CSE547: Discrete Mathematics

Chapter 2, Problem 11

Chapter 2, Problem 11 Question

The general rule (2.56) for summation by parts is equivalent to:

$$\sum_{0 \le k < n} (a_{k+1} - a_k) b_k$$
$$= a_n\, b_n - a_0\, b_0 - \sum_{0 \le k < n} a_{k+1}(b_{k+1} - b_k), for\ n \ge 0$$

Prove this formula directly by using the distributive, associative and commutative laws

Solution

$$\sum_{0 \le k < n} (a_{k+1} - a_k) b_k$$

$$= \sum_{0 \le k < n} (a_{k+1} b_k - a_k b_k) \qquad (Distributive\ Law)$$

$$= \sum_{k=0}^{n-1} a_{k+1} b_k - \sum_{k=0}^{n-1} a_k b_k \qquad (Associative\ Law)$$

We can write,

$$\sum_{k=0}^{n-1} a_k b_k = \sum_{k=0}^{n} a_k b_k - a_n b_n$$

$$= \sum_{k=0}^{n-1} a_{k+1} b_k - \sum_{k=0}^{n} a_k b_k + a_n b_n$$

$$= \sum_{k=0}^{n-1} a_{k+1} b_k - \sum_{k=1}^{n} a_k b_k + a_n b_n - a_0 b_0$$

$$= \sum_{k=0}^{n-1} a_{k+1} b_k - \sum_{k=0}^{n-1} a_{k+1} b_{k+1} + a_n b_n - a_0 b_0$$

$$= \sum_{k=0}^{n-1}(a_{k+1}b_k - a_{k+1}b_{k+1}) + a_n b_n - a_0 b_0$$

$$= \sum_{k=0}^{n-1} a_{k+1}(b_k - b_{k+1}) + a_n b_n - a_0 b_0$$

$$= a_n b_n - a_0 b_0 - \sum_{k=0}^{n-1} a_{k+1}(b_{k+1} - b_k)$$

So we have proved

$$\sum_{0 \le k < n} (a_{k+1} - a_k) b_k$$

$$= a_n\, b_n - a_0\, b_0 - \sum_{0 \le k < n} a_{k+1}(b_{k+1} - b_k), for\ n \ge 0$$