# CSE581 Extra Credit Q1 SOLUTIONS Fall 2024
# (6pts)

Please take your time and write **carefully** your solutions. There is no NO PARTIAL CREDIT. You get **0 pts** for a solution with a formula that is NOT a well formed formula of the given language.

**ONE PROBLEM (6pts)**

**PART 1 (2pts)** Write the natural language statement:

*From the fact that there is a bird that does not fly and* $4 + 4 = 4$*, we deduce the following: it is not possible*

*that all birds fly OR it is not necessary that* $4 + 4 = 4$.

in the following two ways.

**WAY 1 (0.5pts)** As a formula $A_1 \in \mathcal{F}_1$ of a language $\mathcal{L}_{\{\neg,\ \Box,\ \Diamond,\ \cap,\ \cup,\ \Rightarrow\}}$.

**SOLUTION** We ese Propositional Variables $a, b, c$ for consequtive statement where
$a$ denotes statement: *there is a bird that does not fly*
$b$ denotes statement: $4 + 4 = 4$ $c$ denotes statement: *all birds fly*

The formula $A_1 \in \mathcal{F}_1$ is:

$$((a \cap b) \Rightarrow (\neg\Diamond c \cup \neg\Box b))$$

**WAY 2 (1.5pts)** As a formula $A_2 \in \mathcal{F}_2$ of a PREDICATE LANGUAGE language $\mathcal{L}(\mathbf{P}, \mathbf{F}, \mathbf{V})$ with the set $\{\neg, \Box, \Diamond, \cap, \cup, \Rightarrow\}$ of propositional connectives.

**Use the following** Predicates, Functions and Constants:

$B(x)$ for x is a bird, $F(x)$ for x can fly, $E(x, y)$ for $x = y$, $f(x, y)$ for $+$, and $c$ for 4.

**SOLUTION**

**(0.5pts)** Atomic formulas are:

$$B(x), \quad F(x), \quad E(f(c, c), c))$$

**(0.5pts)** Restricted domain formula is

$$((\exists_{B(x)}\neg F(x) \cap E(f(c, c), c)) \Rightarrow (\neg\Diamond\forall_{B(x)}F(x) \cup \neg\Box E(f(c, c), c)))$$

**(0.5pts)** Formula $A_2 \in \mathcal{F}_2$ is:

$$((\exists x(B(x) \cap \neg F(x)) \cap E(f(c, c), c)) \Rightarrow (\neg\Diamond\forall x(B(x) \Rightarrow F(x)) \cup \neg\Box E(f(c, c), c)))$$

**PART 2 (1pts)** Given a formula **A :** $\forall x\exists y\, P(f(x, y), c)$ of the predicate language $\mathcal{L}$, and two **model structures**

$$\mathbf{M_1} = (Z, I_1), \quad \text{and} \quad \mathbf{M_2} = (N, I_2)$$

with the **interpretations** defined as follows.

$$P_{I_1} : =, \quad f_{I_1} : +, \quad c_{I_1} : 0 \quad \text{and} \quad P_{I_2} : >, \quad f_{I_2} : \cdot, \quad c_{I_2} : 0$$

Show that $\mathbf{M_1} \models A$

**Solution**

**$M_1$** $\models A$ because $A_{I_1} : \forall_{x \in Z} \exists_{y \in Z}\; x + y = 0$ is a **true** mathematical statement as we have that each $x \in Z$ exists $y = -x$ and $-x \in Z$ and $x - x = 0$

Show that **$M_2$** $\not\models A$

**Solution**

**$M_2$** $\not\models A$ because $A_{I_2} : \forall_{x \in N} \exists_{y \in N}\; x \cdot y > 0$ is a **false** statement for $x = 0$.

**PART 3 (2pts)**

**(1.0pts)** **Circle** formulas that **are propositional tautologies**

$$\mathcal{S}_1 = \{ ((\neg c \cap c) \Rightarrow (\neg b \Rightarrow (d \cap e))),\quad ((a \Rightarrow b) \cup \neg(a \Rightarrow b)),\quad ((a \cap \neg b) \Rightarrow ((a \cap \neg b) \Rightarrow (\neg d \cup e))),\quad (\neg a \Rightarrow (\neg a \cup b)) \}$$

**Solution**

$$\not\models ((a \cap \neg b) \Rightarrow ((a \cap \neg b) \Rightarrow (\neg d \cup e)))$$

all other formulas are tautologies

**(1.0pts)** **Circle** formulas that **are predicate tautologies**

$$\mathcal{S}_2 = \{ (\exists x\, A(x) \Rightarrow \neg \forall x \neg\, A(x)),\quad (\forall x\, (P(x, y) \cap Q(y)) \Rightarrow \neg \exists x\, \neg(P(x, y) \cap Q(y))),$$

$$((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x\, (A(x) \cap B(x))),\quad (\forall x (A(x) \Rightarrow B) \Rightarrow (\exists x A(x) \Rightarrow B))\ \}$$

**Solution** $\not\models ((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x\, (A(x) \cap B(x)))$, all other formulas are tautologies