CSE581 Intuitive Predicate Logic SELF TEST

VERY SHORT QUESTIONS

Some of them will appear on MIDTERM

Circle proper answer. Write clear justification

This Self Test is created to make you practice what you know and how well and quick you can justify it.

It also will help you to remember basic Laws of Quantifiers. All of them and their counter examples are in the book.

Study also restricted domain quantifiers laws and counter examples. There are only

few of them and I didn't include them here but you must know them too.

For known basic laws of quantifiers you write "basic tautology" or the name- like - "distributivity law ".

If it is not a tautology justification is a mathematical statement that is a counter example.

HAVE FUN!

SHORT QUESTIONS

Circle proper answer. Write justification

1. $(\exists x A(x) \Rightarrow \forall x A(x))$ is a predicate tautology.
   JUSTIFY:
   y n

2. For any predicates A(x), B(x),
   $\neg\forall x(A(x) \cap B(x)) \equiv (\exists x \neg A(x) \cup \exists x \neg B(x))$.
   JUSTIFY:
   y n

3. For any predicates A(x), B, (this means that B does not contain the variable x)
   $\neg\exists x(A(x) \cap B) \equiv \forall x \neg(A(x) \cap \neg B)$.
   JUSTIFY:
   y n

4. $(A(x) \Rightarrow A(x))$ is a predicate tautology.
   JUSTIFY:
   y n

5. $\forall x(A(x) \cap B(x)) \equiv (\forall x A(x) \cap \forall x B(x))$
   JUSTIFY:
   y n

6. $\exists x(A(x) \cup B(x)) \equiv (\exists x A(x) \cup \exists x B(x))$
   JUSTIFY:
   y n

7. $\forall x(x < 0) \Rightarrow 2 + 2 \neq 4$ is a true statement in a set of natural numbers.
   JUSTIFY:
   y n

8. $\forall x \in R(x^2 < 0) \Rightarrow \forall x \in R(x^2 \geq 0)$
JUSTIFY:

y n

9. $x + y > 0$, for $x, y \in N$ is a (mathematical) predicate with the domain N.
JUSTIFY:

y n

10. $\exists x(x < 1) \cup 2 + 2 = 4$ is a true statement in a set of natural numbers numbers.
JUSTIFY:

y n

11. $\forall x \in R(x^2 \geq 0) \Rightarrow \exists x \in R(x^2 \geq 0)$ is a true mathematical statement.
JUSTIFY:

y n

12. $\neg\exists n \exists x(x < \frac{1+n}{n+1}) \equiv \forall n \exists x(x \geq \frac{1+n}{n-1}))$
JUSTIFY:

y n

13. $\neg\exists n \exists x(x < \frac{1+n}{n+1}) \equiv \forall n \forall x(x \geq \frac{1+n}{n-1}))$
JUSTIFY:

y n

14. The formula $\forall x(C(x) \Rightarrow F(x))$ represents sentence: *All trees can fly*
in a domain $X \neq \emptyset$. JUSTIFY:

y n

15. The formula $\exists x(C(x) \cap B(x) \cap F(x))$ represents sentence: *Some blue flowers are yellow* in a domain $X \neq \emptyset$.
JUSTIFY:

y n

16. For any predicates A(x), B(x), the formula
$((\forall x A(x) \cup \forall x B(x)) \Rightarrow \forall x(A(x) \cup B(x)))$ is a predicate tautology.
JUSTIFY:

y n

17. $\exists x A(x) \Rightarrow \forall x A(x)$ is a predicate tautology.
JUSTIFY:

y n

18. $\neg\forall x(A(x) \cap B(x)) \equiv (\neg\forall x A(x) \cup \exists x \neg B(x))$.
JUSTIFY:

y n

19. $\neg\exists x(A(x) \cap B) \equiv \forall x \neg(A(x) \cup \neg B)$.
JUSTIFY:

y n

20. $(A(x) \Rightarrow A(x))$ is a predicate tautology.
JUSTIFY:

y n

21. $\forall x(A(x) \cap B(x)) \equiv (\forall x A(x) \cup \forall x B(x))$
JUSTIFY:

y n

22. $\exists x(A(x) \cup B(x)) \equiv (\exists x A(x) \cup \exists x B(x))$
JUSTIFY:

y n

23. $\forall x(x < 1) \cup 2 + 2 \neq 4$ is a true statement.
JUSTIFY:

y n

24. $x + y > 0$, for $x, y \in N$ is a (mathematical) predicate with the domain N.
JUSTIFY:

25. $\forall x \in R(x^2 < 0) \Rightarrow \exists x \in R(x^2 > 0)$ is a true mathematical statement.
JUSTIFY:

y n

26. $\neg\forall n \exists x(x < \frac{1+n}{n+1}) \equiv \exists n \forall x(x \geq \frac{1+n}{n-1}))$
JUSTIFY:

y n

27. $x + y > 0$, for $x, y \in N$ is a (mathematical) predicate with the domain N.
JUSTIFY:

y n

28. $(\exists x(A(x) \cup B(x))) \equiv (\exists x A(x) \cup \exists x B(x))$
JUSTIFY:

y n

29. $\forall x(x < 1) \cup 2 + 2 \neq 4$ is a true statement.
JUSTIFY:

y n

30. $\forall x \in R(x^2 < 0) \Rightarrow \exists x \in R(x^2 > 0)$ is a true mathematical statement.
JUSTIFY:

y n

31. The formula $\forall x(C(x) \cap F(x))$ represents sentence: *All birds can fly* in in the domain $X \neq \emptyset$.
JUSTIFY:

y n

32. For any propositional function A(x) the formula
$(\forall x A(x) \Rightarrow \exists x A(x))$ is a predicate tautology.
JUSTIFY:

y n

33. For any predicates A(x), B, (this means that B does not contain the variable x) the formula
$(\forall x(A(x) \Rightarrow B) \Rightarrow (\exists x A(x) \Rightarrow B))$ is a predicate tautology.
JUSTIFY:

y n

34. For any predicates A(x), B(x), the formula
$(\exists x((A(x) \cap B(x)) \Rightarrow (\exists x A(x) \cap \exists x B(x)))$ is a predicate tautology.
JUSTIFY:

y n

35. For any propositional functions A(x), B(x), the formula
$(\forall x(A(x) \cup B(x)) \Rightarrow (\forall xA(x) \cup \forall xB(x)))$ is a predicate tautology.
JUSTIFY:

y **n**

36. For any predicates A(x), B(x), the formula
$(\forall x(A(x) \Rightarrow B(x)) \Rightarrow (\forall xA(x) \Rightarrow \forall xB(x)))$ is a predicate tautology.
JUSTIFY:

y **n**

37. For any predicates A(x), B(x), the formula
$(\exists x(A(x) \Rightarrow B(x)) \Rightarrow (\forall xA(x) \Rightarrow \exists xB(x)))$ is a predicate tautology.
JUSTIFY:

y **n**

38. For any predicates A(x), B(x), the formula
$(\forall x((A(x) \cap B(x)) \Rightarrow (\exists xA(x) \cap \exists xB(x)))$ is a predicate tautology.
JUSTIFY:

y **n**

39. For any propositional function A(x) the formula
$(\forall xA(x) \Rightarrow \forall A(x))$ is a predicate tautology.
JUSTIFY:

y **n**

40. For any predicates A(x), B, (this means that B does not contain the variable x) the formula
$\forall x(A(x) \Rightarrow B) \Rightarrow (\exists xA(x) \Rightarrow B)$ is a predicate tautology.
JUSTIFY:

y **n**

41. For any predicates A(x), B(x), the formula
$(\exists x((A(x) \cap B(x)) \Rightarrow (\exists xA(x) \cap \exists xB(x)))$ is a predicate tautology.
JUSTIFY:

y **n**

42. For any predicates A(x), B(x), the formula
$\forall x(A(x) \cup B(x)) \Rightarrow (\forall xA(x) \cup \forall xB(x))$ is a predicate tautology.
JUSTIFY:

y **n**

43. For any predicates A(x), B(x), the formula
$(\forall x(A(x) \Rightarrow B(x)) \Rightarrow (\forall xA(x) \Rightarrow \forall xB(x)))$ is a predicate tautology.
JUSTIFY:

y **n**

44. For any predicates A(x), B(x), the formula
$((\exists xA(x) \cap \exists xB(x)) \Rightarrow \exists x(A(x) \cap B(x)))$ is a predicate tautology.
JUSTIFY:

y **n**