CSE581
Computer Science Fundamentals: Theory

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LOGIC LECTURE 0

GENERAL INFORMATION

Course Book B1

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Logics for Computer Science: Classical and Non-Classical

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You can get the book in Hard cover, or in Electronic form
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https://www.springer.com/us/book/9783319925905

Book **B1**

I wrote **Book B1** with students on my mind so that they can read and learn by themselves, **before** coming to class
For sure, it is also essential to study after the class

We start our Part 1: LOGIC Lectures from **Chapter 2**
It is an intuitive **introduction** to the Classical Propositional and Predicate Logic appropriate for somebody **without** much of previous knowledge of Symbolic Logic

Chapter 3 provides a generalization of classical propositional part of **Chapter 2** Formal Propositional Languages and Extensional Semantics: Classical and Many Valued

Logic Main Tasks

First Task when one builds a symbolic logic, or foundations of mathematics, or foundations of computer science, is to define formally a proper **symbolic language**

We distinguish and define two kind of languages: **propositional** and **predicate**

They are also called also zero and first order languages, respectively

Logic Main Tasks

Second Task is to define formally what does it mean that **formulas** of a symbolic language are considered to be **true**, and **always true** i.e. we have to define a notion of a **tautology**

It means that we **define** what is called a **semantics** for a given **language**

The same languages can have different semantics

For example, the languages for classical and intuitionistic logics can be the same, but their the **semantics** are different

Logic Main Task

Third Tasks is to define a syntactical notion of a **proof** in a **proof system** based on a given **language**

It allows us to find out what can, or cannot be **proved** if certain axioms and rules of inference are assumed

This part of **syntax** is also called a **proof theory**

Logic Main Tasks

Fourth Task is to investigate the relationship between a **syntactical** notion of a **proof system** based on a given language and a **semantics** for that language

It means we establish **formal** relationship between the **syntax** and **semantics** for a given **language**

This **relationship** is established by providing answers to the following **two questions**

Logic Main Tasks

Fourth Task is to pose and answer the following questions

Q1: Is everything one **proves** in a given proof system **tautology** under a given semantics?

The positive answer to the question **Q1** is called **Soundness Theorem** for a given proof system and a given semantics proof system

Such proof system is called a **sound proof system**

Logic Soundness Theorem

We write the Soundness Theorem symbolically as follows

Soundness Theorem (with respect to a semantics **M**)
Let S be a proof system and A any formula of its language, then the following holds

$$\text{IF} \quad \vdash_S A \quad \text{THEN} \quad \models_{\mathbf{M}} A$$

Logic Main Questions

Q2: Is it also possible to guarantee a **provability** in a **sound** proof system of everything we know to be a **tautology** under a given semantics?

The positive answer to the question **Q2** is called **Completeness Theorem** for a proof system under a given semantics

Such proof system is called **complete** **proof system** with respect to the given semantics

Completenss Theorem

We write the Completeness Theorem symbolically as follows

CompletenessTheorem (with respect to a semantics **M**)
Let S be a proof system and A any formula of its language, then the following holds

$$\vdash_S A \quad \text{if and only if} \quad \models_{\mathbf{M}} A$$

Logic Main Tasks

Fifth Task is to develop proof systems in which a process of finding proofs can be carried fully automatically

These are **automated** theorem proving systems

The book presents various Gentzen Type **automated** theorem proving systems and discusses various methods of proving the **Completeness Theorem** for them

In our **Part 1: LOGIC** Lectures we cover, as an example, a theorem proving system RS and and give a constructive and very easy proof of the **Completeness Theorem**

Main Goals

The first set of **Main Goals** of the BOOK is to formally define and develop the above FIVE TASKS in case of **Classical** Propositional and Predicate Logic

The second set of **Main Goals** is to develop and discuss the FIVE TASKS for some **Non-Classical** Propositional Logics, namely for some extensional Many Valued logics, for the Intuitionistic logic, and Modal S4, S5 logics

Main Goals of the Book

The third set of **Main Goals** of the book is to formally define and develop the notion of a **formal theory** based on a given **proof system** for a first order **logic**

It discusses notions of a **model** of a theory, its semantical and syntactical **consistency** and **completeness**

The book presents some **Formal Theories** based on classical predicate logic

In particular it presents the Peano Arithmetic of Natural Numbers **PA** and discusses and proves the **Gödel Incompleteness Theorems**