CSE581 DM - Discrete Mathematics Self STUDY Test

This Self Test is created to make you practice for **Midterm 1**.

Write careful answers many of which you can find in the Lectures and DM Definitions. Some of them were solved in class.

PART 1 YES/NO QUESTIONS

Circle proper answer. Write clear justification

1. Let $f : N \times N \longrightarrow Z$ be given by a formula $f(n, m) = n - m^2$. f is not $1-1$ function.
JUSTIFY:

y n

2. Let $A = N$, $B = \{2k : k \in N\}$. There is a function $f : A \longrightarrow^{1-1}_{onto} B$.
JUSTIFY:

y n

3. If $f : A \longrightarrow^{1-1} B$ and $g : B \longrightarrow^{1-1} A$, then $g \circ f$ and $f \circ g$ exists.
JUSTIFY:

y n

4. $\{(1, 2), (a, \emptyset)\}$ is a binary relation in a set $A = \{1, 2, a, \emptyset\}$.
JUSTIFY:

y n

5. $\{(\emptyset, \emptyset), (\{2\}, \{2\}), (3, 3)\}$ represents a transitive relation in a set $A = \{3, \{2\}, \emptyset\}$.
JUSTIFY:

y n

6. The function $f : N \longrightarrow \mathcal{P}(N)$ given by a formula: $f(n) = \{m \in N : m \leq n\}$ is an *onto* function.
JUSTIFY:

y n

7. Let $f : N \longrightarrow \mathcal{P}(N)$ be given by formula: $f(n) = \{m \in N : m < n^2\}$. We have that $\emptyset \in f(\{0, 1, 2, 3\})$.
JUSTIFY:

y n

8. $f : N \longrightarrow R$ is given by the formula: $f(n) = \frac{ln(n^3+1)}{\sqrt{n+6}}$ is a sequence.
JUSTIFY:

y n

9. Let $f : X \longrightarrow Y$, $B \subset Y$. For any $b \in B$, $f^{-1}(\{b\}) \neq \emptyset$.
JUSTIFY:

y n

10. Let $P(x)$ be a mathematical statement.
If $P(2)$ is true and for all $k \in N$, then implication "if $P(k)$ is true, then if $P(k + 1)$ is true" is a true statement, then $\forall n \in N\ P(n)$ is true.
JUSTIFY:

y n

11. $x \in \bigcap_{t \in T} A_t$ iff $\exists t \in T (x \in A_t)$.
JUSTIFY:

y n

12. $x \in f^{-1}[B]$ iff $f(x) \in B$, for $f : X \longrightarrow Y$ and any $B \subset Y$.
JUSTIFY:

y n

13. $2^{\{1,2\}} \cap \{1,2\} \neq \emptyset$
JUSTIFY:

y n

14. $\{\{a,b\}\} \in 2^{\{a,b,\{a,b\}\}}$
JUSTIFY:

y n

15. Let $A = \{\emptyset, \{a\}, \{\emptyset\}, 1, 2\}$, $B = \{\emptyset, \{\emptyset\}, \emptyset\}$. There is a function $f : A \longrightarrow^{1-1}_{onto} B$. Prove or disprove.
JUSTIFY:

y n

16. $(N \times Q) \cap (Q \times Q) = N \times Q$
JUSTIFY:

y n

17. Let $A_n = \{x \in R : 0 < x < n\}$. The family $\{A_n\}_{n \in N}$ form a partition of R. Prove or disprove.
JUSTIFY:

y n

18. There is an equivalence relation on Z with $\aleph_0$ equivalence classes. Define it or prove it does not exist.
JUSTIFY:

y n

19. There is an equivalence relation on $A = \{x \in R : 1 \leq x < 4\}$ with equivalence 3 equivalence classes: $E_1 = \{x \in R : 1 \leq x < 2\}$, $E_2 = \{x \in R : 2 \leq x < 3\}$, and $E_3 = \{x \in R : 3 \leq x < 4\}$. Define it or prove it does not exist.
JUSTIFY:

y n

20. Let $\approx$ be an equivalence on A. Each equivalence class of $\approx$ is non-empty.Prove or disprove.
JUSTIFY:

y n

21. Let $\approx$ be an equivalence on A. All equivalence classes of $\approx$ are disjoint. Prove or disprove.
JUSTIFY:

y n

22. Let $A = \{0,1,2,3\}$. We define for any $n, m \in A$, $n \approx m$ if and only if $(n)_2 = (m)_2$, where $(n)_2$ denotes a remainder of division n by 2. The relation $\approx$ is an equivalence on A.
JUSTIFY:

y n

item Let $A = \{0,1,2,3\}$.
$A/\approx = \{\{0,2\}, \{1,3\}\}$ where $n \approx m$ if and only if $(n)_2 = (m)_2$, where $(n)_2$ denotes a remainder of division n.
JUSTIFY:

y n

23. All equivalence classes of the relation defined below on Z are infinite.
$n \approx m$ iff $(n)_2 = (m)_2$, where $(n)_2$ denotes a remainder of division n by 2. Prove or disprove.
JUSTIFY:

y n

24. For any $\approx \subset A \times A$ The set $[a] = \{b \in A : a \approx b\}$ is an equivalence class with a representative a.
JUSTIFY:

25. Set A is countable if and only if $N \subseteq A$, where N is the set of natural numbers
JUSTIFY:

y n

26. The set 2^N is infinitely countable
JUSTIFY:

y n

27. There are C of all functions that map N into N
JUSTIFY:

y n

28. $\aleph_0^{\aleph_0} = C$ means that there are C possible sequences that can be form out of an infinitely countable set
JUSTIFY:

y n

29. There is an order relation that is also an equivalence relation and a function.

Justify: **y n**

30. Every non-empty finite poset $(A, \leq)$ (i.e. A is a finite set) has at least one maximal element.

Justify: **y n**

31. Every non-empty finite poset $(A, \leq)$ (i.e. A is a finite set) has at least one minimal element.

Justify: **y n**

32. There is an infinite poset $(A, \leq)$ (i.e. A is an infinite set) has exactly one minimal element and no smallest element.

Justify: **y n**

33. There is an infinite poset $(A, \leq)$ (i.e. A is an infinite set) has infinitely many maximal and minimal elements. You can draw diagram.

Justify: **y n**

34. Every infinite poset $(A, \leq)$ (i.e. A is an infinite set) has at least one minimal element.

Justify: **y n**

35. There i an infinite poset $(A, \leq)$ (i.e. A is an infinite set) has at lagest element.

Justify: **y n**

36. There ia an infinite poset $(A, \leq)$ (i.e. A is an infinite set) has smallest element. You can draw diagram.

Justify: **y n**

37. Each non empty lattice has 0 and 1

Justify: **y n**

38. There is a poset $(N, \leq)$ that has $\aleph_0$ max and and a smallest element. Draw diagram.

Justify: **y n**

39. There is a poset $(N, \leq)$ that has $\aleph_0$ mnimal elements and a SMALLEST element. Draw diagram.

Justify: **y n**

40. Any finite lattice is distributive. Prove or disprove.

Justify: **y n**

41. Any Boolean Algebra is distributive.

Justify: **y n**

PART 2: Yes/No DEFINITIONS QUESTIONS

Here are 10 definitions with **Yes/No** answers about their correctness.

Some of provided answers **are wrong**. Read carefully. **Circle** the **correct** answer.

1. "onto " function $f: A \overset{onto}{\longrightarrow} B$ iff $\forall b \in A \exists a \in B\ f(a) = b$ **n**

2. Generalized Intersection Given a sequence $\{A_n\}_{n \in N}$ of sets, $\bigcap_{n \in N} A_n = \{x : \exists n \in N\ x \in A_n\}$ **n**

3. Partition A family of sets $\mathbf{P} \subseteq \mathcal{P}(A)$ is called a partition of the set A if and only if

1. $\forall X \in \mathbf{P}\ (X \neq \emptyset)$, 2. $\forall X, Y \in \mathbf{P}\ (X \cup Y = \emptyset)$, 3. $\bigcup \mathbf{P} = A$ **n**

4. Maximal $a_0 \in A$ is a maximal element in the poset $(A, \leq)$ iff $\neg \forall a \in A\ (a_0 \leq a \cap a_0 \neq a)$ **n**

5. Smallest (least) $a_0 \in A$ is a smallest (least) element in the poset $(A, \leq)$ iff $\exists a \in A\ (a_0 \leq a)$ **y**

6. Lattice A poset $(A, \leq)$ is a lattice iff For all $a, b \in A$ $lub\{a, b\}$ or $glb\{a, b\}$ exist **y**

7. Lattice as an Algebra An algebra $(A, \cup, \cap)$, where $\cup, \cap$ are two argument operations on A is called a lattice iff the following conditions hold (they are called lattice axioms) For any $a, b, c \in A$

l1 $a \cup b = b \cup a$ and $a \cap b = b \cap a$

l2 $(a \cup b) \cup c = a \cup (b \cup c)$ and $(a \cap b) \cap c = a \cap (b \cap c)$

l3 $a \cap (a \cup b) = a$ and $a \cup (a \cap b) = a$

y

8. Boolean Algebra A distributive lattice with zero and unit such that each element has a complement is called a Boolean Algebra **n**

9. Cardinality Aleph zero We say that a set A has a cardinality $\aleph_0$ ($|A| = \aleph_0$) iff $|A| = |N|$ **n**

10. Power ($\mathcal{M}^{\mathcal{N}}$) $\mathcal{M}^{\mathcal{N}} = card\{f : f : A \longrightarrow B\}$, where A, B are such that $|A| = \mathcal{M}$, $|B| = \mathcal{N}$ **y**