CSE547 HOMEWORK 4 (DISCRETE MATHEMATICS) SOLUTIONS

DEFINITIONS

Check the LIST OF DEFINITIONS (in Downloads) to verify the mistakes in case of NO answer.

PART 1: GENERAL DEFINITIONS

Power Set $\mathcal{P}(A) = \{X : \ A \subseteq X\}$.

n

Relative Complement $A - B = \{a : a \in A \ \cap \ a \ \notin B\}$.

y

(Cartesian) Product of two sets A and B.
$A \times B = \{(a,b) : \ a \in A \ \cap b \ \in B\}$.

y

Domain of R Let $R \subseteq A \times A$, we define domain of R: $D_R = \{a \in A : \ (a,b) \in R\}$.

n

ONTO function $f : \ A \stackrel{onto}{\longrightarrow} B$ iff $\forall b \in A \exists a \in B \ f(a) = b$.

n

Composition Let $f : \ A \longrightarrow B$ and $g : \ B \longrightarrow C$, we define a new function $h : \ A \longrightarrow C$, called a COMPOSITION of f and g, as follows: for any $x \in A$, $h(x) = g(f(x))$.

y

Inverse function Let $f : \ A \longrightarrow B$ and $g : \ B \longrightarrow A$.
g is called an INVERSE function to f iff $\forall a \in A \ ((f \circ g)(a) = g(f(a) = a)$.

y

Sequence of elements of a set A is any function $f : \ N \longrightarrow A$ or $f : \ N - \{0\} \longrightarrow A$.

y

Generalized Intersection of a sequence $\{A_n\}_{n \in N}$ of sets: $\bigcap_{n \in N} A_n = \{x : \ \exists n \in N \ x \in A_n\}$.

n

Equivalence relation $R \subseteq A \times A$ is an equivalence relation in A iff it is reflexive, antisymmetric and transitive.

n

Partition A family of sets $\mathbf{P} \subseteq \mathcal{P}(A)$ is called a partition of the set A iff the following conditions hold.

1. $\forall \ X \in \mathbf{P} \ (X \ = \emptyset)$

2. $\forall\ X, Y \in \mathbf{P}\ (X \cup Y = \emptyset)$

3. $\bigcup \mathbf{P} = A$

n

Partition and Equivalence For any partition $\mathbf{P} \subseteq \mathcal{P}(A)$ of A, there is an equivalence relation on A such that its equivalence classes are some sets of the partition $\mathbf{P}$.

n

Mathematical Induction Let $P(n)$ be any property (predicate) defined on a set N of all natural numbers such that:
Base Case $n = 2$ $P(2)$ is true.
Inductive Step The implication $P(n) \Rightarrow P(n+1)$ can be proved for any $n \in N$
THEN $\forall n \in N P(n)$ is a true statement.

n

PART 2: POSETS

Poset A set $A \neq \emptyset$ ordered by a relation R is called a poset. We write it as a tuple: (A, R), $(A, \leq)$, $(A, \preceq)$ or (A, $\boxed{\leq}$). Name poset stands for "partially ordered set".

y

Smallest (least) $a_0 \in A$ is a smallest (least) element in the poset $(A, \preceq)$ iff $\exists a \in A\ (a_0 \preceq a)$.

y

Greatest (largest) $a_0 \in A$ is a greatest (largest) element in the poset $(A, \preceq)$ iff $\forall a \in A\ (a \preceq a_0)$.

y

Maximal $a_0 \in A$ is a maximal element in the poset $(A, \preceq)$ iff $\neg\ \forall a \in A\ (a_0 \preceq a\ \cap\ a_0 \neq a)$.

n

Minimal $a_0 \in A$ is a minimal element in the poset $(A, \preceq)$ iff $\neg$ $exists a \in A\ (a \preceq a_0\ \cap\ a_0 \neq a)$.

y

Lower Bound Let $B \subseteq A$ and $(A, \preceq)$ is a poset. $a_0 \in A$ is a lower bound of a set B iff $\exists b \in B\ (a_0 \preceq b)$.

n

Upper Bound Let $B \subseteq A$ and $(A, \preceq)$ is a poset. $a_0 \in A$ is an upper bound of a set B iff $\forall b \in B\ (b \preceq a_0)$.

y

Least upper bound of B (lub B) Given: a set $B \subseteq A$ and $(A, \preceq)$ a poset.

An element $x_0 \in B$ is a least upper bound of B, $x_0 = lubB$ iff x_0 is (if exists) the least (smallest) element in the set of all upper bounds of B, ordered by the poset order $\preceq$.

n

Greatest lower bound of B (glb B) Given: a set $B \subseteq A$ of a poset $(A, \preceq)$.

An element $x_0 \in A$ is a greatest lower bound of B, $x_0 = glbB$ iff x_0 is (if exists) the greatest element in the set of all lower bounds of B, ordered by the poset order $\preceq$.

y

PART 3: LATTICES and BOOLEAN ALGEBRAS

Lattice A poset $(A, \preceq)$ is a lattice iff For all $a, b \in A$ $lub\{a, b\}$ or $glb\{a, b\}$ exist.

y

Lattice notation Observe that by definition elements $lubB$ and $glbB$ are always unique (if they exist). For $B = \{a, b\}$ we denote:
$lub\{a, b\} = a \cup b$ and $glb\{a, b\} = a \cap b$.

y

Lattice union (meet) The element $lub\{a, b\} = a \cap b$ is called a lattice union (meet) of a and b. By lattice definition for any $a, b \in A$ $a \cap b$ always exists.

n

Lattice intersection (joint) The element $glb\{a, b\} = a \cup b$ is called a lattice intersection (joint) of a and b. By lattice definition for any $a, b \in A$ $a \cup b$ always exists.

y

Lattice as an Algebra An algebra $(A, \cup, \cap)$, where $\cup, \cap$ are two argument operations on A is called a lattice iff the following conditions hold for any $a, b, c \in A$ (they are called lattice AXIOMS):

l1 $a \cup b = b \cup a$ and $a \cap b = b \cap a$

l2 $(a \cup b) \cup c = a \cup (b \cup c)$ and $(a \cap b) \cap c = a \cap (b \cap c)$

l3 $a \cap (a \cup b) = a$ and $a \cup (a \cap b) = a$.

y

Lattice axioms The conditions **l1- l3** from above definition are called lattice axioms.

y

Lattice orderings Let the $(A, \cup, \cap)$ be a lattice. The relations:

$a \preceq b$ iff $a \cup b = b$, $a \preceq b$ iff $a \cap b = a$
are order relations in A and are called a lattice orderings.

y

Distributive lattice Axioms A lattice $(A, \cup, \cap)$ is called a distributive lattice iff for all $a, b, c \in A$ the following conditions hold

l4 $a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$

l5 $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$.

Conditions l4- l5 from above are called a **distributive lattice axioms**.

y

Lattice special elements The greatest element in a lattice (if exists) is denoted by 1 and is called a lattice UNIT. The least (smallest) element in A (if exists)is denoted by 0 and called a lattice zero.

y

Lattice with unit and zero If 0 (lattice zero) and 1 (lattice unit) exist in a lattice, we will write the lattice as: $(A, \cup, \cap, 0, 1)$ and call is a lattice with zero and unit.

y

Lattice Unit Definition Let $(A, \cup, \cap)$ be a lattice. An element $x \in A$ is called a lattice unit iff for any $a \in A$ $x \cup a = a$ and $x \cap a = x$.

n

Lattice Unit Axioms If lattice unit x exists we denote it by 1 and we write the unit axioms as follows.

l6 $1 \cap a = a$

l7 $1 \cup a = 1.$

n

Lattice Zero Let $(A, \cup, \cap)$ be a lattice. An element $x \in A$ is called a lattice zero iff for any $a \in A$ $x \cup a = x$ and $x \cap a = a$.

n

Lattice Zero Axioms If lattice zero exists we denote it by 0 and write the zero axioms as follows.

l8 $0 \cup a = 0$

l9 $0 \cap a = a.$

n

Complement Definition Let $(A, \cup, \cap, 1, 0)$ be a lattice with unit and zero. An element $x \in A$ is called a complement of an element $a \in A$ iff $a \cap x = 1$ and $a \cup x = 0$. **n**

Complement axioms Let $(A, \cup, \cap, 1, 0)$ be a lattice with unit and zero. The complement of $a \in A$ is usually denoted by $-a$ and the above conditions that define the complement above are called complement axioms. The complement axioms are usually written as follows.

c1 $a \cup -a = 0$

c2 $a \cap -a = 1.$

n

Boolean Algebra A distributive lattice with zero and unit such that each element has a complement is called a Boolean Algebra.

y

Boolean Algebra Axioms A lattice $(A, \cup, \cap, 1, 0)$ is called a Boolean Algebra iff the operations $\cap, \cup$ satisfy axioms **l1 -l5**, $0 \in A$ and $1 \in A$ satisfy axioms **l6 - l9** and each element $a \in A$ has a complement $-a \in A$, i.e.

l1o $\forall a \in A \ \exists -a \in A \ ((a \cup -a = 1) \ \cap \ (a \cap -a = 0)).$

y

PART 4: CARDINALITIES OF SETS, Finite and Infinite Sets.

Cardinality definition Sets A and B have the same cardinality iff $\exists f(\ f : \ A \stackrel{1-1, onto}{\longrightarrow} B)$.

y

Cardinality notations $|A| = |B|$ or $cardA = cardB$, or $A \sim B$ all denote that the sets A and B have the same cardinality.

y

Finite We say: a set A is finite iff $\exists n \in N(|A| = n)$.

y

Infinite A set A is infinite iff A is NOT finite.

y

Cardinality Aleph zero We say that a set A has a cardinality $\aleph_0$ $(|A| = \aleph_0)$ iff $|A| = |N|$.

y

Countable A set A is countable iff $|A| = \aleph_0$.

n

Uncountable A set A is uncountable iff A is NOT countable.

y

Cardinality Continuum We say that a set A has a cardinality $\mathcal{C}$ ($|A| = \mathcal{C}$) iff $|A| = |R|$.

y

Cardinality $A \leq$ **Cardinality** B $|A| \leq |B|$ iff $A \sim C$ and $C \subseteq B$.

y

Cardinality $A <$ **Cardinality** B $|A| < |B|$ iff $|A| \leq |B|$ or $|A| \neq |B|$.

n

Cantor Theorem For any set A, $|A \leq \mathcal{P}(A)|$.

n

PART 5: ARITHMETIC OF CARDINAL NUMBERS

Sum ($\mathcal{N} + \mathcal{M}$) We define:
$\mathcal{N} + \mathcal{M} = |A \cup B|$, where A, B are such that $|A| = \mathcal{N}$, $|B| = \mathcal{M}$.

n

Multiplication ($\mathcal{N} \cdot \mathcal{M}$) We define:
$\mathcal{N} \cdot \mathcal{M} = |A \times B|$, where A, B are such that $|A| = \mathcal{N}$, $|B| = \mathcal{M}$.

y

Power ($\mathcal{M}^{\mathcal{N}}$) $\mathcal{M}^{\mathcal{N}} = card\{f : \; f : \; A \longrightarrow B\}$, where A, B are such that $|A| = \mathcal{M}$, $|B| = \mathcal{N}$.

y

Power $2^{\mathcal{N}}$ We define:

$$2^{\mathcal{N}} = card\{f : \; f : \; A \longrightarrow \{0, 1\} \; \}, \text{ where } |A| = \mathcal{N}.$$

y

PART 4: ARITHMETIC OF n, $\aleph_0$, $\mathcal{C}$

Union 1 $\aleph_0 + \aleph_0 = \aleph_0$.
Union of two countable sets is a countable set.

n

Union 2 $\aleph_0 + n = \aleph_0$.
Union of a finite (cardinality n) and a countable set is an infinitely countable set. **n**

Union 3 $\aleph_0 + \mathcal{C} = \mathcal{C}$.
Union of an infinitely countable set and an uncountable set is an uncountable set.

n

Cartesian Product 1 $\aleph_0 \cdot \aleph_0 = \aleph_0$.
Cartesian Product of two countable sets is a countable set.

n

Cartesian Product 2 $n \cdot \aleph_0 = \aleph_0$.
Cartesian Product of a finite set and an infinite set is an infinite set.

n

Cartesian Product 3 $\aleph_0 \cdot \mathcal{C} = \mathcal{C}$.

Cartesian Product of an infinitely countable set and a set of the same cardinality as Real numbers has the same cardinality as the set of Real numbers.

y

Cartesian Product 4 $\mathcal{C} \cdot \mathcal{C} = \mathcal{C}$.

Cartesian Product of two uncountable sets is an uncountable set.

n

Power 1 $2^{\aleph_0} = \mathcal{C}$.

y

Power 2 $\aleph_0^{\aleph_0} = \mathcal{C}$ means that

$card\{f: \quad f: N \longrightarrow N\} = \mathcal{C}$.

y

Power 3 $\mathcal{C}^{\mathcal{C}} = 2^{\mathcal{C}}$ means that there are $2^{\mathcal{C}}$ of all functions that map R into R.

y

Inequalities $n < \aleph_0 \leq \mathcal{C}$.

n

QUESTIONS

Circle proper answer. WRITE a short JUSTIFICATION. NO JUSTIFICATION, NO CREDIT.

Here are YES/NO answers with FEW JUSTIFICATIONS as examples

1. If $f: \quad A \longrightarrow^{1-1}_{onto} \; B$ and $g: B \longrightarrow^{1-1}_{onto} A$, then g is an inverse to f.

   JUSTIFY: The statement guarantee only that INVERSE function EXISTS. **n**

2. Let $f: \quad N \times N \longrightarrow \; N$ be given by a formula $f(n,m) = n + m^2$. f is a $1-1$ function.

   JUSTIFY: $f(1,2) = 5 = f(4,1)$ **n**

3. Let $A = \{a, \{\emptyset\}, \emptyset\}$, $B = \{\emptyset, \{\emptyset\}, \emptyset\}$. There is a function $f: \quad A \longrightarrow^{1-1}_{onto} \; B$.

   JUSTIFY: $|A| = 3, \; |B| = 2$ **n**

4. If $f: A \longrightarrow^{1-1} \; B$ and $g: B \longrightarrow^{onto} \; A$, then $f \circ g$ and $g \circ f$ are *onto*.

   JUSTIFY: $g \circ f$ no; take $|A| = 2, \; |B| = 3$ **n**

5. $f: R - \{0\} \longrightarrow^{1-1} \; R$ is given by a formula: $f(x) = \frac{1}{x}$ and $g: R - \{0\} \longrightarrow \; R - \{0\}$ given by $g(x) = \frac{1}{x}$.
   g is inverse to f.

   JUSTIFY: f is not "onto"; inverse does not exist. **n**

6. $\{(1,2),(a,1)\}$ is a binary relation in $\{1,2,3,\}$.

JUSTIFY: $a \notin \{1,2,3,\}$. **n**

7. The function $f:\ N \longrightarrow \mathcal{P}(N)$ given by formula: $f(n) = \{m \in N:\ \ m \leq n\}$ is a $1{-}1$ function.

JUSTIFY: $n_1 \neq n_2$ then obviously $f(n_1) \neq f(n_2)$ **y**

8. The function $f:\ N \times N \longrightarrow \mathcal{P}(N)$ given by formula: $f(n,m) = \{m \in N:\ \ m+n=1\}$ is is a sequence.

JUSTIFY: Domain of f is not N. **n**

9. The function $f:\ N \times N \longrightarrow \mathcal{P}(N)$ given by formula: $f(n,m) = \{m \in N:\ \ m+n=1\}$ is is 1-1.

JUSTIFY: $f(n,m) = \emptyset$ for all n,m such that $m+n \neq 1$. **n**

10. The $f:\ N \longrightarrow \mathcal{P}(N)$ given by formula: $f(n) = \{m \in N:\ \ m+n=1\}$ is a family of sets.

JUSTIFY: Values of f are sets. **y**

11. Let P be a predicate. If $P(0)$ is true and for all $k \leq n$, $P(k)$ is true implies $P(n+1)$ is true, then $\forall n \in N\ P(n)$ is true.

JUSTIFY: Principle of mathematical Induction. **y**

12. Let $A_n = \{x \in R:\ n \leq x \leq n+1\}$. Consider $\{A_n\}_{n\in N}$. $\bigcap_{n\in N} A_n = \emptyset$.

JUSTIFY: $A_n \cap A_{n+1} = n$ **n**

13. Let $A_n = \{x \in R:\ n+1 \leq x \leq n+2\}$. Consider $\{A_n\}_{n\in N}$. $\bigcup_{n\in N} A_n = R$.

JUSTIFY: $\bigcup_{n\in N}\{x \in R:\ n+1 \leq x \leq n+2\} = [1,\infty) \neq R$ **n**

14. $x \in \bigcup_{t\in T} A_t$ iff $\exists t \in T(x \in A_t)$

JUSTIFY: definition **y**

15. Let $A_n = \{x \in N : 0 < x < n\}$. The family $\{A_n\}_{n\in N}$ form a partition of N.

JUSTIFY: $A_0 = \{x \in N : 0 < x < 0\} = \emptyset$. **n**

16. Let $A_t = \{x \in \{1,2,3\} : x > t\}$ for $t \in \{0,1,2\}$. $\bigcap_{t\in T} A_t = \{1\}$.

JUSTIFY: $A_0 = \{1,2,3\},\ A_1 = \{2,3\}, A_2 = \{3\}$ and $\bigcap_{t\in T} A_t = \emptyset$. **n**

17. There is an equivalence relation on N with infinite number of equivalence classes.

JUSTIFY: Equality on N. **y**

18. There is an equivalence relation on $A = \{x \in R : 1 \leq x < 4\}$ with equivalence classes: $[1] = \{x \in R : 1 \leq x < 2\}$, $[2] = \{x \in R : 2 \leq x < 3\}$, and $[3] = \{x \in R : 3 \leq x < 4\}$.

JUSTIFY: {[1], [2], [3] } is a partition of A. **y**

19. Each element of a partition of a set A = { 1,2,3} is an equivalence class of a certain equivalence relation.

JUSTIFY: True for any set $A \neq$. **y**

20. Set of all equivalence classes of a given equivalence relation is a partition.

JUSTIFY: Partition Theorem. **y**

21. Let $R \subseteq A \times A$ The set $[a] = \{b \in A : (a, b) \in R\}$ is an equivalence class with a representative a.

JUSTIFY: ONLY when R is an equivalence relation. **n**

22. Let $A = \{a, b, c, d\}$. There are 4^3 words of length 3 in A^*.

JUSTIFY: Counting the functions theorem. **y**

23. If a set A has n elements ($n \in N$), then every subset of A is finite.

JUSTIFY: Any subset of a finite set is a finite set. **y**

24. Let $\sum$ be an alphabet $\sum = \{\%, \$, \&\}$. Denote $\sum^k = \{w \in \sum^* : lenghth(w) = k\}$. The set $\sum^3$ has 29 elements.

JUSTIFY: $3^3 = 27$ **n**

25. There is an order relation that is also an equivalence relation and a function.

JUSTIFY: Equality on any set. **y**

26. $R = \{(N, \{1, 2, 3, \}), (Z, \{1, 2, 3, \}), (1, N), (-1, N), (3, Z)\}$ is a function defined on a set $\{N, Z, 1, -1, 3\}$ with values in the set Z.

JUSTIFY: Elements of the range (values) of R are SUBSETS, not elements of Z. **n**

27. If $f : R \longrightarrow R$ and $g : R \longrightarrow^{1-1} R$, then $g \circ f$ and $f \circ g$ exists.

JUSTIFY: Corresponding domains and ranges agree. **y**

28. $\{(1, 2), (a, 1), (a, a)\}$ is a transitive binary relation defined in $A = \{1, 2, a\}$.

JUSTIFY: $(a, 2) \notin R$. **n**

29. $f : N \longrightarrow \mathcal{P}(R)$ is given by the formula: $f(n) = \{x \in R; \; x \leq \frac{-n^3+1}{\sqrt{n+3+6}}\}$ is a sequence.

JUSTIFY: Domain of f is N. **y**

30. There is an order relation R defined in $A \neq \emptyset$ such that (A, R) is a poset.

JUSTIFY: Definition of Poset. **y**

31. Let $A = \{\emptyset, N, \{1\}, \{a, b, 3\}\}$. There are no more then 50 words of length 4 in A^*.

JUSTIFY: $|A| = 4|$, $3^4 > 50$. **n**

32. There is an equivalence relation on Z with infinitely countably many equivalence classes.

JUSTIFY: Equality on Z. **y**

33. A is uncountable iff $|A| = |R|$ where R is the set of real numbers.

JUSTIFY: $A = \mathcal{P}(R)$ is uncountable and by Cantor theorem $|R| < |\mathcal{P}(R)|$. **n**

34. A is infinite iff some subsets of A are infinite.

JUSTIFY: All subsets of a finite set are finite. **y**

35. There exists an equivalence relation on N with $\aleph_0$ equivalence classes.

JUSTIFY: Equality; $[n] = \{n\}$. **y**

36. A is finite iff some subsets of A are finite.
JUSTIFY: all subsets are finite; $\{1\} \subseteq N$ and N is infinite. **n**

37. If A is a countable set, then any subset of A is countable.

JUSTIFY: Theorem **y**

38. If A is uncountable set, then any subset of A is uncountable.

JUSTIFY: $N \subseteq R$. **n**

39. $\{x \in Q : 1 \leq x \leq 2\}$ has the same cardinality as $\{x \in Q : 5 \leq x \leq 10\}$.

JUSTIFY: both sets are of cardinality $\aleph_0$. **y**

40. If A is infinite set and B is a finite set, then $((A \cup B) \cap A)$ is infinite set.

JUSTIFY: $((A \cup B) \cap A) = A$. **y**

41. The set of all squares centered in the origin has the same cardinality as R.

JUSTIFY: All such circles are uniquely defined by the radius r and $r \in R$. **y**

42. If A, B are infinitely countable sets, then $A \cap B$ is a countable set.

JUSTIFY: $A \cap B$ is finite or infinitely countable. **y**

43. A is uncountable iff there is a subset B of A such that $|B| = |A|$.

JUSTIFY: $N \subseteq Q$, $|N| = |Q|$ and Q is NOT uncountable. **n**

44. A is uncountable iff $|A| = C$.

JUSTIFY: $\mathcal{P}(R)$ is uncountable and $|\mathcal{P}(R)| \neq \mathcal{C}$.

n

45. $\aleph_0 + \aleph_0 = \aleph_0$ means that the union of two infinitely countable sets is an infinitely countable set.

JUSTIFY: The fact that the union of two infinitely countable sets is an infinitely countable set is true (theorem), but does not reflect the definition of sum of cardinal numbers; two DISJOINT infinitely countable sets. **n**

46. $|\mathcal{P}(N)| = \aleph_0$

JUSTIFY: $|\mathcal{P}(N)| = \mathcal{C}$.

n

47. $card(N \cap \{1, 3\}) = card(Q \cap \{1, 2\})$

JUSTIFY: both sets have 2 elements. **y**

48. A relation in N defined as follows: $n \approx m$ iff $n + m \in EVEN$ has $\aleph_0$ equivalence classes. in N.

JUSTIFY: two equivalence classes. **n**

49. $cardA < card\mathcal{P}(A)$

JUSTIFY: Cantor Theorem

y

50. A is infinite set iff there is $f : N \longrightarrow^{1-1}_{onto} A$.

JUSTIFY: this is definition of the infinitely countable set.

n

51. $\mathcal{P}(A) = \{B : B \subset A\}$

JUSTIFY: $B \subseteq A$

n

52. $|Q \cup N| = \aleph_0$

JUSTIFY: $Q \cup N = Q$.

y

53. $|R \times Q| = \mathcal{C}$

JUSTIFY: $\mathcal{C} \cdot \aleph_0 = \mathcal{C}$. **y**

54. $|N| \leq \aleph_0$

JUSTIFY: $|A| \leq |A|$.

y

55. Any non empty POSET has at least one MAX element.

JUSTIFY: $(N, \leq)$ has no max element for $\leq$ natural order. **n**

56. If $(A, \preceq)$ is a finite poset (i.e. A is a finite set), then a unique maximal is the largest element and a unique minimal is the least element.

JUSTIFY: Theorem **y**

57. There is a non empty POSET that has no Max element.

JUSTIFY: $(N, \leq)$ has no max element for ($\leq$ natural order. **y**

58. Any lattice is a POSET.

JUSTIFY: definition **y**

59. It is possible to order N in such a way that $(N, \leq)$ has $\aleph_0$ MAX elements and no MIN elements.

JUSTIFY: diagram (lecture) **y**

60. In any poset $(A, \preceq)$, the greatest and least elements are unique.

JUSTIFY: Theorem **y**

61. If a non empty poset is finite, then unique MAX element is the smallest.

JUSTIFY; in a finite poset unique MAX element is the greatest.

n

62. Each non empty lattice has 0 and 1.

JUSTIFY: $(Z, \leq)$ **n**

63. In any poset $(A, \preceq)$, if a greatest and a least elements exist, then they are unique.

JUSTIFY: Theorem **y**

64. Each distributive lattice has zero and unit elements.

JUSTIFY: diagram **n**

65. It is possible to to order the set of Natural numbers N in such a way that the poset $(N, \preceq)$ has a unique maximal element (minimal element) and no largest element (least element).

JUSTIFY: diagram **n**

66. It is possible to to order the set of rational numbers Q in such a way that the poset $(Q, \preceq)$ has a unique minimal element and no smallest (least) element.

JUSTIFY: diagram **n**

67. In any poset, the largest element is a unique maximal element and the least element is the unique minimal element.

JUSTIFY: Theorem **y**

68. If $(A, \cup, \cap)$ is an infinite lattice (i.e. the set A is infinite), then 1 or 0 might or might not exist.

JUSTIFY: always true **y**

69. There is a poset $(A, \preceq)$ and a set $B \subseteq A$ and that B has none upper bounds.

JUSTIFY: $(N, \leq)$, $B = N - \{0\}$. **y**

70. There is a poset $(A, \preceq)$ and a set $B \subseteq A$ and that B has infinite number of lower bounds.

JUSTIFY: $(N, \geq), B = \{0, 1\}$. **y**

71. If $(A, \cup, \cap)$ is a finite lattice (i.e. A is a finite set), then 1 and 0 always exist.

JUSTIFY: Theorem **y**

72. Any finite lattice is distributive.

JUSTIFY: example in the lecture of 5elemst non-distributive lattice **n**

73. Every Boolean algebra is a lattice.

JUSTIFY: definition **y**

74. Any infinite Boolean algebra has unit (greatest) and zero (smallest) elements.

JUSTIFY: by definition every Boolean algebra has unit (greatest) and zero (smallest) elements **y**

75. A non- generate Finite Boolean Algebras always have 2^n elements ($n \geq 1$.

JUSTIFY: Theorem **y**

76. Sets A and B have the same cardinality iff $\exists f(\ f: \quad A \stackrel{1-1}{\longrightarrow} B)$.

JUSTIFY: f must be also "onto. **n**

77. We say: a set A is finite iff $\exists n \in N(|A| = n)$.

JUSTIFY: definition **y**

78. A set A is infinite iff A is NOT finite.

JUSTIFY: definition **y**

79. $\aleph_0$ (Aleph zero) is a cardinality of only N (Natural numbers).

JUSTIFY: definition **y**

80. A set A is countable iff $|A| = \aleph_0$.

JUSTIFY: A set A is countable iff is FINITE or $|A| = \aleph_0$. **n**

81. $\mathcal{C}$ (Continuum) is a cardinality of Real Numbers, i.e. $\mathcal{C} = |\mathcal{R}|$.

JUSTIFY: definition **y**

82. For any set A, $|A| < |\mathcal{P}(A)|$.

JUSTIFY: Cantor Theorem **y**

83. $\mathcal{M}^{\mathcal{N}}$ is the cardinality of all functions that map a set A (of cardinality $\mathcal{N}$) into a set B (of cardinality $\mathcal{M}$).

JUSTIFY: definition **y**