CSE 541 - Logic in Computer Science - Spring 2009 Homework 9, Due on Thursday, April 23rd (in class)

Consider the statement that if R is a equivalence relation and two elements are not equivalent with respect to R, then their equivalence classes are disjoint.

1. First formalize the statement in predicate logic, as a sentence $\phi \rightarrow \psi$, where ψ is:

$$\forall x \forall y [\neg R(x, y) \to \forall z (R(x, z) \to \neg R(y, z))].$$

Give a sentence ϕ that expresses that R is a reflexive, symmetric, and transitive binary relation.

- 2. Next convert the negation $\neg(\phi \rightarrow \psi)$ to prenex form, and then apply skolemization to obtain a universal sentence α .
- 3. Show the set of clauses N that corresponds to α .
- 4. Finally, show that there is a finite (minimal) refutation tree T with respect to the ground instances of clauses in N; thereby proving that α is Herbrand unsatisfiable. For each failure node of T, specify an instance of a clause in N that is falsified at that node.