

cse547/ams547 ONE QUESTION Quiz 1 Spring 2017
(25 points)

NAME

ID:

ams/cs

Given Recursive Formula **RF**:

$$\begin{aligned} f(i) &= \alpha_i, & i &= 1, \dots, d-1 \\ f(dn+j) &= cf(n) + \beta_j, & n \geq 1, 0 \leq j < d \end{aligned}$$

We proved the following closed formula **CF**:

$$f((b_m, b_{m-1}, \dots, b_1, b_0)_d) = (\alpha_{b_m}, \beta_{b_{m-1}}, \dots, \beta_{b_1}, \beta_{b_0})_c$$

where β_{b_j} are defined by

$$\beta_{b_j} = \begin{cases} \beta_0 & b_j = 0 \\ \beta_1 & b_j = 1 \end{cases} ; \quad j = 0, \dots, m-1,$$

QUESTION Part 1: 5pts

Given Recursive Formula RF:

$$f(1) = 34, \quad f(2) = 5,$$

$$f(3n) = 10f(n) + 76, \quad f(3n+1) = 10f(n) - 2, \quad f(3n+2) = 10f(n) + 8$$

Evaluate $f(19)$. Write proper formula- do not need to evaluate numbers!

Part 2: 20pts

Use the **repertoire method** to solve the general four-parameter recurrence **RF**:

$$f(1) = \alpha,$$

$$f(2n) = 3f(n) + \gamma n + \beta_0, \quad f(2n+1) = 3f(n) + \gamma n + \beta_1$$

Solve means to FIND a system of equations needed to evaluate the closed formula **CF**:

$$f(n) = \alpha A(n) + \gamma B(n) + \beta_0 C(n) + \beta_1 D(n).$$

To do so follow the steps below

Step 1. Use the proper form of the closed formula solution from **Part 1**

Step 2. $A(n) = 3^k$, $n = 2^k + \ell$, $0 \leq \ell < 2^k$, $n \in N - \{0\}$

Step 3. Use **repertoire function** $f(n) = 1$, for all $n \in N - \{0\}$

Step 4. Use **repertoire function** $f(n) = n$, for all $n \in N - \{0\}$

Step 5. write your system of equations. Do not solve it.

Write carefully your solution on pages provides. Indicate which STEP are you solving.

Solution

Step 1:

Solution Space

Solution Space