CSE547
Discrete Mathematics

Chapter 1, Problem 7

P4: Problem 7 on Page 17

Let $H(n) = J(n+1) – J(n)$. Equation (1.8) tells us that $H(2n) = 2$, and $H(2n + 1) = J(2n+2) – J(2n + 1) = (2J(n+1) – 1) – (2J(n) + 1) = 2H(n) – 2$, for all $n \geq 1$. Therefore it seems possible to prove that $H(n) = 2$ for all n, by induction on n. What's wrong here?

Detailed Solution

From the problem, we know the following:

(1) $H(n) = J(n+1) - J(n)$

(2) Equation (1.8) from the textbook page 10:

$$J(1) = 1$$

$$J(2n) = 2J(n) - 1, \quad \text{for } n \geq 1;$$

$$J(2n + 1) = 2J(n) + 1, \quad \text{for } n \geq 1.$$

But how do we get $H(2n)$ and $H(2n + 1)$?

Detailed Solution - continue

Let's check whether $H(2n) = 2$, for all $n \geq 1$ as the problem stated:

$$H(2n) = J(2n + 1) - J(2n) \qquad H(n) = J(n+1) - J(n)$$

$$= [2J(n) + 1] - [2J(n) - 1] \qquad \text{Eq. (1.8)}$$

$$= 2J(n) + 1 - 2J(n) + 1 \qquad \text{Take the [ ] out}$$

$$= 2 \qquad \text{YES !} \qquad \text{Algebra}$$

Eq. (1.8):

$$J(1) = 1$$

$$J(2n) = 2J(n) - 1, \qquad \text{for } n \geq 1;$$

$$J(2n + 1) = 2J(n) + 1, \qquad \text{for } n \geq 1.$$

Detailed Solution - continue

Let's check whether $H(2n + 1) = J(2n+2) – J(2n + 1) = (2J(n+1) – 1) – (2J(n) + 1) = 2H(n) – 2$, for all $n \geq 1$ as the problem stated:

$H(2n + 1)$	$= J(2n + 1 + 1) – J(2n + 1)$	$H(n) = J(n+1) – J(n)$
	$= J(2n + 2) – J(2n + 1)$	Algebra
	$= J(2(n + 1)) – J(2n + 1)$	Algebra
	$= [2J(n +1) – 1] – [2J(n) + 1]$	Eq. (1.8)
	$= 2J(n + 1) – 1 – 2J(n) – 1$	Take the [ ] out
	$= 2[J(n + 1) – J(n)] – 2$	Algebra
	$= 2H(n) – 2$ YES !	$H(n) = J(n+1) – J(n)$

Eq. (1.8):

$$J(1) = 1$$
$$J(2n) = 2J(n) – 1, \text{ for } n \geq 1;$$
$$J(2n + 1) = 2J(n) + 1, \text{ for } n \geq 1.$$

Detailed Solution - continue

Now, we proved the following is true.

$H(2n) = 2$, for all $n \geq 1$;

$H(2n + 1) = 2H(n) – 2$, for all $n \geq 1$.

What is missing?

The base case when $n = 1$.

What is $H(1) = ?$

Detailed Solution - continue

What is the base case H(1) = ?

$H(1) = J(1 + 1) – J(1)$	$H(n) = J(n+1) – J(n)$, when $n = 1$
$= J(2) – J(1)$	Algebra
$= [2J(1) – 1] – J(1)$	Eq. (1.8)
$= 2(1) – 1 – 1$	Eq. (1.8)
$= 0$	Algebra

Eq. (1.8):

$J(1) = 1$

$J(2n) = 2J(n) – 1$, for $n \geq 1$;

$J(2n + 1) = 2J(n) + 1$, for $n \geq 1$.

Conclusion

Review the problem: Let $H(n) = J(n+1) – J(n)$. Equation (1.8) tells us that $H(2n) = 2$, and $H(2n + 1) = J(2n+2) – J(2n + 1) = (2J(n+1) – 1) – (2J(n) + 1) = 2H(n) – 2$, for all $n \geq 1$. Therefore it seems possible to prove that $H(n) = 2$ for all n, by induction on n. What's wrong here?

Conclusion: We proved that the following are true:

$H(2n) = 2$, for all $n \geq 1$;

$H(2n + 1) = 2H(n) – 2$, for all $n \geq 1$.

However, the original problem does not have the base case $H(1) = 0$, which we solved.

And, $H(1) \neq 2$ for $n = 1$, which is a counter example for $H(n) = 2$ for all n. Therefore, $H(n) = 2$ for all n is not true.