

CSE 541 - Logic in Computer Science
Sample Natural Deduction Proofs for First-Order Logic

Exercise 2.3.10(b). Proof of $\exists xP(x) \vdash \neg\forall x\neg P(x)$:

1	$\exists xP(x)$	premise
2	$x_0 \quad P(x_0)$	assumption
3	$\forall x\neg P(x)$	assumption
4	$\neg P(x_0)$	$\forall e \ 3$
5	$\perp$	$\neg e \ 2,4$
6	$\neg\forall x\neg P(x)$	$\neg i \ 3-5$
7	$\neg\forall x\neg P(x)$	$\exists e \ 1,2-6$

Exercise 2.3.12. Proof of $S \rightarrow \forall xQ(x) \vdash \forall x(S \rightarrow Q(x))$:

1	$S \rightarrow \forall xQ(x)$	premise
2	x_0	
3	S	assumption
4	$\forall xQ(x)$	$\rightarrow e \ 1, \ 3$
5	$Q(x_0)$	$\forall e \ 4$
6	$S \rightarrow Q(x_0)$	$\rightarrow i \ 3-5$
7	$\forall x(S \rightarrow Q(x))$	$\forall i \ 2-6$

Exercise 2.3.13(g). Proof of $\exists x\exists y(S(x, y) \vee S(y, x)) \vdash \exists x\exists yS(x, y)$:

1	$\exists x\exists y(S(x, y) \vee S(y, x))$	Premise
2	$x_0 \quad \exists y(S(x_0, y) \vee S(y, x_0))$	assumption
3	$y_0 \quad S(x_0, y_0) \vee S(y_0, x_0)$	assumption
4	$S(x_0, y_0)$	assumption
5	$\exists yS(x_0, y)$	$\exists i \ 4$
6	$\exists x\exists yS(x, y)$	$\exists i \ 5$
7	$S(y_0, x_0)$	assumption
8	$\exists yS(y_0, y)$	$\exists i \ 7$
9	$\exists x\exists yS(x, y)$	$\exists i \ 8$
10	$\exists x\exists yS(x, y)$	$\vee e \ 3,4-6,7-9$
11	$\exists x\exists yS(x, y)$	$\exists e \ 2,3-10$
12	$\exists x\exists yS(x, y)$	$\exists e \ 1,2-11$

Exercise 2.3.13(h). Proof of $\exists x(P(x) \wedge Q(x)), \forall x(P(x) \rightarrow R(x)) \vdash \exists x(R(x) \wedge Q(x))$:

1	$\exists x(P(x) \wedge Q(x))$	premise
2	$\forall x(P(x) \rightarrow R(x))$	premise
3	$x_0 \quad P(x_0) \wedge Q(x_0)$	assumption
4	$P(x_0)$	$\wedge e$ 3
5	$Q(x_0)$	$\wedge e$ 3
6	$P(x_0) \rightarrow R(x_0)$	$\forall e$ 2
7	$R(x_0)$	$\rightarrow e$ 4, 6
8	$R(x_0) \wedge Q(x_0)$	$\wedge i$ 7, 5
9	$\exists x(R(x) \wedge Q(x))$	$\exists i$ 8
10	$\exists x(R(x) \wedge Q(x))$	$\exists e$ 1,3-9