CSE548, Spring 2006

Homework 4

Due 5/2, Tuesday

For each problem, show your complete work, not just the final answer.


Question 3. Let \( X \) be the number of times we need to flip a coin until we get heads twice in a row? What are the expected value and variance of \( X \) in terms of \( p = \Pr(\text{head}) \)? Solve this problem using the linearity of the expected value. (The textbook has a solution by the method of probability generating functions.)

Question 4. Call a point \((x, y, z)\) in the three-dimensional plane a lattice point if \(x\), \(y\) and \(z\) are all integers. Determine the minimum value \(n\) such that any set \(S\) of \(n\) lattice points must have the following property: There exist two points in \(S\) such that the midpoint of the line segment connecting these two points is also a lattice point. [Hint: Use the pigeonhole principle. Also, if \(x + y\) is an even integer, then the midpoint between \(x\) and \(y\) is an integer.]

Question 5. A 3-coloring of a complete graph of \(n\) vertices is an assignment of one of the three colors Red, Blue or Yellow to each edge. Let \(R_3(i, j, k)\) be the minimum number \(n\) such that, for any 3-coloring of a complete graph \(G\) of \(n\) vertices, \(G\) must contain a Red complete subgraph of \(i\) vertices, or a Blue complete subgraph of \(j\) vertices, or a Yellow complete subgraph of \(k\) vertices.

(a) Prove that for \(i, j, k \geq 3\),
\[
R_3(i, j, k) \leq R_3(i - 1, j, k) + R_3(i, j - 1, k) + R_3(i, j, k - 1) - 1.
\]

(b) Apply part (a) above to find an upper bound for \(R_3(3, 3, 3)\).

(c) Use probabilistic method to find a lower bound for \(R(k, k, k)\).