L = {I2:7, I1:6, I3:6, I4:2, I5:2}

- The items in each transaction are **processed** in L order (i.e. sorted order)

- A **branch** is created for each transaction with items having their support count separated by colon

FP-Growth Method: Construction of FP-Tree

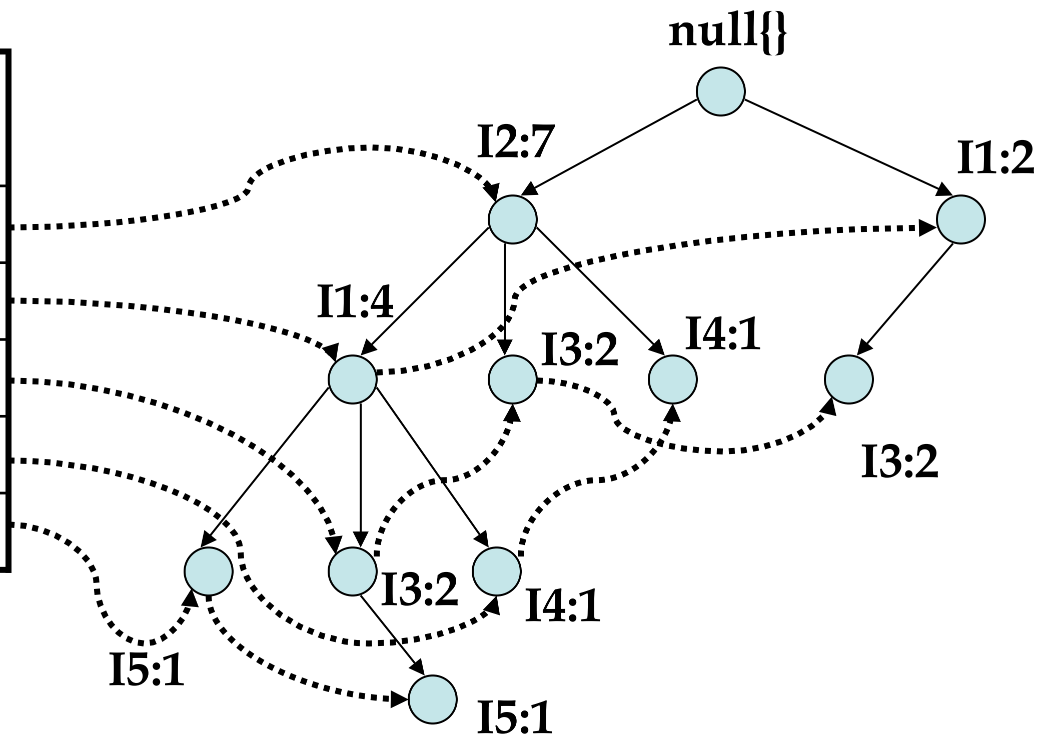
- Whenever the same node is encountered in another transaction, we just increment the support count
- of the common node or Prefix

- To facilitate tree traversal, an item header table is built
- so that each item **points** to its occurrences in the
- tree via a chain of node-links

- The problem of mining frequent patterns in database is
- transformed to that of mining the FP-Tree

FP-Growth Method: Construction of FP-Tree

Item Id	Sup Count	Node-link
I2	7	
I1	6	
I3	6	
I4	2	
I5	2	



An FP-Tree that registers compressed, frequent pattern information

Mining the FP-Tree by Creating Conditional (sub) pattern bases

Steps:

1. Start from each frequent length-1 pattern (as an initial suffix pattern).
2. Construct its conditional pattern base which consists of the set of prefix paths in the FP-Tree co-occurring with suffix pattern.
3. Then, Construct its conditional FP-Tree & perform mining on such a tree.
4. The pattern growth is achieved by concatenation of the suffix pattern with the frequent patterns generated from a conditional FP-Tree
5. The **union** of all frequent patterns (generated by step 4) gives the required frequent itemset

FP-Tree Example

Item	Conditional pattern base	Conditional FP-Tree	Frequent pattern generated
I5	{(I2 I1: 1),(I2 I1 I3: 1)}	<I2:2 , I1:2>	I2 I5:2, I1 I5:2, I2 I1 I5: 2
I4	{(I2 I1: 1),(I2: 1)}	<I2: 2>	I2 I4: 2
I3	{(I2 I1: 1),(I2: 2), (I1: 2)}	<I2: 4, I1: 2>,<I1:2>	I2 I3:4, I1, I3: 2 , I2 I1 I3: 2
I2	{(I2: 4)}	<I2: 4>	I2 I1: 4

Mining the FP-Tree by creating conditional (sub) pattern bases

Now, Following the above mentioned steps:

- Lets start from I5. The I5 is involved in 2 branches namely {I2 I1 I5: 1} and
- {I2 I1 I3 I5: 1}
- Therefore considering I5 as **suffix**, its 2 corresponding **prefix paths** would be {I2 I1: 1} and {I2 I1 I3: 1}, which forms its conditional pattern base.

FP-Tree Example

- {I2 I1: 1}, {I2 I1 I3: 1} form the conditional pattern base

- Out of these, only I1 and I2 is selected in the conditional FP-Tree because I3 is **not** satisfying the minimum support count.

  For I1 , support count in conditional pattern base = 1 + 1 = 2
  For I2 , support count in conditional pattern base = 1 + 1 = 2
  For I3, support count in conditional pattern base = 1

  Thus support count for I3 is **less than** required min_sup which is 2 here

FP-Tree Example

- Now , we have conditional FP-Tree with us
- All frequent patterns corresponding to **suffix** I5 are generated by considering all possible combinations of
- I5 and conditional FP-Tree.
- The same procedure is applied to **suffixes** I4, I3 and I1
- Note:
- I2 is **not** taken into consideration **for suffix** because it doesn't have any **prefix** at all

Why Frequent Pattern Growth Method?

- **Performance study shows**
  - FP-growth is an order of magnitude **faster** than Apriori, and is also **faster** than tree-projection
- Reasoning
  - **No** candidate generation, **no** candidate test
  - Use **compact** data structure
  - **Eliminate** repeated database scan
  - Basic **operation** is counting and FP-tree building