CSE581 Extra Q3 Solutions FALL 2024
(5pts extra credit)

YES/NO QUESTIONS

Circle proper answer and write A SHORT **justification**. No justification- no points.

1. $\{2, \{\emptyset\}, \emptyset\} \cap 2^{\emptyset} = \emptyset$
   Justify: $2^{\emptyset} = \{\emptyset\}$ and $\{2, \{\emptyset\}, \emptyset\} \cap \{\emptyset\} \neq \emptyset$ as $\emptyset \in \{2, \{\emptyset\}, \emptyset\}$ **n**

2. Set A is countable if and only if $|A| = |N|$, where N is the set of Natural numbers
   Justify: This is definition of infinitely countable set. Definition of a countable set is: a set A is countable iff is finite or infinitely countable. **n**

3. Let $A = \{a, \{\emptyset\}, \emptyset\}$, $B = \{\emptyset, \{\emptyset\}, \emptyset\}$. There is a function $f: A \longrightarrow^{1-1}_{onto} B$
   Justify: $|A| = 3$, $|B| = 2$ **n**

4. A is uncountable iff $|A| = C$
   Justify: the set $\mathcal{P}(R)$ of all subsets of Real numbers is uncountable and by Cantor theorem, $|\mathcal{P}(R)| > C$ **n**

5. There is an order relation that is also an equivalence relation and a function
   Justify: Equality on any set **y**

6. Let $f : N \times N \longrightarrow Z$ be given by a formula $f(n, m) = n - m^2$. f is not $1-1$ function.
   JUSTIFY: Take $(1, 0) \neq (2, 1)$. We get $f(1, 0) = 1 = f(2, 1)$. For example, we get $f(m^2, m) = m^2 - m^2 = 0$ for all $m \in N$. **y**

7. Let $A = N$, $B = \{2k : k \in N\}$. There is a function $f : A \longrightarrow^{1-1}_{onto} B$.
   JUSTIFY: Take $f(n) = 2n$ for $n \in N$ **y**

8. If $f: A \longrightarrow^{1-1} B$ and $g : B \longrightarrow^{1-1} A$, then $g \circ f$ and $f \circ g$ exists.
   JUSTIFY: In both cases g, f share the middle set. The $1-1$ property is irrelevant that th to the existence composition. **y**

9. $\{(1, 2), (a, \emptyset)\}$ is a binary relation in a set $A = \{1, 2, a, \emptyset\}$.
   JUSTIFY: $\{(1, 2), (a, \emptyset)\} \subseteq AxA$ **y**

10. $\{(\emptyset, \emptyset), (\{2\}, \{2\}), (3, 3)\}$ reprezents a transitive relation in a set $A = \{3, \{2\}, \emptyset\}$.
    JUSTIFY: Vacuously TRUE **y**

11. The function $f : N \longrightarrow \mathcal{P}(N)$ given by a formula: $f(n) = \{m \in N : m \leq n\}$ is an *onto* function.
    JUSTIFY: Observe that $f(0) = \{0\}$, $f(1) = \{0, 1\}$, ... etc. Can't get $\emptyset \in \mathcal{P}(N)$. Can't also get one element subsets of N except $f(0) = \{0\}$, and many others like a for example $\{3, 10, 5\}$, etc. **n**

12. Let $f : N \longrightarrow \mathcal{P}(N)$ be given by formula: $f(n) = \{m \in N : m < n^2\}$. We have that $\emptyset \in f(\{0, 1, 2, 3\}$.
    JUSTIFY: $f(0) = \emptyset$, so $\emptyset \in f(\{0, 1, 2, 3\}$. **y**

13. $f : N \longrightarrow R$ is given by the formula: $f(n) = \frac{ln(n^3+1)}{\sqrt{n+6}}$ is a sequence.
    JUSTIFY: By definition of a sequence as the domain of f is N. **y**

14. Let $f : X \longrightarrow Y$, $B \subset Y$. For any $b \in B$, $f^{-1}(\{b\}) \neq \emptyset$.
    JUSTIFY: Only if f is "onto" the set B **n**

15. Let $P(x)$ be a mathematical statement.
    If $P(2)$ is true and for all $k \in N$, $P(k)$ is true, then $\forall n \in N$ $P(n)$ is true.
    JUSTIFY: " all $k \in N$, $P(k)$ is true" is the same as "$\forall n \in N$ $P(n)$ is true". **y**

16. $x \in \bigcap_{t\in T} A_t$ iff $\exists t \in T(x \in A_t)$.
JUSTIFY: Definition of $\bigcap_{t\in T} A_t$ is $x \in \bigcap_{t\in T} A_t$ iff $\forall t \in T(x \in A_t)$ **n**

17. $x \in f^{-1}(B)$ iff $f(x) \in B$, for $f : X \longrightarrow Y$ and any $B \subset Y$.
JUSTIFY: Definitioin of the counter Image under a functioin f. **y**

18. $2^{\{1,2\}} \cap \{1, 2\} \neq \emptyset$
JUSTIFY: $2^{\{1,2\}} = \{\emptyset,\ \{1\},\ \{2\},\ \{1, 2\}\} \cap \{1, 2\} = \emptyset$. **n**

19. $\{\{a, b\}\} \in 2^{\{a,b,\{a,b\}\}}$
JUSTIFY: Observe $2^{\{a,b,\{a,b\}\}} = \{\emptyset,\ \{a\},\ \{b\},\ \{\{a, b\}\}\ \dots\}$ **y**

20. Let $A = \{\emptyset, \{a\}, \{\emptyset\}, 1, 2\}$, $B = \{\emptyset, \{\emptyset\}, \emptyset\}$. There is a function $f : A \longrightarrow^{1-1}_{onto} B$.
JUSTIFY: A has 5 elements, $B = \{\emptyset, \{\emptyset\}, \emptyset\} = B = \{\{\emptyset\}, \emptyset\}$ has 2 elements **n**

21. $(N \times Q) \cap (Q \times Q) = N \times Q$
JUSTIFY: $N \subseteq Q$ **y**

22. Let $A_n = \{x \in R : 0 < x < n\}$. The family $\{A_n\}_{n\in N}$ form a partition of R.
JUSTIFY: $\bigcup_{n\in N} A_n \neq R$ **n**

23. There is an equivalence relation on Z with 123 equivalence classes.
JUSTIFY: (example) Write any PARTITION **P** containing 123 partition sets!
By Partition Theorem (Theorem 2 DM Lecture 2) any PARTITION defines the Equivalence relation you need. It is defined in the Proof of the Theorem 2: Given a partition $\mathbf{P} \subseteq \mathcal{P}(A)$, we define a binary relation $R \subseteq A \times A$ as follows $R = \{(a, b) :\ a, b \in X \text{ for some } X \in \mathbf{P}\}$ **y**

24. There is an equivalence relation on $A = \{x \in R : 1 \leq x < 4\}$ with equivalence 3 equivalence classes: $E_1 = \{x \in R : 1 \leq x < 2\}$, $E_2 = \{x \in R : 2 \leq x < 3\}$, and $E_3 = \{x \in R : 3 \leq x < 4\}$.
JUSTIFY: Observe that t he set $\mathbf{P} = E_1,\ E_2,\ E_3\}$ forms as partition of $A = \{x \in R : 1 \leq x < 4\}$, so by the **Partition Theorem** (Theorem 2 DM Lecture 2) it defines the Equivalence Relation on $A = \{x \in R : 1 \leq x < 4\}$ with exactly $E_1,\ E_2,\ E_3$ as its equivalence classes **y**

25. Let $\approx$ be an equivalence on A. Each equivalence class of $\approx$ is non-empty.
JUSTIFY: True by the **Partition Theorem** (Theorem 1 DM Lecture 2) - **Proof :** All equivalence classes are nonempty, This holds as $a \in [a]$ for all $a \in A$ by reflexivity of equivalence relation $\approx$. **y**

26. All equivalence classes of the relation defined below on Z are infinite.
$n \approx m$ iff $(n)_2 = (m)_2$, where $(n)_2$ denotes a remainder of division n by 2
JUSTIFY: Take for example $0, 1 \in Z$ and evaluate by definition of equivalence class
$[0] = \{m \in Z :\ 0 \approx m\} = \{m \in Z :\ (0)_2 = (m)_2\}$
$= \{m \in Z :\ 0 = (m)_2\} = \{m \in Z :\ (m)_2\ = 0\}\{m \in Z :$ m is EVEN $\}$
$[1] = \{m \in Z :\ 1 \approx m\} = \{m \in Z :\ (1)_2 = (m)_2\} = \{m \in Z :\ 1 = (m)_2\}$
$= \{m \in Z :\ (m)_2\ = 1\} =$ m is ODD $\}$
I choose $0, 1 \in Z$ - as an example of an EVEN (zero is EVEN!) and odd numbers repeat this process for any two other odd and even numbers **y**

27. For any $\approx \subset A \times A$ The set $[a] = \{b \in A : a \approx b\}$ is an equivalence class with a representative a.
JUSTIFY: True only when $\approx \subset A \times A$ is an equivalence relation. **n**

28. Set A is countable if and only if $N \subseteq A$, where N is the set of natural numbers
JUSTIFY: Set $A = \{a, b\}$ is countable, but N is not a subset of A **n**

29. The set 2^N is infinitely countable
JUSTIFY: We stated in DM Lecture 3, Theorem 1 that that the set 2^N of all subsets of natural numbers is uncountable - The proof is given (as Cantor Theorem 2) in Lecture 4 **n**

30. There are C of all functions that map N into N
JUSTIFY: DM Lecture 3 **y**

31. Every non-empty finite poset $(A, \leq)$ (i.e. A is a finite set) has at least one maximal element
Justify: Proof by Mathematical Induction in DMLecture 2 **y**

32. Each non empty lattice has 0 and 1

Justify: $(Z, \leq)$ or $(N, \leq)$, where $\leq$ is natural order - or draw your own diagram **n**

33. $(N, \leq)$ has $\aleph_0$ MAX elements and no MIN elements

Justify: draw diagram **y**

34. Any finite lattice is distributive

Justify: draw a diagram of the example of 5 elements non-distributive lattice **n**