cse547/ams547 PRACTICE MIDTERM 2
Spring 2010

5 extra points

NAME **ID:** **ams/cs**

ONE PROBLEM WILL BE CORRECTED for 5pts.

QUESTION 1 Prove that

1. the series
$$\Sigma_{n=1}^{\infty} \frac{n!}{n^n}$$
converges

2. but the inharmonic series
$$\Sigma_{n=1}^{\infty} \ (-1)^{n+1} \ \frac{1}{n}$$
conditionally converges.

Hint. Use the following Theorem

THEOREM 3 The alternating infinite sum $\Sigma_{n=1}^{\infty}(-1)^{n+1}a_n, (a_n \geq 0)$ such that
$$a_1 \geq a_2 \geq a_3 \geq .... \text{ and } \lim_{n\to\infty} a_n = 0$$
always CONVERGES.

Definition The series
$$\Sigma_{n=1}^{\infty} a_n$$
converges **conditionally** if and only if the series
$$\Sigma_{n=1}^{\infty} |a_n|$$
converges, but not absolutely.

1. SOLUTION:

2. SOLUTION

QUESTION 2 Prove that for any predicates $P(m), Q(k)$ such that sets $\{m \in Z : P(m)\}$, $\{k \in Z : P(k)\}$ are finite, the following equalities hold.

1. $\sum_k [P(m)] = |P(m)|$,

2. $\sum_{k,m} [P(m)][Q(k)] = \sum_k |P(m)|[Q(k)]$,

where $|P(m)|$ denotes the cardinality of $\{m \in Z : P(m)\}$.

QUESTION 3 Write a detailed solution explaining and justifying EACH step of the following fact.

FACT There are 172 integers n, such that $1 \leq n \leq 1000$ and $\lfloor \sqrt[3]{n} \rfloor | n$.

Reminder
We define: $m|n$ if and only if $m > 0 \cap \exists k \in Z(n = mk)$.

QUESTION 4 Write a detailed pro0f of

$$spec(\sqrt{2}) \cap spec(2+\sqrt{2}) = \emptyset$$

QUESTION 5 Prove or disprove

$$(x \bmod ny) \bmod y = x \bmod y$$

integer n

QUESTION 6 Prove or disprove:

1. $gcd(km, kn) = k \cdot gcd(m, n)$

2. $lcm(km, kn) = k \cdot lcm(m, n)$

Useful Properties

$$\lfloor x \rfloor = x \iff x \in Z, \qquad \lceil x \rceil = x \iff x \in Z$$

$$x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$$

$$\lfloor -x \rfloor = -\lceil x \rceil, \qquad \lceil -x \rceil = -\lfloor x \rfloor$$

$$\lceil x \rceil - \lfloor x \rfloor = 0 \; if \; x \in Z, \qquad \lceil x \rceil - \lfloor x \rfloor = 1 \; if \; x \notin Z$$

$$\lfloor x \rfloor = n \iff n \leq x < n + 1$$

$$\lfloor x \rfloor = n \iff x - 1 < n \leq x$$

$$\lceil x \rceil = n \iff n - 1 < x \leq n$$

$$\lceil x \rceil = n \iff x \leq n < x + 1$$

$$\lfloor x + n \rfloor = \lfloor x \rfloor + n$$

$$x < n \iff \lfloor x \rfloor < n$$

$$n < x \iff n < \lceil x \rceil$$

$$x \leq n \iff \lceil x \rceil \leq n$$

$$n \leq x \iff n \leq \lfloor x \rfloor$$

$$[\alpha \ldots \beta] \; contains \; \lfloor \beta \rfloor - \lceil \alpha \rceil + 1 \; integers, \; for \; \alpha \leq \beta$$

$$[\alpha \ldots \beta) \; contains \; \lceil \beta \rceil - \lceil \alpha \rceil \; integers, \; for \; \alpha \leq \beta$$

$$(\alpha \ldots \beta] \; contains \; \lfloor \beta \rfloor - \lfloor \alpha \rfloor \; integers, \; for \; \alpha \leq \beta$$

$$(\alpha \ldots \beta)\ contains\ \lceil \beta \rceil - \lfloor \alpha \rfloor - 1\ intergers,\ for\ \alpha < \beta$$

$$x = y \left\lfloor \frac{x}{y} \right\rfloor + x \bmod y$$

extra space

extra space

extra space