

cse643

Data Mining

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Course Textbook

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DATA MINING

Concepts and Techniques

Morgan Kaufmann

Second or Third Edition

Book 2nd edition and Course Content

We will cover, some in more detail than the others, the following **book chapters** from the **2nd edition**

- **Chapter 1** Introduction and overview
- **Chapter 2** Preprocessing
- **Chapter 3** Data warehouse and OPLAP
- **Chapter 5** Association
- **Chapter 6** Classification
- **Chapter 7** Clustering

Book 2nd Edition and Course Content

- Chapter 8 Stream, series, sequence data
- Chapter 10 Multimedia, text, web data
- Chapter 11 Applications and trends

- We will also cover in some level of detail the following subjects
 - Genetic Algorithms
 - Types of Neural Networks
 - Protein Secondary Structure Prediction
 - Descriptive Granularity - DM Model

Book and Course Content

We will cover, some in more detail than the others, the following **book chapters** from the **3rd edition**

- **Chapter 1** Introduction
- **Chapter 2** Know your Data
- **Chapter 3** Data Preprocessing
- **Chapter 4** Data Warehousing
- **Chapter 6** Mining Frequent Patterns; Association and Correlation: Basic Concepts
- **Chapter 7** Advanced Frequent Patterns

Book 3rd Edition and Course Content

- Chapter 8 Classification: Basic Concepts
- Chapter 9 Classification: Advanced Concepts
- Chapter 10 Cluster Analysis: Basic Concepts
- Chapter 13 Trends and Research Frontiers in DM

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Chapter 1: Introduction

- **Data Mining Main Objectives:**
- **Indentification** of **data as a source** of useful **information**
- **Use** of discovered information for **competitive advantages** when working in **business enviroment**

Data – Information - Knowledge

- **Data** – as in databases
- **Information** or **knowledge** is a meta information **ABOUT** the **patterns hidden** in the data
- **The patterns** must be discovered **automatically**

Why Data Mining?

- **Data explosion problem**

Automated data collection tools and mature database technology lead to tremendous amounts of data stored in **databases, data warehouses** and other data **repositories**

Why Data Mining

- **Data explosion problem**
- We are drowning in **data**, but starving for **knowledge**
- **Solution:** Data warehousing and data mining
- **Data Mining:**
Extraction of interesting knowledge (rules, regularities, patterns, constraints) from data in large databases

What is Data Mining?

- There are many activities with the same name: CONFUSION
- **DM**: Huge volumes of data
- **DM**: Potential hidden knowledge
- **DM**: Process of **discovery** of hidden **patterns** in data

DM: Intuitive Definition

DM is a process to extract previously unknown knowledge from large volumes of data

Requires both new technologies and new methods

Data Mining

- **DM** creates models (algorithms):
- **classification** (chapter 6)
- **association** (chapter 5)
- **prediction** (chapter 6)
- **clustering** (chapter 7)

Data Mining

- **DM** often **presents** the knowledge as a set of **rules** of the form

IF.... THEN ...

In this case it is called a **Descriptive DM**

- **DM** finds **other** relationships in data
- **DM** detects deviations

DM: Some Historical Applications

- **Market analysis and management**
 - target marketing, customer relation management, market basket analysis, cross selling, market segmentation
- **Risk analysis and management**
 - Forecasting, customer retention, improved underwriting, quality control, competitive analysis

DM: Some Historical Applications

- **More Applications**
- Text mining
- News groups, emails, documents
- Web analysis
- Intelligent query answering
- Scientific Applications

DM: Business Advantages

- **Data Mining** uses collected data to
- **Predict** tendencies and waves
- **Classifiy** new data
- **Find** previously unknown patterns for the use for business advantages
- **Discover** unknown relationships

DM: Technologies

- Many commercially available tools
- Many **methods** (models, algorithms) for the same task
- **TOOLS ALONE ARE NOT THE SOLUTION**
- **The user must often be able to interpret the results**

DM: Technologies

- One of the **requirements** of **DM** is:
“the results must be easily comprehensible to the user”
- **Strenght** of **Descriptive Methods**
- Most often, especially when dealing with **statistical methods**
- separate analysts are needed to **interpret** the results (knowledge)
- **Weakness** of **Statistical** Methods

Data Mining vs Statistics

- Some **statistical methods** are considered as a part of **Data Mining** i.e.
- they are used **as Data Mining algorithms**, or as a **part of Data Mining algorithms**
- Some, like **statistical prediction** methods, different types of **regression**, and **clustering** methods are now considered as an **integral part of Data Mining** research and applications

Bussiness Applications

- Buying patterns
- Fraud detection
- Customer Campaings
- Decision support
- Medical aplications
- Marketing
- and more

Fraud Detection and Management (B1)

- **Applications**

- widely used in health care, retail, credit card services, telecommunications (phone card fraud), etc.

- **Approach**

- use historical data to **build models of fraudulent behavior** and use data mining to help identify similar instances

Fraud Detection and Management (B2)

- **Examples**

- **auto insurance**: detect **characteristics of group of people** who stage accidents to collect on insurance
- **money laundering**: detect **characteristics of suspicious money transactions** (US Treasury's Financial Crimes Enforcement Network)
- **medical insurance**: detect **characteristics of fraudulent** patients and doctors

Fraud Detection and Management (B3)

- **Detecting inappropriate medical treatment**
 - **Australian Health Insurance** Commission detected that in many cases blanket screening tests were requested (save Australian \$1m/yr)
- **Detecting telephone fraud**
 - **DM builds telephone call model**: destination of the call, duration, time of day or week.
 - **Detects patterns** that deviate from an expected norm.
 - **British Telecom** identified discrete groups of callers with frequent intra-group calls, especially mobile phones, and broke a **multimillion dollar fraud**

Fraud Detection and Management (B4)

- **Retail**

- Analysts used Data Mining techniques to estimate that **38% of retail shrink** is due to **dishonest employees**
- and more....

Data Mining vs Data Marketing

- **Data Mining** methods apply to many domains
- **Data Marketing:**
- Applications of Data Mining methods in which the goal is to find **buying patterns** in **Transactional Data Bases**
- **Association Rules- APRIORI Algorithm**

Market Analysis and Management (MA1)

- **Data sources for analysis**
 - Credit card transactions, loyalty cards, discount coupons, customer complaint calls, plus (public) lifestyle studies
- **Target marketing**
 - **DM** finds **clusters** of “model” customers who share the same **characteristics**: interest, income level, spending habits, etc.

Market Analysis and Management (MA2)

- Determine customer **purchasing patterns** over time
 - Conversion of single to a joint bank account: when marriage occurs, etc.
- **Cross-market analysis**
 - **Associations/co-relations** between **product sales**
 - **Prediction** based on the **association** information

Market Analysis and Management (MA3)

- **Customer profiling**
 - **DM** can tell you what types of customers buy what products (clustering or classification)
- **Identifying customer requirements**
- identifying the best products for different customers

Corporate Analysis and Risk Management (CA1)

- **Finance planning and asset evaluation**
 - cash flow analysis and prediction
 - contingent claim analysis to evaluate assets
 - cross-sectional and time series analysis (financial-ratio, trend analysis, etc.)
- **Resource planning:**
 - summarize and compare the resources and spending

Corporate Analysis and Risk Management (CA2)

- **Competition:**
 - monitor competitors and market directions
 - group customers into classes and a class-based pricing procedure
 - set pricing strategy in a highly competitive market

Business Summary

- **Data Mining** helps **to improve competitive advantage** of organizations in dynamically changing environment;
- it improves clients **retention** and **conversion**
- Different **Data Mining** methods are **required** for different kind of **data** and different kinds of **goals**

Scientific Applications

- Networks failure detection
- Controllers
- Geographic Information Systems
- Genome- Bioinformatics
- Intelligent robots
- Intelligent rooms
- etc... etc

What is **NOT** Data Mining

- Once **patterns** are found **Data Mining process** is **finished**
- Use of the patterns is **not** Data Mining
- **Monitoring** is **not** Data Mining
- **Queries** to the database are **not** DM

Evolution of Database Technology

- 1960s:
 - Data collection, database creation, **IMS** and network **DBMS**
- 1970s:
 - Relational data model, **relational DBMS** implementation

Evolution of Database Technology

- 1980s:
 - **RDBMS**, advanced data models (extended-relational, OO, deductive, etc.) and application-oriented **DBMS** (spatial, scientific, engineering, etc.)
- 1990s—2000s:
 - **Data mining** and **data warehousing**, multimedia databases, and Web database
- 2000 ---- **Big Data**

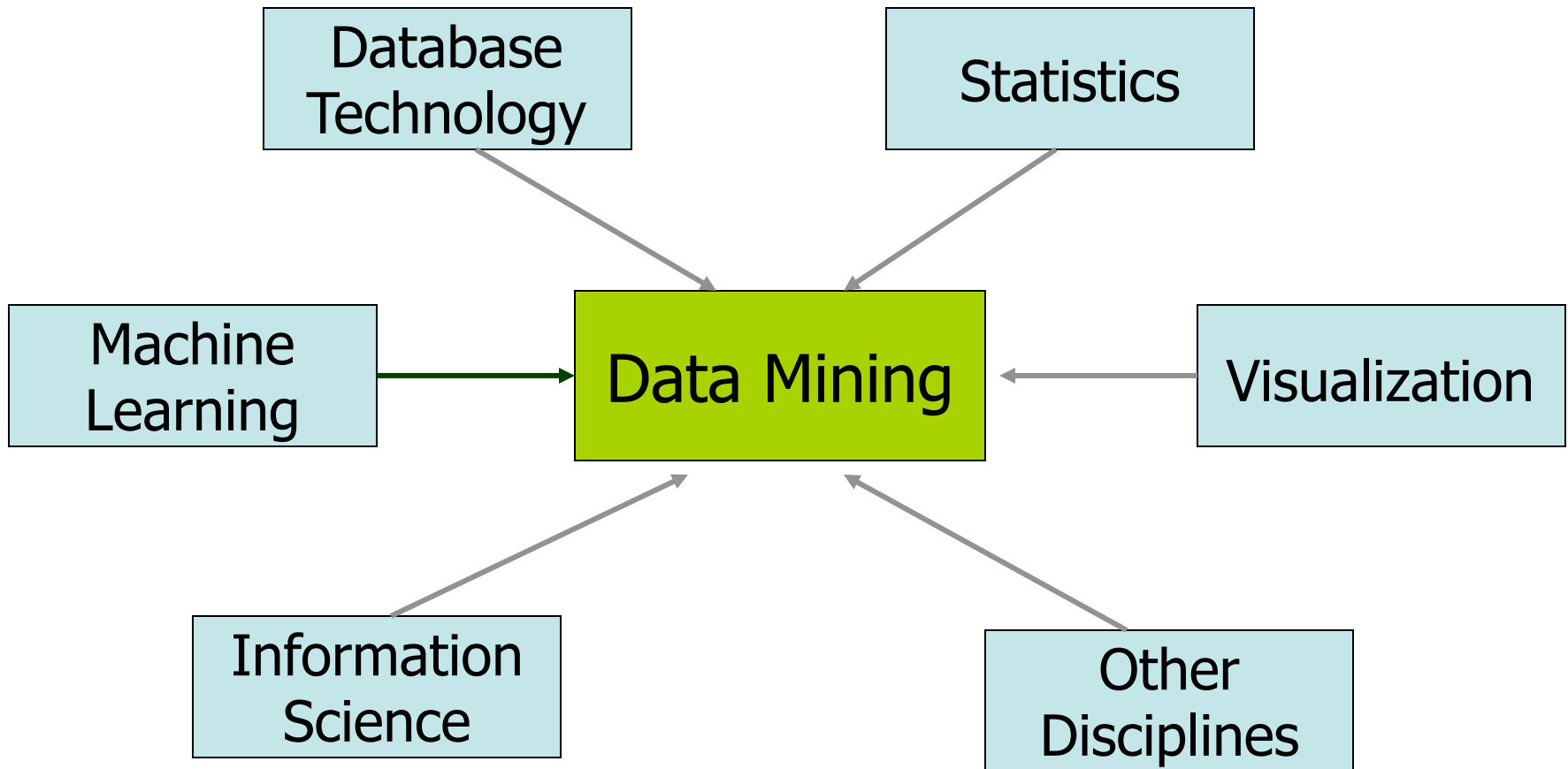
Short History of Data Mining

- **1989** - **KDD** term: **Knowledge Discovery in Databases** appears in (IJCAI Workshop)
- **1991** - a collection of **research papers** edited by Piatetsky-Shapiro and Frawley
- **1993** – **Association Rule Mining Algorithm** **APRIORI** proposed by **Agraval, Imielinski and Swami**

Short History of Data Mining

- **1996 –2000**
- **KDD** evolves as a conjunction of different knowledge areas:
 - data bases, machine learning, statistics, artificial intelligence
- **2000- present**
- the term **Data Mining** becomes established and evolves into **Big Data**

Data Mining: Confluence of Multiple Disciplines

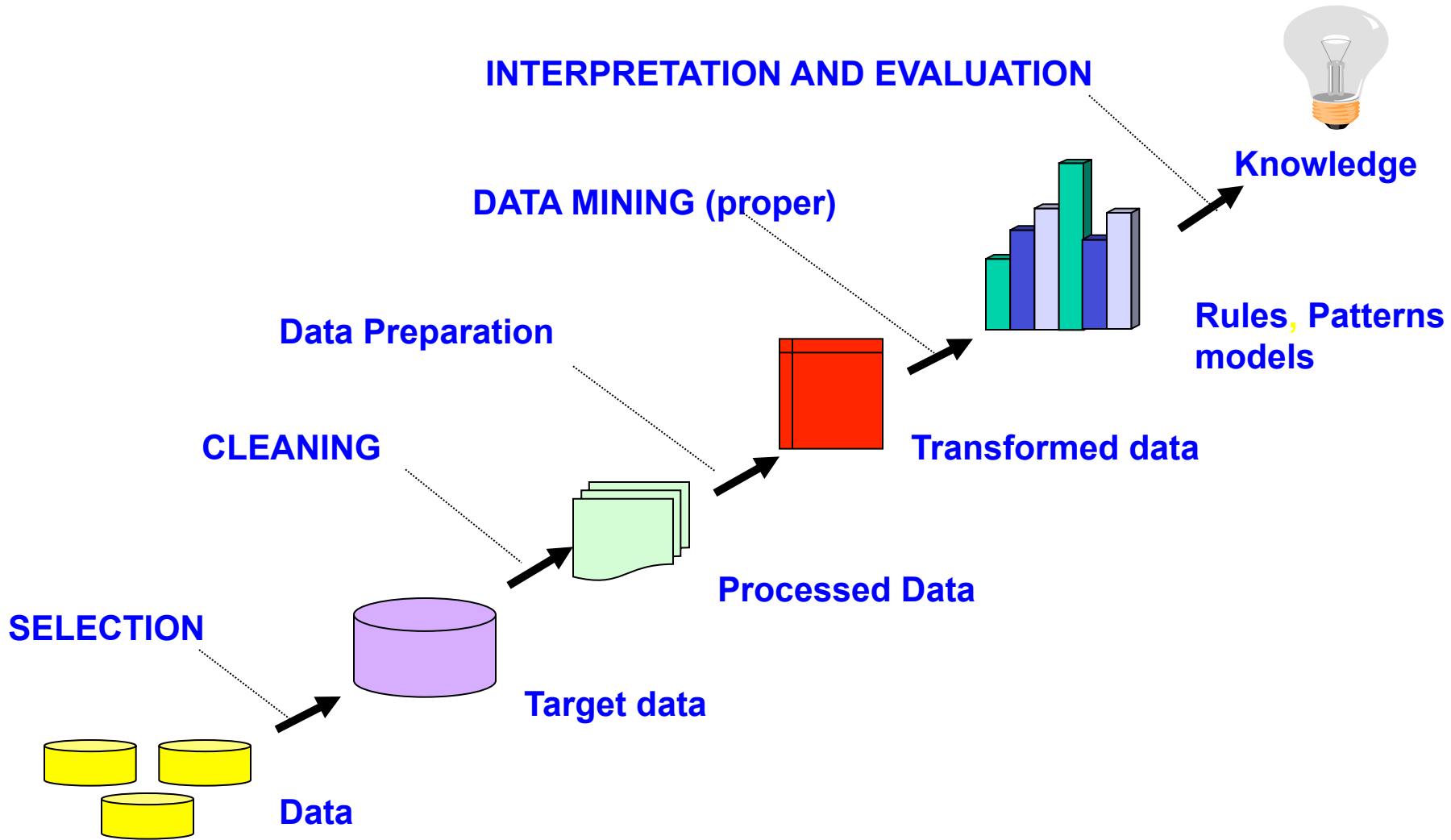


KDD process Definition

[Piatetsky-Shapiro 97]

- **KDD** is a non trivial **process** for identification of :
 - Valid
 - New
 - Potentially useful
 - Understandable
 - **patterns in data**

KDD Process



DM: Data Mining

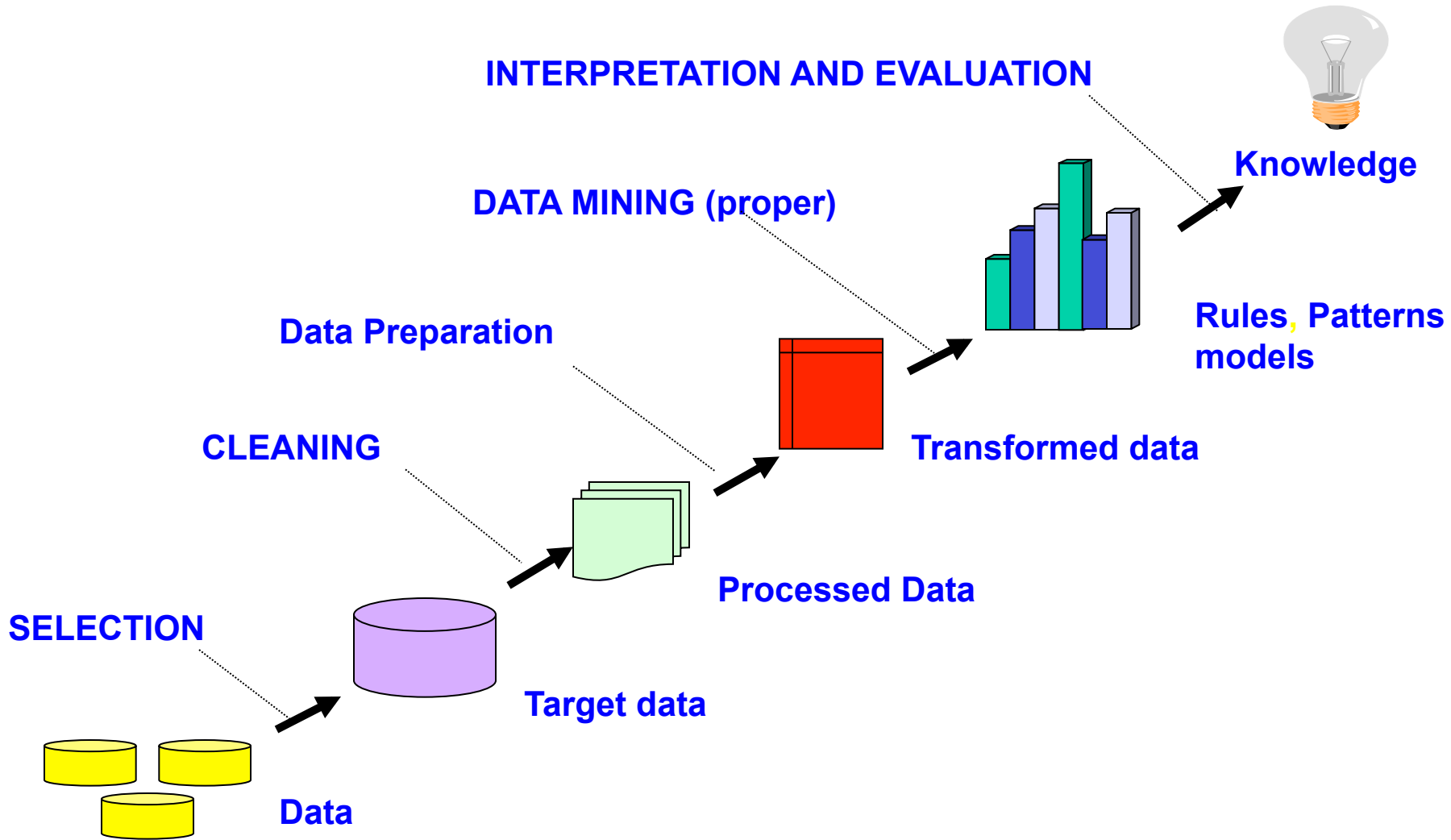
- **DM** is a **step** in the **KDD process**
- in which algorithms are applied to look for patterns in data

- We use term **DATA MINING PROPER** for **DM step** in KDD Process
- We usually use term **DM process** term for **KDD process**

DM: Data Mining Process

- **Remember**
- It is necessary to apply first
- the **preprocessing** operations to clean and preprocess the data in order to obtain **significant patterns**
- **DM Process** can be re-iterated- and usually is

Data Mining Process



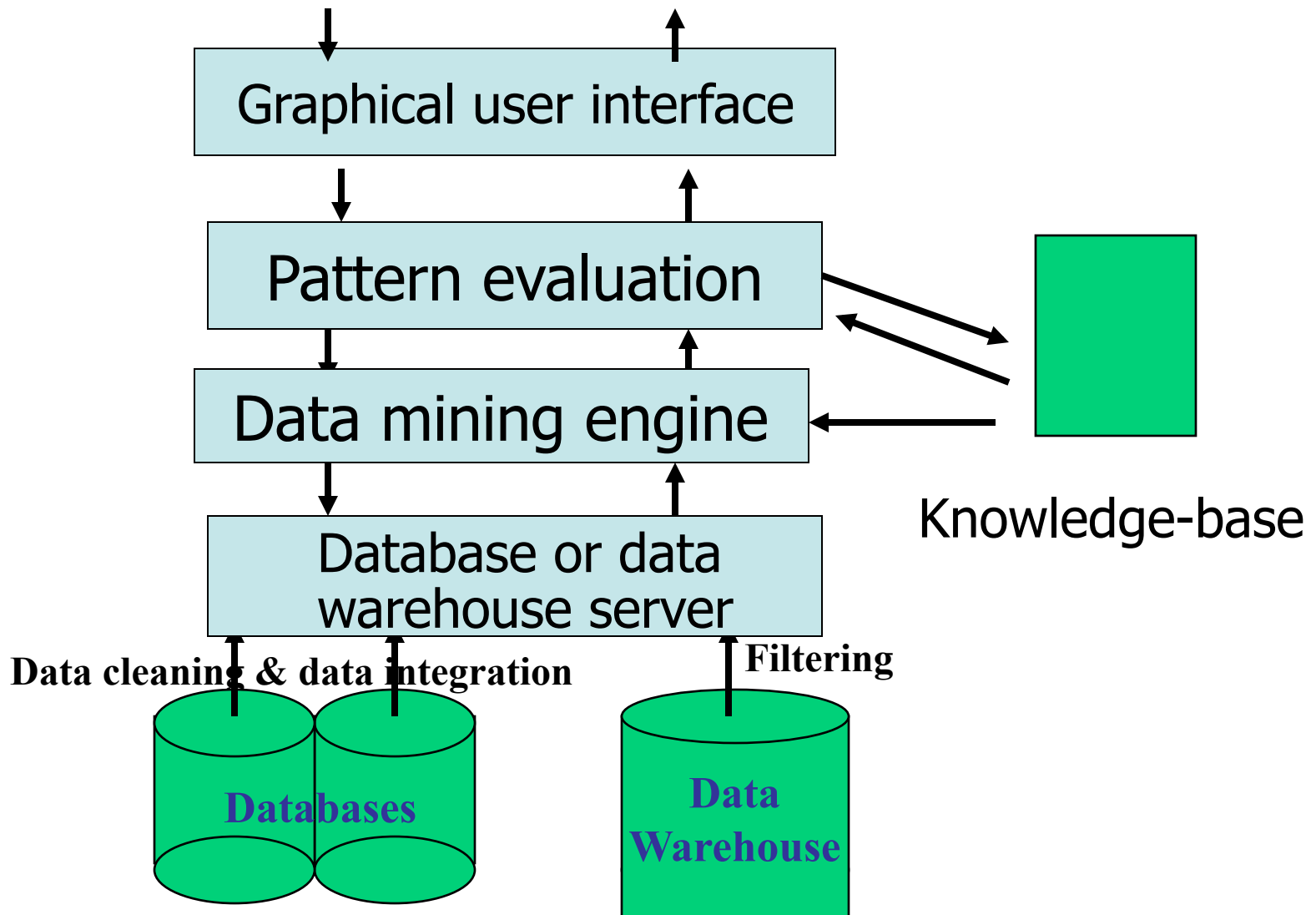
KDD vs DM

- **KDD** was a term used by **academia**
- **DM** was often used as a **commercial** term
- **DM** term is now being used in academia, as it has become a “**brand name**” for both **KDD process** and its **DM sub-process**
- The important point is to see **DM** as a **process** with **Data Mining Proper** as part of it
- **BIG DATA** – a new widely use term

Steps of the DM process

- **Preprocessing:** includes all the operations that have to be performed before a data mining algorithm is applied
- **Data Mining (proper):** knowledge discovery algorithms are applied in order to obtain the **patterns**
- **Interpretation:** discovered patterns are presented in a proper format and the user decides if it is necessary to **re-iterate** the algorithms

Architecture of a Typical Data Mining System (book slide)



What Kind of Data?

- **Relational** Databases
- Data **warehouses**
- **Transactional** databases
- Advanced DB and information repositories
 - Object-oriented and object-relational databases
 - Spatial databases
 - Time-series data and temporal data
 - Text databases and multimedia databases
 - Heterogeneous and legacy databases
 - WWW

Descriptive Data Mining:

Concept Description

- **Concept** – is defined **semantically** as **any subset of records**
- We often define the **concept** by distinguishing in our database an **attribute c** and **its value v**
- In this case the **concept description** is written **syntactically** as : **$c=v$**
- We define a concept with the description **$c=v$** as
 $CONCEPT = \{records: c=v\}$

Descriptive Data Mining:

Concept Description

- Let **C** be a concept with the description **c=v**, i.e.

$$C = \{records: c=v\}$$

- We call such attribute **c** a **class** attribute, or a **decision** attribute, or a **classification** attribute
- The description **c=v** is called a **class**, or **decision** description

Descriptive Data Mining:

Concept, Class Description

- **For example:**
- *climate=wet* is a *description* of the concept of WET CLIMATE and
- ***WET CLIMATE = {records: climate=wet}***
- **We use words:** decision attribute, class attribute, concept attribute
- We talk about **decision** or **class description**
-
- **REMEMBER:** all definitions are relative to the **database** we deal with.

Descriptive DM

Decision, Concept, Class Characteristics

- Let **C** be a **class** (concept) with a **description** **c=v**, i.e.
 $C = \{records: c=v\}$
- The **class C characteristics** is a set of attributes **a1, a2, ... ak**, and their respective values **v1, v2, vk** that are **characteristic** for a given class **C**, i.e. such that
- **$\{records: a1=v1 \ \& \ a2=v2 \ \& \ \dots \ ak=vk\} \wedge C = \text{non empty set}$**
- **Characteristics description** of **C** is then written as
 $a1=v1 \ \& \ a2=v2 \ \& \ \dots \ ak=vk$

Characterization

- *Describes the process which aim is to **find rules** that describe **characteristic** properties of a class (concept)*

- They take the form

If class then characteristics

If $c = v$ then $a_2=v_2 \& \dots a_k=v_k$

$C=1 \rightarrow A=1 \ \& \ B=3$ **25%** (support: there are 25% of the records for which the rule is true)

• $C=1 \rightarrow A=1 \ \& \ B=4$ **17%**

• $C=1 \rightarrow A=0 \ \& \ B=2$ **16%**

Discrimination

- *It is the process which aim is to **find rules** that allow us to **discriminate** the objects (records) belonging to a given concept (one class) from the rest of records*

If characteristics then concept

If $a_2=v_2 \& \dots a_k=v_k$ then $c = v$

- **A=0 & B=1 → C=1** 33% 83% (support, confidence: the conditional probability of the concept given the characteristics)
- **A=2 & B=0 → C=1** 27% 80%
- **A=1 & B=1 → C=1** 12% 76%

Classification - Supervised Learning

– Classification

- Finding models (**rules**) that describe (**characterize**) or/ and distinguish (**discriminate**) classes or concepts for **future prediction**
- **Example: classify** countries based on climate (**characteristics**)
- **classify** cars based on gas mileage and use it to **predict classification** of a new car

Classification Algorithms

Models, Basic Classifiers

- **Decision Trees (ID3, C4.5)** –descriptive
- **Neural Networks-** statistical
- **Bayesian Networks** - statistical
- **Rough Sets** - descriptive
- **Genetic Algorithms** – descriptive or statistical but mainly an optimization method
- **Classification by Association** – descriptive

Classification Algorithms

Models, Basic Classifiers

- **Presentation of results:**
 - characteristic and /or discriminant rules
 - In case of **descriptive DM**
- converged network (Neural, Bayes) in case of **statistical DM**

Statistical DM, Clustering

- **Statistical Prediction** - predict some unknown or missing numerical values
- **Cluster analysis (statistical)**
 - Class label is unknown
 - Goal: group data to form new classes
 - It is called **unsupervised learning**
 - **For example:** cluster houses to find distribution patterns
 - **Clustering** is based on the principle:
 - **maximizing** the **intra-class similarity** and **minimizing** the **interclass similarity**

Statistical DM

- **Outlier analysis**

- **Outlier:** a data object that does not comply with the general behavior of the data
- It can be considered as **noise** or **exception** but is quite **useful** in fraud detection, rare events analysis and others

Statistical DM

- **Trend and evolution analysis (statistical)**
 - Trend and deviation: regression analysis
 - Sequential pattern mining, periodicity analysis
 - Similarity-based analysis
- **Other pattern-directed or statistical analyses**

Classification

Supervised Learning

- Given a set of objects (**concept, class**) described by a **class attribute**, a **classification algorithm** builds a set of **discriminant and /or characterization rules** (or other descriptions in case of statistical methods) in order to be able,
- as the next step, to **classify unknown sets of objects**
- This is also called a **supervised learning**

Classification: Chapter 6

- Decision Trees (ID3, C4.5) – **descriptive**
- Neural Networks - **statistical**
- Rough Sets – **descriptive**
- Bayesian Networks- **statistical**
- Genetic Algorithms- can be both, but is mainly an **optimization method**

Association: Chapter 5

Problem Statement

- $I = \{i_1, i_2, \dots, i_n\}$ a set of **items**
- **Transaction T**: set of items, T is subset of I
- **Data Base**: set of transactions
- An association rule is an implication of the form : $X \rightarrow Y$, where **X, Y** are **disjoint** subsets of T
- **Problem**: Find association rules that have **support** and **confidence** greater than user-specified **minimum support** and **minimum confidence**

Association Rules

- **Confidence:** a rule $X \rightarrow Y$ holds in the database D with a confidence c if the $c\%$ of transactions in D that contain X also contain Y
- **Support:** a rule $X \rightarrow Y$ has a support s in D if $s\%$ of transactions contain XUY

Association Rules

- Association rules presentation (predicate presentation)

Multi-dimensional:

age(X, "20..29") ^ income(X, "20..29K") → buys(X, "PC") [support = 2%, confidence = 60%]

Single-dimensional:

– **buys(x, "computer") → buys(x, "software")** [1%, 75%]

Association rules presentation (non-predicate presentation)

Age = 20..29 ∧ income=20..29K → buys=PC (2%, 60%)

Buys=computer → buys=software (1%,75%)

Clustering- Chapter 7

- Database segmentation
- Given a set of objects (records) the algorithm obtains a division of the objects into clusters in which the distance of objects inside a cluster is minimal and the distance among objects of different clusters is maximal
- Unsupervised learning

Other Statistical Methods

chapter 6

- Regression
- Temporal Series
- Lazy learners
- Support Vector Machines

.....

Major Issues in Data Mining (1)

Book Slide

- Mining methodology and user interaction
 - Mining **different kinds of knowledge** in databases
 - Interactive **mining of knowledge at multiple levels** of abstraction
 - **Incorporation** of background knowledge
 - **Data mining query languages** and ad-hoc data mining
 - Expression and **visualization** of data mining results

Major Issues in Data Mining (2)

Book Slide

- Handling **noise** and **incomplete data**
- **Pattern evaluation**: the interestingness problem
 - Performance and scalability
 - Efficiency** and **scalability** of data mining algorithms
 - Parallel, distributed and incremental mining methods

Major Issues in Data Mining (3)

Book Slide

- Issues relating to the diversity of data types
 - Handling **relational** and **complex types** of data
 - Mining information from **heterogeneous databases** and **global** information systems (WWW)
- Issues related to applications and social impacts
 - **Application of discovered knowledge**
 - Domain-specific data mining tools
 - Intelligent query answering
 - Process control and decision making
 - **Integration** of the discovered knowledge with existing knowledge: A **knowledge fusion** problem
 - Protection of data **security, integrity, and privacy**

Summary

- **Data mining:** discovering comprehensible, interesting patterns from large amounts of data
- **A natural evolution of database technology, in great demand, with wide applications**
- **A KDD process**, or **DM process** includes data cleaning, data integration, data selection, transformation, data mining proper, pattern evaluation, and knowledge presentation
- **Mining can be performed in a variety of information repositories**

Summary

- **Data mining functionalities:**
characterization, discrimination,
association, classification, clustering,
outlier and trend analysis
- **Classification** of data mining systems
- Major issues in data mining

Preprocessing

Introduction to chapter 2

Preprocessing

- **Select, integrate,** and **clean** the data
- **Decide** which kind of **patterns** are needed
- **Decide** which **algorithm** is the best for your **goal**
It depends on many factors
- **Prepare data** for algorithms
- Different **algorithms** accept different **data format**

Implementaion Preparation

- **Identify** the **problem** to be solved.
- **Study** **problem** it in detail
- **Explore** the solution **space**
- **Find** one acceptable **solution** (feasible to implement)
- **Specify** the **solution**
- **Prepare** the data

Preparation

- **Remember** GIGO! (garbage in gabage out)
- Add some data, if necessary
- **Structure** the data in a proper form
- Be careful with **incomplete** and **noisy** data

Some implementation preparation rules to follow

- **Select** the **problem**
- **Specify** the **problem**
- **Study** the data
- The **problem** must guide the search for **tools** and **technologies**
- **Search** for the **simplest** model (algorithm, method)
- **Define** for each data the solution is **valid**, where it is **not valid** at all and where it is **valid** with some **constraints**

Studying the data

- The surrounding world consists of objects (**data**) and the **DM** problem is to find the **relationships** among **objects** (**data**)
- The objects (**data**) are characterized by the following properties:
attributes, values of attributes

The results (**rules, descriptions**) are **valid** (**true**) under certain circumstances (**data**) and in certain moments (**available data** at the moment)

Measures

- Type of data decides a way in which data are analyzed and preprocessed
 - **Names** (attributes)
 - **Categories**, classes, class attributes
 - Ordered values of **attributes**
 - Intervals of values of **attributes**
 - Types of values of **attributes**

Types of data

- Generally we distinguish:
 - Quantitative Data
 - Qualitative Data
- Bivaluated: often very useful
- Null Values are not applicable
- Missing data usually not acceptable
- NNetworks, and Bayes accept some missing data.

What to take into account

- **Eliminate** redundant records
- **Eliminate** out of range values of attributes
- **Decide a generalization level**
- **Consistency**

Other preprocessing tasks

- **Generalization** vs specification
- **Discretization**
- **Sampling**
- Reducing number of **attributes** at the preprocessing stage

Summary

- **The preprocessing** is required and is an **essential** part of the **DM process**
- If preprocessing is not performed patterns obtained could be of no use
- **Preprocessing** is a tedious task that could even take **more time** than **DM proper**

APPROACHES TO DATA MINING

Approaches

- **Mathematics:** Consist in the creation of mathematical models, algorithms, methods, to extract rules, regularities and patterns
- **Rough Sets** is the most precise model
- **Statistics:** They are focused in the creation of statistical models to analyse data
- Regression, Bayesian networks, NN, Clustering

Statistical methods

- Numerical data are needed
- Statistical methods are also often used in preprocessing steps to study the sample
- Hypothesis validation and regression analysis are used in data mining steps of the process

Decision trees

- Discovering **discriminant** rules
- **Descriptive Data Mining**
- Method: successive division of the set of data
- This is a **classification** algorithm
- Works better when **attributes**
have a small set of values

Apriori Algorithm

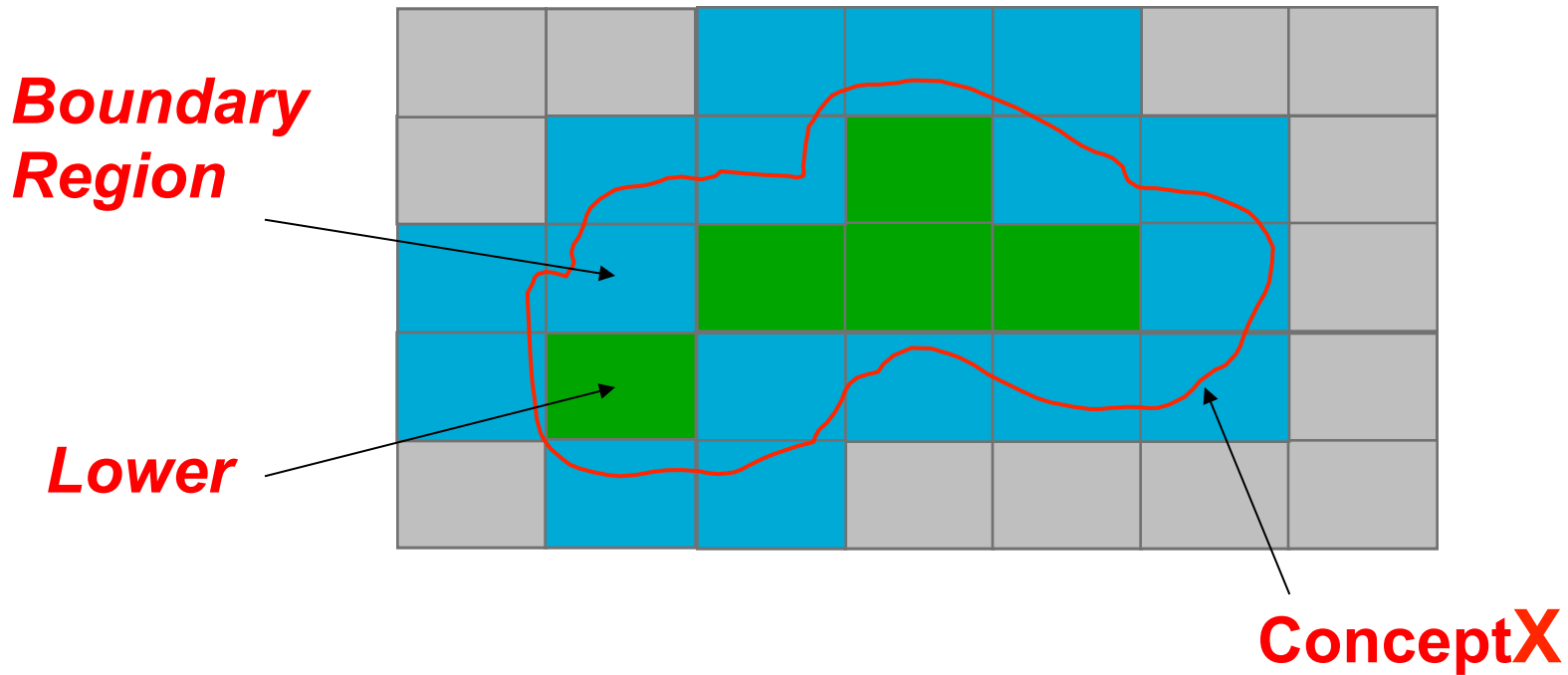
- Agrawal, Imielinski (IBM S. José. California)
- It is an intuitive and efficient algorithm to extract associations from transactions
- Also used as classification algorithm
- classification by association
- **Method:**
- Iterates until the associations obtained don't have the required support

Rough Sets

Descriptive Classification

- Approximation space $A=(U,IND(B))$:
 - *Lower Approximation* $\underline{X}_B = \{o \in U / [o] \subseteq X\}$
 - *Upper Approximation* $\overline{X}_B = \{o \in U / [o] \cap X \neq \emptyset\}$
 - *Boundary Region* $Bnd(X)_B = \overline{X}_B - \underline{X}_B$
 - *Positive Region*: $POS_B(D) = \bigcup \{\overline{X} : X \in IND(D)\}$

Rough Sets



$$\text{Boundary} + \text{Lower} = \text{Upper}$$

Variable Precision Rough Set Model



$$c(X, Y) = \begin{cases} 0 \\ 1 - \text{card}(X \cap Y) / \text{card}(X) \end{cases} \quad \text{if } \begin{cases} \text{card}(X) = 0 \\ \text{card}(X) > 0 \end{cases}$$

Rough Sets in SQL

```
Begin UPPER
  setdb(dbName);
  exec(conn, "BEGIN");

  "DECLARE classes CLASSES FOR
  SELECT C1, . . . . ., CN, D, COUNT (*) AS cnt
  FROM R
  GROUP BY C1, . . . . ., CN, D
  ORDER BY C1, . . . . ., CN, D, CNT desc");

  while not_end_records() do
    equ_class=exec("FETCH 1 IN cursor");
    first_decision_value=get_value(equ_class("D"));
    insert(equ_class, upper[first_decision_value]);
    while (equ_class == exec("FETCH 1 IN cursor")) do
      decision_value=get_value(equ_class("D"));
      insert(equ_class, upper[first_decision_value]);
    end while
  end while
End UPPER
```

Statistical Methods

- **Neural Network:** statistical CLASSIFICATION algorithm
- the **network is trained** to obtain classification patterns

- **Clustering:** form groups of objects without any previous hypothesis

Genetic Algorithms

- **Optimization method**
- They should be used when the **goal is to find an optimal solution** in solution space
- They often are used together with **neural networks**, or other methods to produce more understable (optimal) outputs
- They also are used to find the **optimal set of discriminant and/or characteristic rules** for a given database and a given class

Classification: requirements

- **Decision attribute**; called also **class attribute**, concept attribute
- **Condition attributes**: rest of the attributes or its subset
- Some require numerical data but there are algorithms to deal with **any kind of data**

Asociation: requirements

- **Transactional** data
- There is not needed to specify right and left side of the rules
- There are algorithms to tackle any kind of data
- **Minimum support**
- **Maximum** number of rules to be obtained

Clustering: requirements

- Set of attributes
- **Maximum** number of clusters
- **Number** of iterations
- **Mimimun** number of elements in any cluster