CSE541 Practice Midterm 1 Spring 2015
25 pts extra credit

NAME **ID:**

QUESTION 1

Write the following natural language statement:

One likes to play bridge, or from the fact that the weather is good we conclude the following: one does not like to play bridge or one likes not to play bridge

as a formula of 2 different languages

1. Formula $A_1 \in \mathcal{F}_1$ of a language $\mathcal{L}_{\{\neg,\ \mathbf{L},\ \cup,\ \Rightarrow\}}$, where **L** A represents statement "one likes A", "A is liked".

2. Formula $A_2 \in \mathcal{F}_2$ of a language $\mathcal{L}_{\{\neg,\ \cup,\ \Rightarrow\}}$.

Write carefully all steps of your translation.

QUESTION 2

Write the formal definition of the language $\mathcal{L}_{\{\neg,\ \mathbf{L},\ \cup,\ \Rightarrow\}}$ and give examples if is formulas of the degrees 0, 1, 2, 3, and 4.

QUESTION 3

Define formally your OWN 3 valued extensional semantics **M** for the language $\mathcal{L}_{\{\neg,\ \mathbf{L},\ \cup,\ \Rightarrow\}}$ under the following assumptions

1. Assume that the third value is **intermediate** between truth and falsity, i.e. the set of logical values is **ordered** and we have the following

Assumption 1 $F < \perp < T$

Assumption 2 T is the **designated value**

2. Model the situation in which one "likes" only truth; i.e. in which $\mathbf{L}T = T$ and $\mathbf{L} \perp = F$, $\mathbf{L}F = F$

3. The connectives $\neg$, $\cup$, $\Rightarrow$ can be defined as you wish, but you have to define them in such a way to make sure that

$$\models_{\mathbf{M}} (\mathbf{L}A \cup \neg \mathbf{L}A)$$

REMINDER

Formal definition of many valued extensional semantics follows the pattern of the classical case and consists of giving **definitions** of the following main components:

1. Logical Connectives

2. Truth Assignment

3. Satisfaction Relation, Model, Counter-Model

4. Tautology

QUESTION 3

1. Verify whether the formulas A_1 and A_2 from the **QUESTION 1** have a model/ counter model under your semantics **M**. You can use **shorthand notation**

2. Verify whether the following set **G** is **M**-consistent. You can use **shorthand notation**

$$\mathbf{G} = \{\ \mathbf{L}a,\ \ (a \cup \neg \mathbf{L}b),\ \ (a \Rightarrow b),\ b\ \}$$

3. Give an **example** on an infinite, **M**-consistent set of formulas of the language $\mathcal{L}_{\{\neg,\ \mathbf{L},\ \cup,\ \Rightarrow\}}$

QUESTION 4

Let S be the following **proof system**

$$S = (\ \mathcal{L}_{\{\neg,\mathbf{L},\cup,\Rightarrow\}},\ \mathcal{F},\ \ \{\mathbf{A1}, \mathbf{A2}\},\ \ \{r1,\ r2\}\ )$$

for the logical axioms and rules of inference defined for any formulas $A, B \in \mathcal{F}$ as follows

Logical Axioms

A1 $(\mathbf{L}A \cup \neg \mathbf{L}A)$

A2 $(A \Rightarrow \mathbf{L}A)$

Rules of inference:

$$(r1)\ \frac{A\ ;B}{(A \cup B)}, \qquad (r2)\ \frac{A}{\mathbf{L}(A \Rightarrow B)}$$

1. Write a proof in S with 2 applications of rule (r1) and one application of rule (r2)

You must write comments how each step pot the proof was obtained

2. Show, by constructing a formal proof that

$$\vdash_S ((\mathbf{L}b \cup \neg\mathbf{L}b) \cup \mathbf{L}((\mathbf{L}a \cup \neg\mathbf{L}a) \Rightarrow b)))$$

3. Verify whether the inference rules r1, r2 are **M**-sound. You can use **shorthand notation**

4. Verify whether the system S is **M**-sound. You can use **shorthand notation**

EXTRA QUESTION

If the system S **is not sound** under your semantics **M** then **re-define the connectives** in a way that such obtained new semantics **N** would make S sound.

You can use **shorthand notation**