Association Analysis
Chapter 5

cse634

Data Mining

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Association Rules Mining
An Introduction

- This is an intuitive (more or less) **introduction**
- It contains explanation of the main ideas:
- Frequent item sets, association rules, how we construct the association rules
- How we **judge** the goodness of the rules
- **Example** of an intuitive “run” of the **Appriori Algorithm** and association rules generation
- Discussion of the **relationship** between the Association and Correlation analysis

What Is Association Mining?

Association rule mining:

Finding frequent patterns called associations, among sets of items or objects in transaction databases, relational databases, and other information repositories

- **Applications:**
  - Basket data analysis, cross-marketing, catalog design, loss-leader analysis, clustering, classification, etc.

Association Rules

- Rule **general** form:

  "Body → Head [support, confidence]"

  Rule **Predicate** form:

  buys(x, "diapers") → buys(x, "beer")

  [0.5%, 60%]

  major(x, "CS") ^ takes(x, "DB") → grade(x, "A") [1%, 75%]

  Rule **Attribute** form:

  Diapers → beer [1%, 75%]

Association Analysis: Basic Concepts

- **Given: a** database of transactions, where each transaction is a list of items

- **Find:** all rules that associate the presence of one set of items with that of another set of items
- **Example**

  98% of people who purchase tires and auto accessories also get automotive services done

Association Model

- **I** ={i1, i2, ..., in} a set of **items**
- **J** = P(**I**) set of **all subsets** of the set of items, elements of **J** are called **itemsets**
- **Transaction T:** **T** is **subset** of set **I** of **items**
- **Data Base:** set of transactions
- **An association rule** is an implication of the form : **X-> Y,** where **X, Y** are disjoint subsets of items **I** (elements of **J**)
- **Problem:** Find rules that have support and confidence greater that user-specified minimum support and minimun confidence

Apriori Algorithm

- **Apriori Algorithm:**
- **First Step:** we find all **frequent** item-sets
- An item-set is **frequent** if it has a **support** greater or equal a fixed minimum support
- We fix minimum support usually low
- **Rules generation** from the frequent item-sets is a **separate problem** and we will cover it as a part of Association Process

Apriori Algorithm

- In order **to calculate** efficiently frequent item-sets:
- 1-item-sets (one element item-sets)
- 2-item-sets (two elements item-sets)
- 3-item-sets (three elements item-sets), etc..
- **we use** a **principle**, called an Apriori Principle (hence the name: Apriori Algorithm):
- **Apriori Principle**
- ANY **SUBSET** OF A **FREQUENT** ITEMSET **IS** A **FREQUENT** ITEMSET

The Apriori Algorithm (Han Book)

- **Pseudo-code**:

  C_k: Candidate itemset of size k

  L_k : frequent itemset of size k

  L_1 = {frequent items};

  for (k = 1; L_k !=$\emptyset$; k++) **do begin**

  C_{k+1} = candidates generated from L_k;

  for each transaction t in database do

  increment the count of all candidates in C_{k+1} that are contained in t

  L_{k+1} = candidates in C_{k+1} with min_support

  end

  return $\cup_k L_k$;

Appriori Process:
Rules Generation

- **Appriori Algorithm stops** after the **First Step**
- **Second Step** in the **Appriori Proces** (item-sets generation AND rules generation) is the **rules generation:**
- We calculate, from the frequent item-sets a set of the **strong rules**
- **Strong rules:** rules with at least minimum support (low) and minimum confidence (high)
- **Apriori Process** is then **finished** .

Apriori Process
Rules Generation

- The Apriori Process problem is:
- How do we form the association rules (A =>B) from the frequent item sets?

- **Remember:** A, B are disjoint subsets of the set I of items in general, and of the set 2- frequent, 3-frequent item sets ….. etc, … as generated by the Apriori Algorithm

How we find the rules?

- 1-frequent item set: {i1}- no rule
- 2-frequent item set {i1, i2}: there are two rules:
- {i1} => {i2} and {i2} => {i2}
- We write them also as
- i1 => i2 and i2 => i2
- We **decide** which rule we **accept** by calculating its support (greater= minimum support) and confidence (greater= minimum confidence)

How we find the rules?

- 3-frequent item set: {i1, i2, i3}
- **The rules**, by definition are of the form (A =>B) where A and B are **disjoint subsets** of {i1, i2, i3}, i.e.
- we have to find all subsets A,B of {i1, i2, i3} such that $A \cup B = \{i1, i2, i3\}$ and $A \cap B = \Phi$
- **For example**,
- let A= {i1, i2} and B= {i3}
- **The rule** is
- {i1, i2} => {i3}
- and we write it in a form:

  $i1 \cap i2 \Rightarrow i3$ or milk ∩ bread => vodka

if item i1 is milk, item i2 is bread and item i3 is vodka

How we find the rules?

- Another choice for A and B is, for example:
- A= {i1} and B= {12, i3}.
- **The rule** is
- {i1} => {i2, i3} and we write it in a form:

i1 => i2∩ i3 or milk => bread ∩ vodka

if item i1 is milk, item i2 is bread and item i3 is vodka

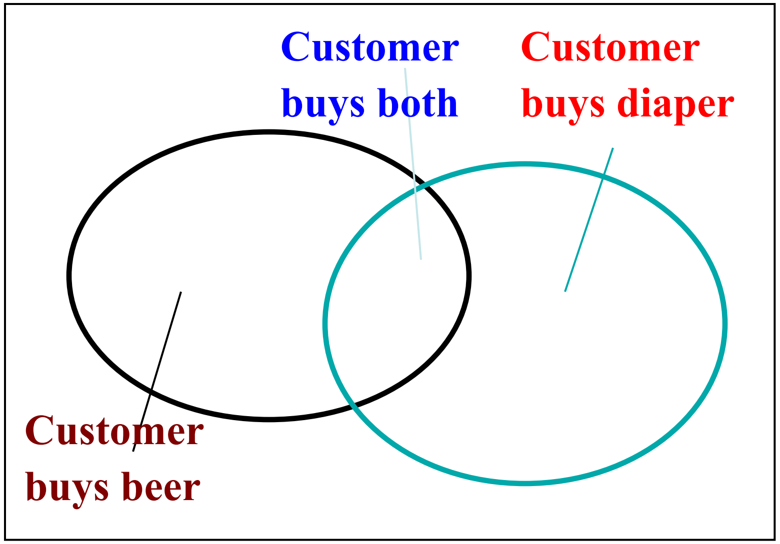
- REMEMBER:
- We have to cover all the choices for A and B!
- **Which rule we accept is being decided by calculating its support (greater = minimum support) and confidence (greater = minimum confidence)**

Rules Confidence and Support

- **Confidence:**
- the rule **X->Y** **holds** in the database **D** with confidence c if the c% of the transactions in **D** that contain **X** also contain **Y**

- **Support:** The rule **X->Y** has **support** s in **D** if s% of the transaction contain **XUY**

Support and Confidence



- **Find** all the rules $X \& Y \Rightarrow Z$ with minimum confidence and support
  - **Support *s:*** **probability** that a transaction contains {X, Y, Z}
  - **confidence *c:*** **conditional probability** that a transaction containing {X, Y} also contains *Z*

Support Definition

- **Support** of a rule A=>B in the database **D** of transactions is given by formula (where sc=support count)
- Support(A => B) = P(A U B) =

$$\frac{sc(A \cup B)}{\#D}$$

Frequent Item sets: sets of items with a support support >= MINIMAL support
We (user) **fix** MIN support usually **low** and
Min Confidence **high**

Confidence Definition

- **Confidence** of a rule A=>B in the database **D** of **transactions** is given by formula (where sc=support count)

- **Conf(A => B)** = P(B|A) = $\boxed{\dfrac{P(A \cup B)}{P(A)}}$

- $= \boxed{\dfrac{sc(AUB)}{\#D} \text{ divided by } \dfrac{scA}{\#D}} = \boxed{\dfrac{sc(AUB)}{scA}}$

Example

- Let consider a data base **D** ={ T1, T2, …. T9}, where
- T1={ 1,2,5} (we write k for item ik)
- T2= {2, 4}, T3={2, 3}, T4={1, 2, 4}, T5={1, 3}
- T6={2, 3}, T7={1, 3}, T8={1, 2, 3,5}, T9={1,2,3}
- **To find association rules** we follow the **following steps**
- **STEP 1**: **Count** occurrences of items in **D**
- **STEP2:** Fix Minimum support (usually **low**)
- **STEP 3:** Calculate frequent 1-item sets
- **STEP 4:** Calculate frequent 2-item sets
- **STEP 5:** Calculate frequent 3-item sets
- **STOP** when **there is no more** frequent item sets
- This is the **end** of **Apriori Algorithm** phase

Example

- **How to** generate **all** frequent 3-item sets (in **Step 5**)
- FIRST: use the frequent 2-item sets to **generate** all **3-item set candidates**
- SECOND: use **Apriori Principle** to **prune** the candidates set
- THIRD: **Evaluate** the count of the **pruned set**
- FOUR: **list** the frequent 3-item sets
- **STEP 6:** repeat the procedure for 4-item sets etc (if any)

Example- Apriori Pocess

- **Apriori Process Steps:**

- **STEP 7:** Fix the **minimum** confidence (usually **high**)

- **STEP 8:** Generate **strong rules** (support >min support and confidence> min confidence)

- **END** of rules generation phase
- **END** of the **Apriori Process**

Example- Apriori Pocess

- Lets now **calculate all steps** of our **Apriori Process** for a data base
- **D** ={ T1, T2, …. T9}, where
- T1={ 1,2,5} (we write k for item ik)
- T2= {2, 4}, T3={2, 3}, T4={1, 2, 4}, T5={1, 3}
- T6={2, 3}, T7={1, 3}, T8={1, 2, 3,5}, T9={1,2,3}
- **Here is our Step 1**
- We represent our **transactional** data base as **relational** data base (a table) and put the occurrences of items as an extra row, on the bottom

Example: Step 1

STEP 1: items occurrences=sc

its	1	2	3	4	5
T1	+	+	0	0	+
T2	0	+	0	+	0
T3	0	+	+	0	0
T4	+	+	0	+	0
T5	+	0	+	0	0
T6	0	+	+	0	0
T7	+	0	+	0	0
T8	+	+	+	0	+
T9	+	+	+	0	0
sc	**6**	**7**	**6**	**2**	**2**

Example; Step 2

- **STEP 2:** Fix minimal support count, for example
- msc = 2
- **Minimal support** = msc/#D= 2/9=**22%**
- ms=22%

- **Observe:** minimal support of an item set is **determined uniquely** by the minimal support count (msc) and we are going to use only **msc** to choose our frequent k-itemsets

Example: steps 3, 4

- **STEP 3:** calculate frequent 1-item sets: look at the sc count – we get that all 1-item sets are **frequent**

- **STEP 4:** calculate frequent 2-item sets
- **First** we calculate **2-item sets candidates from frequent 1-item sets.**

- As our all 1-item sets are frequent so all **subsets** of any 2-item set **are frequent** and we have to find **counts** of **all** 2-item sets

Observation

- If **for example** we set our **msc=6**, i.e we would have only {1}, {2} and {3} as **frequent item** sets

- Then by **Apriori Principle:**
- **"if A is a frequent item set, then each of its subsets is a frequent item set"**

- we would be examining **only those** 2-item sets that have {1}, {2}, {3} as subsets

- **Apriori Principle reduces the complexity of the algorithm**

Example: Step 4

- **STEP 4 :** All 2-item sets = all 2-element subsets of {1,2,3,4,5} are **candidates** and we evaluate their sc=support counts (in red). They are called **2-item set candidates**
- T1={ 1,2}, T2= {1, 3}, T3={1, 4}, T4={1, 5},
- T5={2, 3}, T6={2, 4}, T7={2, 5}, T8={3,4}, T9={3,5}, T10={4,5}

- {1,2,} **(4)**, {1,3} **(4)**, {1,4} **(1)**, {1,5} **(2)**,
- {2,3} **(4)**, {2,4} **(2)**, {2,5} **(2)** **,**
- {3,4} **(0)**, {3,5} **(1)**,
- {4,5} **(0)**

- **msc=2** and we choose **candidates** with **sc >= 2** and get the following
- **Frequent 2- item sets:**
- {1,2}, {1,3}, {1,5}, {2,3}, {2,4}, {2,5}

Example: Step 5

- **STEP 5 :** generate all frequent 3-item sets
- We use **frequent 2- item sets:**
- {1,2}, {1,3}, {1,5}, {2,3}, {2,4}, {2,5}
- and proceed as follows
- FIRST: we calculate from the **frequent** 2- item sets a set of all 3-item set **candidates:**
- {1,2,3}, {1,2,4}, {1,2,5}, {1,3,4}, {1,3,5}, {2,3,4}, {2,3,5}, {2,4,5}
- **Observe** that the **candidates**
- {1,3,4}, {1,3,5}, {2,3,4}, {2,3,5}, {2,4,5}
- **do not** follow **Apriori Principle:**
- "if A is a frequent item set, then each of its subsets is a frequent item set"

Example: Step 5

- **Frequent 2- item sets are:**
- {1,2}, {1,3}, {1,5}, {2,3}, {2,4}, {2,5}
- We reject {1,3,4} as its subset {3,4} **is not** a frequent 2- item set
  We reject {1,3,5} as its subset {3,5} **is not** a frequent 2- item set
  We reject {2,3, 4} as its subset {3,4} **is not** a frequent 2- item set
  We reject {2,3, 5} as its subset {3,5} **is not** a frequent 2- item set
  We reject {2, 4, 5} as its subset {4,5} **is not** a frequent 2- item set

This **rejection process** is called **pruning**

The following form of the Apriori Algorithm is called

- Prune Step: Any (k-1)-item set that is not frequent **cannot** be a subset of a frequent k-item set

Example: Step 5

- SECOND: we perform the **Prune Step** and write the **pruned** frequent 3-item set **candidates**
- {1,2,3}, {1,2,5}, {1,2, 4}
- THIRD: we calculate the sc=support count for the **pruned** frequent 3-item **candidates**
- {1,2,3} **(2)**, {1,2,5} **(2)**, {1,2, 4} **(1)**

- FOUR:
- **msc=2** and we choose the 3-item **candidates** with **sc >= 2** and get the following list of
- **Frequent 3-item sets**
- {1,2,3}, {1,2,5}

Example: Steps 6, 7

- **STEP 6:** there is no 4-item sets
- We **STOP** when **there is no more** frequent item sets

- This is the **end** of **Apriori Algorithm phase**

- **STEP 7:**
- We fix **minimum confidence** (usually high) as

- **min conf = 70%**

- We use the **confidence** to generate Apriori Rules

Example: Step 8
Association Rules Generation

- **Step 8:** Strong Rules Generation
- We will generate, as **an example** rules only from **one** frequent 2-item set: {1,2}
- Rule generation for other 2-item sets is **similar**
- **Reminder:** conf(A=>B) =

$$\frac{sc(A \cup B)}{scA}$$

- We split {1,2} into disjoint subsets A and B as follows: A={1} and B={2} or A={2} and B={1} and get two possible rules:
- {1}=>{2} or {2}=>{1}

Example: Association Rules Generation

- Conf(1=>2)= $\boxed{\frac{sc\{1,2\}}{sc\{1\}}}$ = 4/6 = 66%

The rule is **not accepted** (min conf= 70%)

- Conf(2=>1) = $\boxed{\frac{sc\{1,2\}}{sc\{2\}}}$ = 4/7= 57%

The rule is **not accepted**

Example: Step 8

- Now we use one frequent 3-item set
- {1,2,5} to show how to generate **strong rules**

- **First** we evaluate all possibilities how to **split** the set {1,2,5} into to disjoint subsets A,B to obtain **all possible** rules A=>B
- For each rule we **evaluate** its confidence and choose only those with conf ≥ 70% (our **minimal** confidence)

- The minimal support condition is **fulfilled** as we **deal only** with frequent items
- The rules such obtained are **strong rules**

Example: Association Rules Generation

- The rules for {1,2,5} are the following:
- **R1:** {1,2}=>{5}
- conf(R1)=sc{1,2,5}/sc{1,2}= 2/4 = ½ = 50%
- **R1 is rejected**
- **R2:** {1,5} => {2}
- conf(R2)=sc{1,2,5}/sc{1,5}= 2/2 = 100%
- **R2 is a strong rule (keep)**
- **R3:** {2,5} => {1}
- conf(R3)=sc{1,2,5}/sc{2,5}= 2/2 = 100%
- **R3 is a strong rule (keep)**
- **R4:** {1} => {2,5}
- conf(R4)=sc{1,2,5}/sc{1}= 2/6 = 33%
- **R4 is rejected**

Example: Association Rules Generation

- The next set of rules for {1,2,5} are the following:
- **R5:** {2}=>{1,5}
- conf(R5)=sc{1,2,5}/sc{2}= 2/7 = 27%
- **R5 is rejected**
- **R6:** {5} => {1,2}
- conf(R6)=sc{1,2,5}/sc{5}= 2/2 = 100%
- **R6 is a strong rule (keep)**
- As the last step we evaluate the **exact support** for the **strong rules**
- We **know** already that it is **greater** or **equal** to minimum support, as rules were obtained from the frequent item sets

Example: Association Rules Generation

- **Exact support** for the **strong rules** is:

- Sup({1,5}=>{2})=sc{1,2,5}/#D=2/9= 22%
- We write:
- $1 \cap 5 \Rightarrow 2$ [22%, 100%]
- Sup({2,5}=>{1}) =sc{1,2,5}/#D=2/9= 22%
- We write:
- $2 \cap 5 \Rightarrow 1$ [22%, 100%]
- Sup({5}=>{1,2}) =sc{1,2,5}/#D=2/9= 22%
- We write:
- $5 \Rightarrow 1 \cap 2$ [22%, 100%]

- **THE END of Apriori Process**

Association and Correlation

- As we can see the support-confidence framework can be misleading;
- it can identify a rule A=>B as interesting (strong) when, in fact the occurrence of A might not imply the occurrence of B
- **Correlation Analysis** provides an alternative framework for finding **interesting relationships**,
- or to **improve** understanding of meaning of some association rules (a lift of an association rule)

Correlation and Association

- **Definition:** Two item sets A and B are **independent** (the occurrence of A is independent of the occurrence of item set B) iff probability P fulfills the condition
- $P(A \cup B) = P(A) \cdot P(B)$
- Otherwise A and B are **dependent** or **correlated**
- The **measure of correlation**, or **correlation** between A and B is given by the formula:
- $\mathrm{Corr}(A,B) = \boxed{\dfrac{P(A \cup B)}{P(A)P(B)}}$

Correlation and Association

- **corr(A,B) >1** means that A and B are positively correlated i.e. the occurrence of one implies the occurrence of the other
- **corr(A,B) < 1** means that the occurrence of A is negatively correlated with B
- or **discourages** the occurrence of B
- **corr(A,B) =1** means that A and B are independent

Correlation and Association

- The correlation formula can be re-written as
- Corr(A,B) = $\boxed{\dfrac{P(B|A)}{P(B)}}$
- Supp(A=>B)= P(AUB)
- Conf(A=>B)= P(B|A), i.e.
- **Conf(A=>B)= corr(A,B) P(B)**
- So correlation, support and confidence are all different, but the correlation provides an **extra information** about the **association rule** (A=>B)
- We say that the correlation corr(A,B) provides the **LIFT** of the **association rule** (A=>B), i.e.
- A is said to **increase** or to **LIFT** the likelihood of B by the factor of the value returned by the formula for corr(A,B)

Correlation Rule (HAN Book)

- **A correlation rule** is a set of items
- {i1, i2, ….in}, where the items occurrences are **correlated**
- The correlation value is given by the **correlation formula** and we use X square test to determine if correlation is statistically significant
- The X square test can also determine the negative correlation
- We can also form minimal correlated item sets, etc…
- Limitations: X square test is less accurate on the data tables that are sparse and can be misleading for the contingency tables larger then 2x2

Criticism to Support and Confidence
(Han book)

- **Example 1:** (Aggarwal & Yu, PODS98)
  - Among 5000 students
    - 3000 play basketball
    - 3750 eat cereal
    - 2000 both play basket ball and eat cereal

RULE: play basketball ⇒ eat cereal [40%, 66.7%] is misleading because the overall percentage of students eating cereal is 75% which is higher than 66.7%.

RULE: play basketball ⇒ not eat cereal [20%, 33.3%] is far more accurate, although with lower support and confidence

	basketball	not basketball	sum(row)
cereal	2000	1750	3750
not cereal	1000	250	1250
sum(col.)	3000	2000	5000

EXTRTA Slides

- ADDITIONAL MATERIAL
- Read, explore; much of it I already covered in our slides

Mining Association Rules in Large Databases

Slightly modified HAN Book slides follow from now

Mining Association Rules in Large Databases

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
- Mining multilevel association rules from transactional databases
- Mining multidimensional association rules from transactional databases and data warehouse
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

Association Rule Mining: A Road Map

- **Boolean (Qualitative) vs. quantitative associations** (Based on the types of values handled)

  buys(x, “SQLServer”) ^ income(x, “DMBook”) => buys(x, “DBMiner”) [0.2%, 60%] (**Boolean/Qualitative**)

  age(x, “30..39”) ^ income(x, “42..48K”) => buys(x, “PC”) [1%, 75%] **(quantitative)**

- Single dimension (one predicate) vs. multiple dimensional associations (multiple predicates)

Association Rule Road Map (c.d)

- **Single level vs. multiple-level analysis**
  - What brands of beers are associated with what brands of diapers – **single level**
  - **Various extensions**

1. Correlation analysis (just discussed)
2. **Association** does not necessarily imply **correlation** or causality
3. Constraints enforced

   Example:

   smallsales (sum < 100) implies bigbuys (sum >1,000)?

Chapter 5: Mining Association Rules

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
- Mining multilevel association rules from transactional databases
- Mining multidimensional association rules from transactional databases and data warehouse
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

An Example

Transaction ID	Items Bought
2000	A,B,C
1000	A,C
4000	A,D
5000	B,E,F

Min. support 50%
Min. confidence 50%

Frequent Itemset	Support
{A}	75%
{B}	50%
{C}	50%
{A,C}	50%

For rule ***A*** $\Rightarrow$ ***C***:

support = support({A, C}) = 50%

confidence = sc({A, C})/sc({A}) = 66.6%

The Apriori principle:

Any subset of a frequent itemset must be frequent

Mining Frequent Itemsets: the Key Step

- **Find the frequent item sets:** the sets of items that have minimum support
  - A subset of a frequent item set must also be a frequent item set
    - **i.e., if {*A, B*} is a frequent item set, both {*A*} and {*B*} should be a frequent item set**
  - Iteratively find frequent item sets with cardinality from 1 to k (k-item set)
- **Use the frequent item sets to generate association rules.**

Apriori Algorithm — Book Example of frequents items sets generation

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

Scan D

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

L_2

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

C_3

itemset
{2 3 5}

Scan D

L_3

itemset	sup
{2 3 5}	2

Generating Candidates: C_k

- Join Step: C_k is generated by **joining** L_{k-} with itself
- Prune Step: Any (k-1)-item set that is not frequent **cannot** be a subset of a frequent k-item set

Example of Generating Candidates

- L_3={*abc, abd, acd, ace, bcd*}
- We write abc for {a,b,c}, etc…
- **Self-joining**: $L_3 * L_3$
  - *abcd* from *abc* and *abd*
  - *acde* from *acd* and *ace*
- **Pruning**:
  - *acde* is removed because *ade* is **not frequent**: is not in $\boldsymbol{L_3}$
- C_4={*abcd*}

Appriori Performance Bottlenecks

- The **core** of the Apriori algorithm:
  - Use frequent (k – 1)-item sets to generate **candidate** frequent k-item sets
  - Use database scan and pattern matching to collect counts for the **candidate** item sets
- The **bottleneck** of Apriori: **candidate generation**
  - Huge candidate sets:
    - 10^4 frequent 1-itemset will generate 10^7 **candidate** 2-itemsets
    - To discover a frequent pattern of size 100, e.g.,
    - $\{a_1, a_2, \ldots, a_{100}\}$, one needs to generate $2^{100} \approx 10^{30}$ **candidates**
  - Multiple scans of database:
    - Needs (n +1) scans, n is the length of the longest pattern

How to Count Supports of Candidates?

- **Why counting supports of candidates is a problem?**
  - The **total number** of candidates can be **very huge**
  - **One transaction** may contain **many candidates**
- **Method:**
  - **Candidate itemsets** are stored in a **hash-tree**
  - *Leaf* node of hash-tree contains a list of itemsets and counts
  - *Interior* node contains a hash table
  - *Subset function*: finds all the candidates contained in a transaction

Methods to Improve Apriori’s Efficiency

- Hash-based itemset counting: A *k*-itemset whose corresponding **hashing bucket** count is below the threshold cannot be frequent
- Transaction reduction: A **transaction** that does not contain any frequent k-itemset is **useless** in subsequent scans
- Partitioning: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
- Sampling: mining on a subset of given data, lower support threshold + a method to determine the completeness
- Dynamic itemset counting: add new candidate itemsets only when all of their subsets are estimated to be frequent

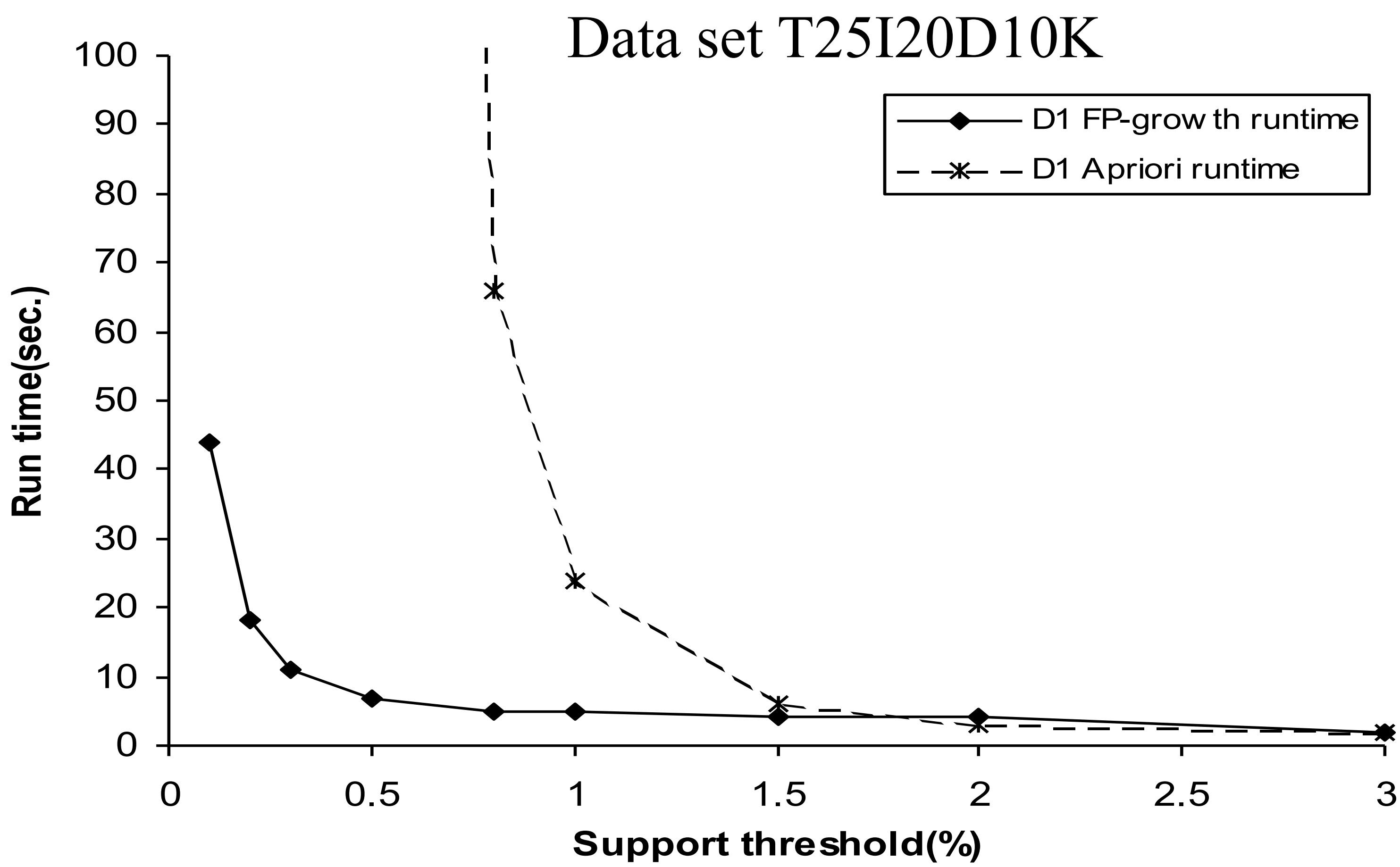
An Alternative: Mining Frequent Patterns Without Candidate Generation

- **Compress** a large database into a compact,
- **Frequent-Pattern tree (FP-tree)** structure
  - highly condensed, but complete for frequent pattern mining
  - avoid costly database scans
- **Develop** an efficient, FP-tree-based frequent pattern mining method
  - A **divide-and-conquer** methodology: decompose mining tasks into smaller ones
  - **Avoid candidate generation**: sub-database test only!

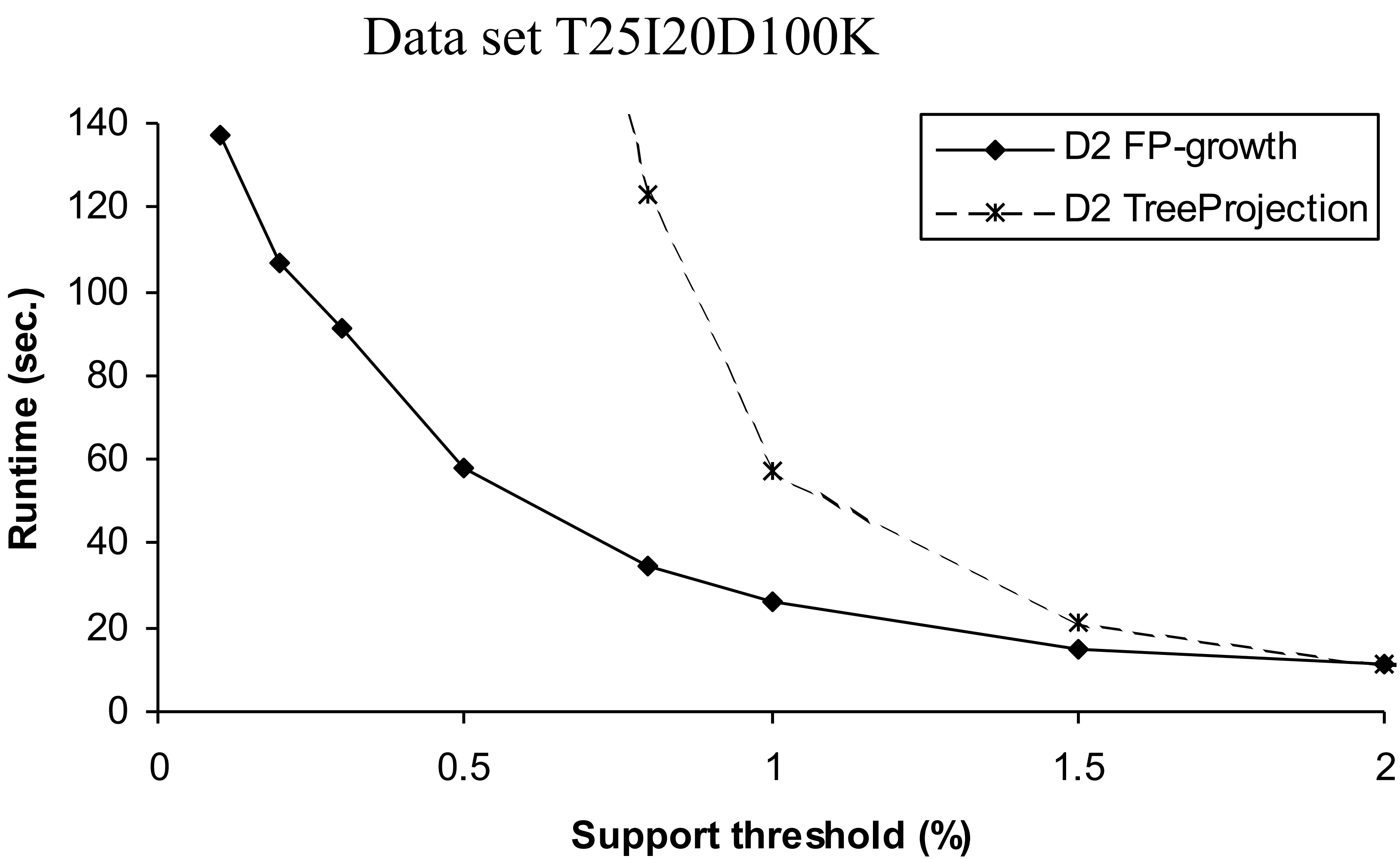
Why Is Frequent Pattern Growth Fast?

- **Performance study shows**
  - FP-growth is an order of magnitude faster than Apriori, and is also faster than tree-projection
- **Reasoning**
  - No candidate generation, no candidate test
  - Use compact data structure
  - Eliminate repeated database scan
  - Basic operation is counting and FP-tree building

FP-growth vs. Apriori: Scalability With the Support Threshold



FP-growth vs. Tree-Projection: Scalability with Support Threshold



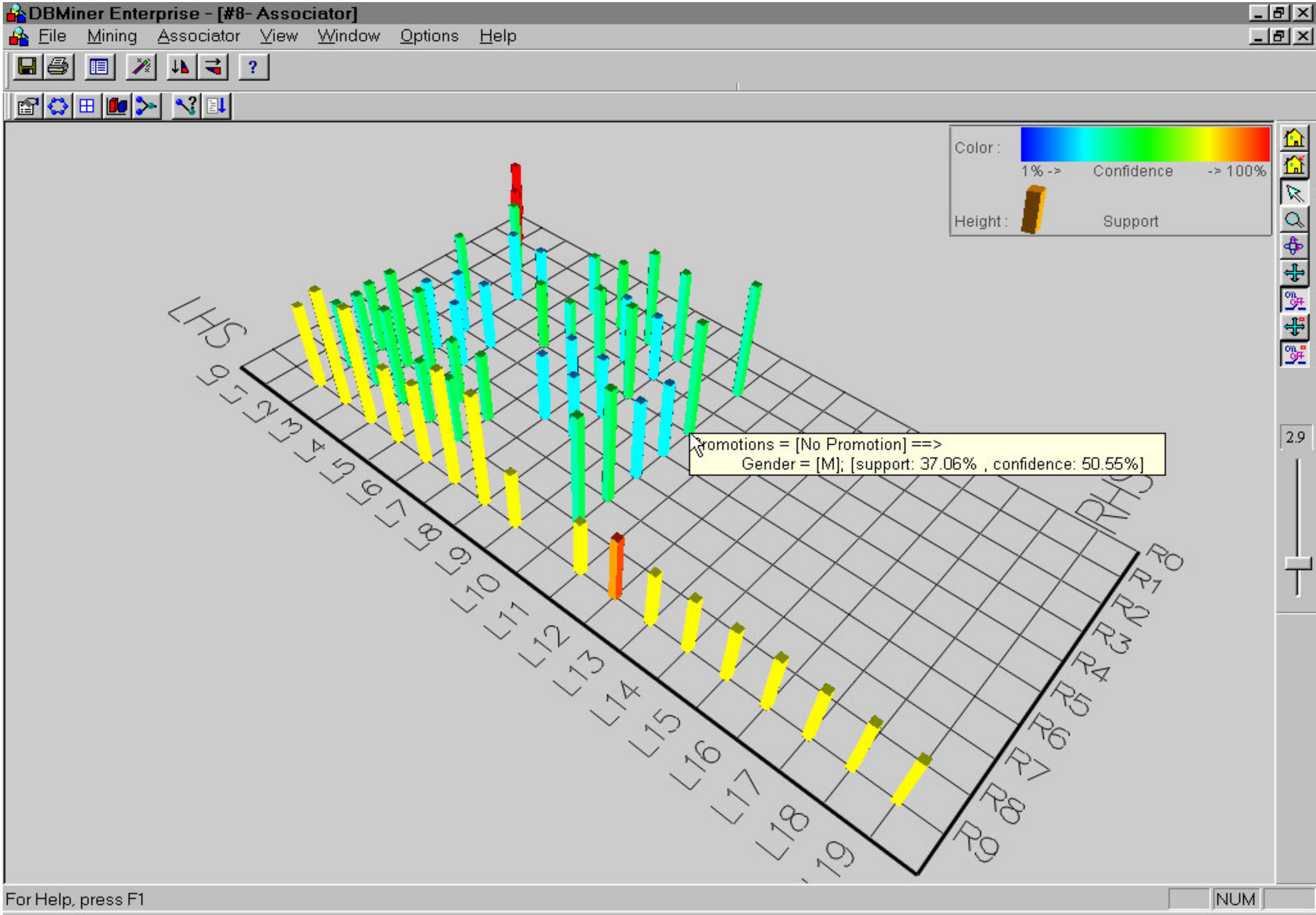
Presentation of Association Rules

(Table Form)

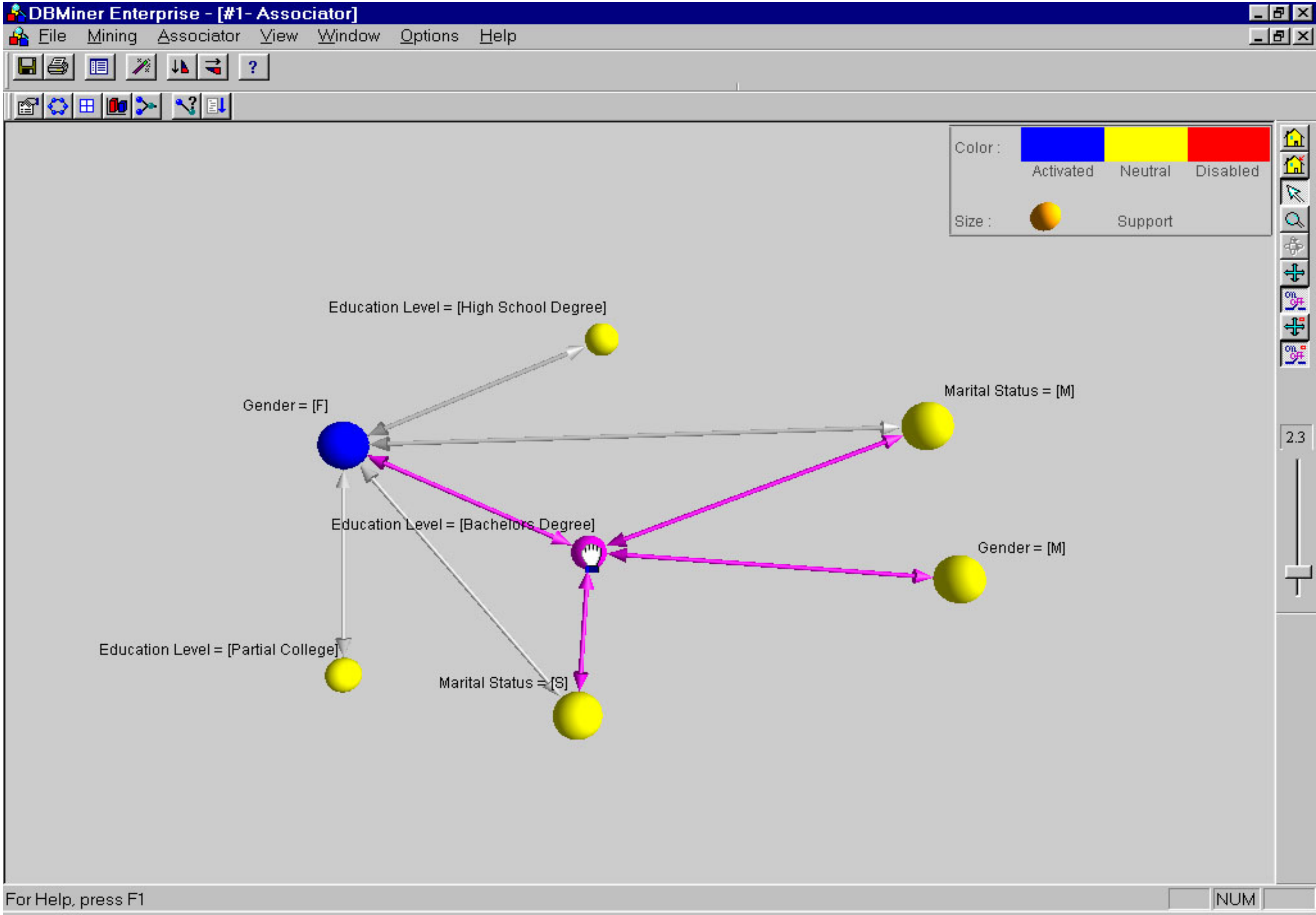
	Body	Implies	Head	Supp (%)	Conf (%)	F	G	H	I
1	cost(x) = '0.00~1000.00'	==>	revenue(x) = '0.00~500.00'	28.45	40.4				
2	cost(x) = '0.00~1000.00'	==>	revenue(x) = '500.00~1000.00'	20.46	29.05				
3	cost(x) = '0.00~1000.00'	==>	order_qty(x) = '0.00~100.00'	59.17	84.04				
4	cost(x) = '0.00~1000.00'	==>	revenue(x) = '1000.00~1500.00'	10.45	14.84				
5	cost(x) = '0.00~1000.00'	==>	region(x) = 'United States'	22.56	32.04				
6	cost(x) = '1000.00~2000.00'	==>	order_qty(x) = '0.00~100.00'	12.91	69.34				
7	order_qty(x) = '0.00~100.00'	==>	revenue(x) = '0.00~500.00'	28.45	34.54				
8	order_qty(x) = '0.00~100.00'	==>	cost(x) = '1000.00~2000.00'	12.91	15.67				
9	order_qty(x) = '0.00~100.00'	==>	region(x) = 'United States'	25.9	31.45				
10	order_qty(x) = '0.00~100.00'	==>	cost(x) = '0.00~1000.00'	59.17	71.86				
11	order_qty(x) = '0.00~100.00'	==>	product_line(x) = 'Tents'	13.52	16.42				
12	order_qty(x) = '0.00~100.00'	==>	revenue(x) = '500.00~1000.00'	19.67	23.88				
13	product_line(x) = 'Tents'	==>	order_qty(x) = '0.00~100.00'	13.52	98.72				
14	region(x) = 'United States'	==>	order_qty(x) = '0.00~100.00'	25.9	81.94				
15	region(x) = 'United States'	==>	cost(x) = '0.00~1000.00'	22.56	71.39				
16	revenue(x) = '0.00~500.00'	==>	cost(x) = '0.00~1000.00'	28.45	100				
17	revenue(x) = '0.00~500.00'	==>	order_qty(x) = '0.00~100.00'	28.45	100				
18	revenue(x) = '1000.00~1500.00'	==>	cost(x) = '0.00~1000.00'	10.45	96.75				
19	revenue(x) = '500.00~1000.00'	==>	cost(x) = '0.00~1000.00'	20.46	100				
20	revenue(x) = '500.00~1000.00'	==>	order_qty(x) = '0.00~100.00'	19.67	96.14				
21									
22									
23	cost(x) = '0.00~1000.00'	==>	revenue(x) = '0.00~500.00' AND order_qty(x) = '0.00~100.00'	28.45	40.4				
24	cost(x) = '0.00~1000.00'	==>	revenue(x) = '0.00~500.00' AND order_qty(x) = '0.00~100.00'	28.45	40.4				
25	cost(x) = '0.00~1000.00'	==>	revenue(x) = '500.00~1000.00' AND order_qty(x) = '0.00~100.00'	19.67	27.93				
26	cost(x) = '0.00~1000.00'	==>	revenue(x) = '500.00~1000.00' AND order_qty(x) = '0.00~100.00'	19.67	27.93				
27	cost(x) = '0.00~1000.00' AND order_qty(x) = '0.00~100.00'	==>	revenue(x) = '500.00~1000.00'	19.67	33.23				

Sheet1

Visualization of Association Rule Using Plane Graph



Visualization of Association Rule Using Rule Graph



Iceberg Queries

- Iceberg query: Compute aggregates over one or a set of attributes only for those whose aggregate values is above certain threshold
- Example:

```
select P.custID, P.itemID, sum(P.qty)
from purchase P
group by P.custID, P.itemID
having sum(P.qty) >= 10
```

- Compute iceberg queries efficiently by Apriori:
  - First compute lower dimensions
  - Then compute higher dimensions only when all the lower ones are above the threshold

Chapter 5: Mining Association Rules in Large Databases

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
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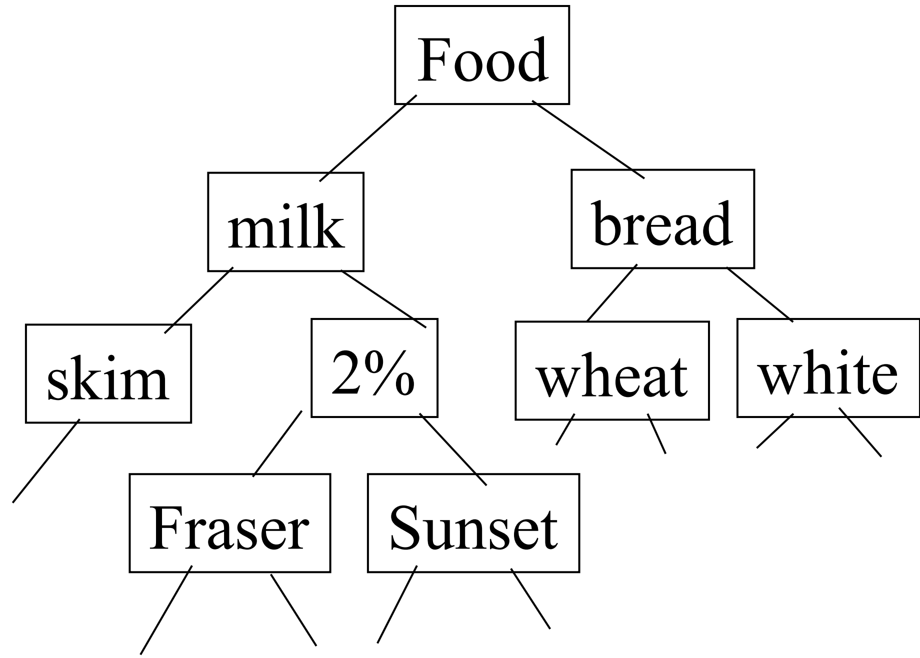
Multiple-Level Association Rules



- **Items often form hierarchy**
- Items at the lower level are expected to have lower support.
- Rules regarding itemsets at appropriate levels could be quite useful.
- Transaction database can be encoded based on dimensions and levels
- We can explore shared multi-level mining

TID	Items
T1	{111, 121, 211, 221}
T2	{111, 211, 222, 323}
T3	{112, 122, 221, 411}
T4	{111, 121}
T5	{111, 122, 211, 221, 413}

Mining Multi-Level Associations

- A top_down, progressive deepening approach:
  - **First find high-level strong rules**:

    milk → bread [20%, 60%]
  - **Then find their lower-level "weaker" rules**:

    2% milk → wheat bread [6%, 50%]
- **Variations at mining multiple-level association rules**.
  - **Level-crossed association rules:**

    *2% milk → **Wonder** wheat bread*
  - **Association rules with multiple, alternative hierarchies:**

    *2% milk → **Wonder** bread*

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Multi-Dimensional Association (1)

- **Single-dimensional rules**:
- buys(X, “milk”) $\Rightarrow$ buys(X, “bread”)
- **Multi-dimensional rules**: Involve 2 or more dimensions or predicates
  - **Inter-dimension association rules (*no repeated predicates*)**
    - age(X,”19-25”) $\wedge$ occupation(X,“student”) $\Rightarrow$ buys(X,“coke”)

Multi-Dimensional Association

- **Hybrid-dimension association rules (*repeated predicates*)**
  - age(X,"19-25") $\wedge$ buys(X, "popcorn") $\Rightarrow$ buys(X, "coke")
- **Categorical (qualitative) Attributes**
  - finite number of possible values, no ordering among values
- **Quantitative Attributes**
  - numeric, implicit ordering among values

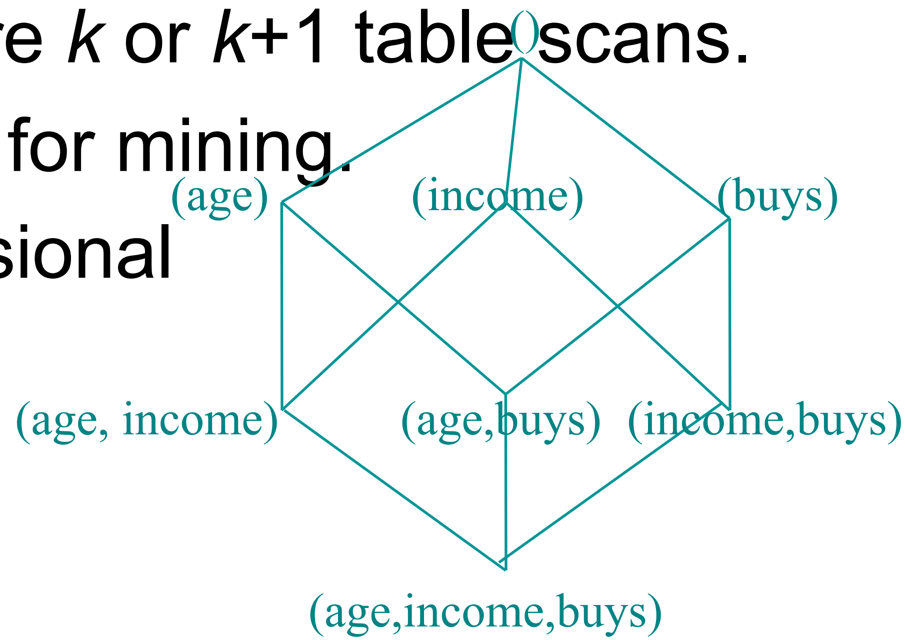
Techniques for Mining MD Associations

- Search for frequent *k*-predicate set:
  - **Example:**
  - {age, occupation, buys} is a **3-predicate set**.
  - Techniques can be categorized by how age are treated.

1. Using static discretization of quantitative attributes
   - Quantitative attributes are statically discretized by using predefined concept hierarchies.
2. Quantitative association rules
   - Quantitative attributes are dynamically discretized into "bins" based on the distribution of the data.
3. Distance-based association rules
   - This is a dynamic discretization process that considers the distance between data points.

Static Discretization of Quantitative Attributes

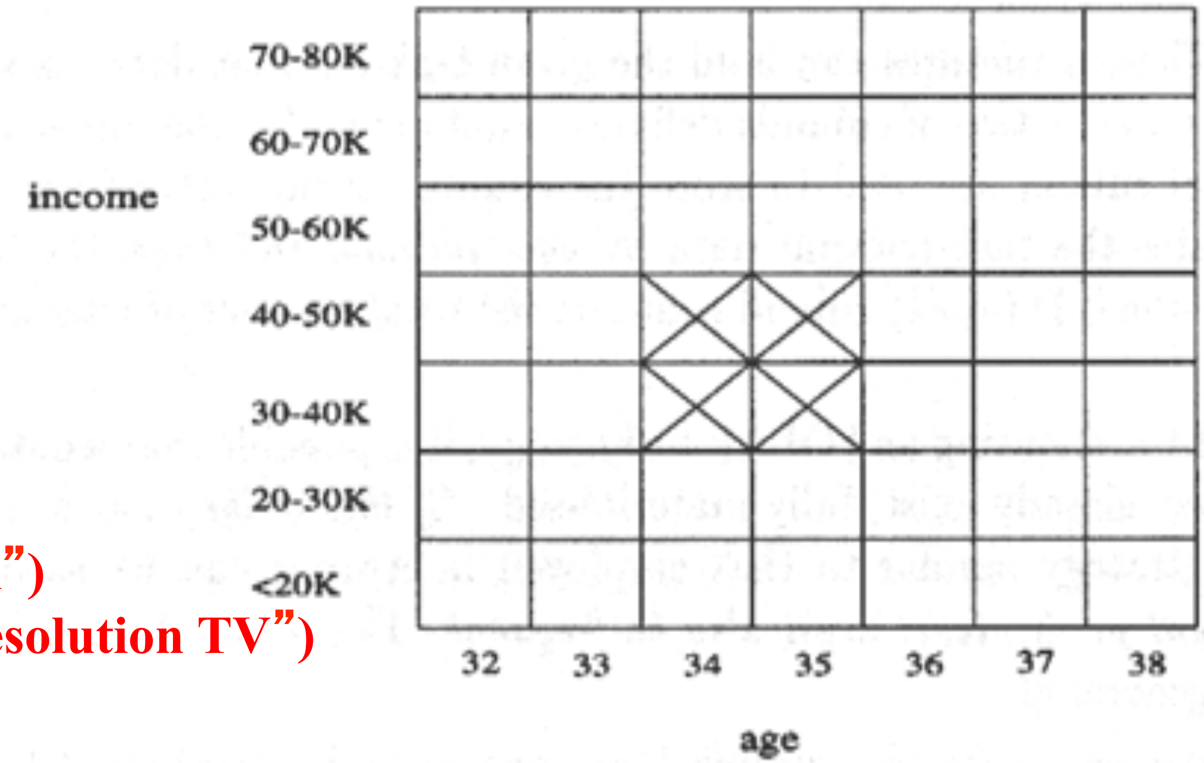
- **Discretized prior** to mining using concept hierarchy.
- Numeric values are replaced by ranges
- In relational database, finding all frequent k-predicate sets will require *k* or *k*+1 table scans.
- Data cube is well suited for mining.
- The cells of an n-dimensional cuboid correspond to the predicate sets.
- Mining from data cubes can be much faster.



Quantitative Association Rules

- Numeric attributes are *dynamically* discretized
  - Such that the confidence or compactness of the rules mined is maximized.
- 2-D quantitative association rules: $A_{quan1} \wedge A_{quan2} \Rightarrow A_{cat}$
- Cluster "adjacent"

  association rules to form general rules using a 2-D grid.
- Example:



age(X,"30-34") ∧
income(X,"24K - 48K")
⇒ buys(X,"high resolution TV")

ARCS (Association Rule Clustering System)

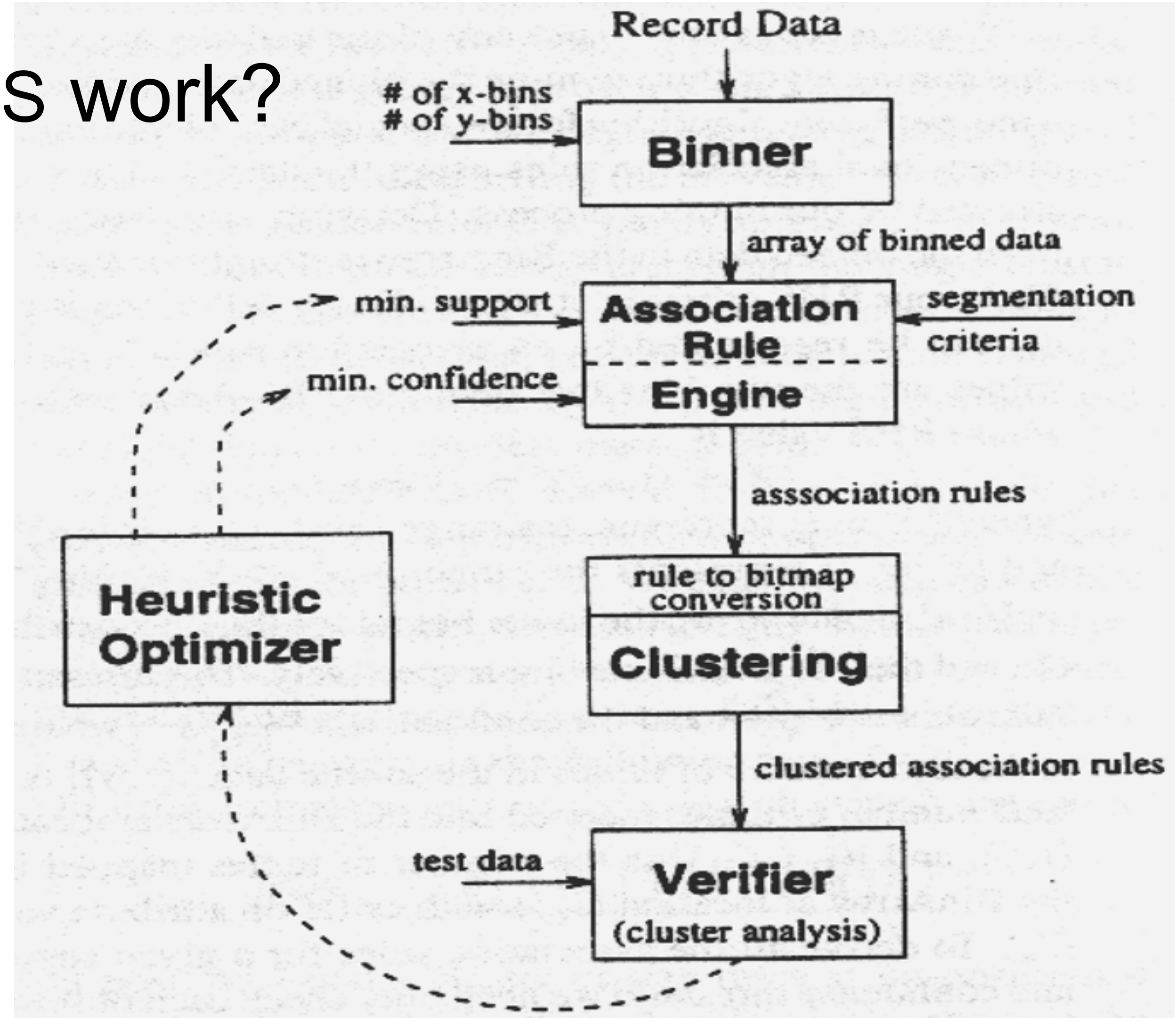
How does ARCS work?

1. Binning

2. Find frequent predicateset

3. Clustering

4. Optimize



Limitations of ARCS

- Only quantitative attributes on LHS of rules.
- Only 2 attributes on LHS. (2D limitation)
- An alternative to ARCS
  - Non-grid-based
  - equi-depth binning
  - clustering based on a measure of *partial completeness*.
  - "***Mining Quantitative Association Rules in Large Relational Tables***" by R. Srikant and R. Agrawal.

Clusters and Distance Measurements

- The diameter, *d,* assesses the density of a cluster C_X, where

$$d(C_X) \leq d_0^{\ X}$$

$$|C_X| \geq s_0$$

- Finding clusters and distance-based rules
  - the density threshold, d_0, replaces the notion of support
  - modified version of the BIRCH clustering algorithm

Mining Distance-based Association Rules

- Binning methods do not capture the semantics of interval data

Price($)	Equi-width (width $10)	Equi-depth (depth 2)	Distance-based
7	[0,10]	[7,20]	[7,7]
20	[11,20]	[22,50]	[20,22]
22	[21,30]	[51,53]	[50,53]
50	[31,40]		
51	[41,50]		
53	[51,60]		

- Distance-based partitioning, more meaningful discretization considering:
  - density/number of points in an interval
  - “closeness” of points in an interval

Clusters and Distance Measurements

- S[X] is a set of N tuples t_1, t_2, …, t_N , projected on the attribute set X
- The diameter of S[X]:

$$d(S[X]) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} dist_X(t_i[X], t_j[X])}{N(N-1)}$$

  – *dist*$_x$:distance metric, e.g. Euclidean distance or Manhattan

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Interestingness Measurements

- Objective measures

  Two popular measurements:

  ➊ *support;* and

  ➋ *confidence*

- Subjective measures (Silberschatz & Tuzhilin, KDD95)

  A rule (pattern) is interesting if

  ➊ it is *unexpected* (surprising to the user); and/or

  ➋ *actionable* (the user can do something with it)

Criticism to Support and Confidence

- Example 2:
  - X and Y: positively correlated,
  - X and Z, negatively related
  - support and confidence of X=>Z dominates

X	1	1	1	1	0	0	0	0
Y	1	1	0	0	0	0	0	0
Z	0	1	1	1	1	1	1	1

Rule	Support	Confidence
X=>Y	25%	50%
X=>Z	37.50%	75%

Other Interestingness Measures: Interest

- **Interest** $$\frac{P(A \wedge B)}{P(A)P(B)}$$
  - taking both $P(A)$ and $P(B)$ in consideration
  - $P(A \wedge B)=P(B)*P(A)$, if A and B are independent events
  - A and B negatively correlated, if the value is less than 1; otherwise A and B positively correlated.

X	1	1	1	1	0	0	0	0
Y	1	1	0	0	0	0	0	0
Z	0	1	1	1	1	1	1	1

Itemset	Support	Interest
X,Y	25%	2
X,Z	37.50%	0.9
Y,Z	12.50%	0.57

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Constraint-Based Mining

- Interactive, exploratory mining giga-bytes of data?
  - Could it be real? — Making good use of constraints!
- What kinds of constraints can be used in mining?
  - Knowledge type constraint: classification, association, etc.
  - Data constraint: SQL-like queries
  - Dimension/level constraints:
    - in relevance to **region, price, brand, customer category**
    - small sales (price $< \$10$) triggers big sales (sum $> \$200$).
  - Interestingness constraints:
    - strong rules (min_support $\geq 3\%$, min_confidence $\geq 60\%$).

Rule Constraints in Association Mining

- Two kind of rule constraints:
  - Rule form constraints: meta-rule guided mining.
    - $P(x, y) \wedge Q(x, w) \rightarrow$ takes(x, "database systems").
  - Rule (content) constraint: constraint-based query optimization (Ng, et al., SIGMOD'98).
    - sum(LHS) < 100 ^ min(LHS) > 20 ^ count(LHS) > 3 ^ sum(RHS) > 1000
- 1-variable vs. 2-variable constraints (Lakshmanan, et al. SIGMOD'99):
  - 1-var: A constraint confining only one side (L/R) of the rule, e.g., as shown above.
  - 2-var: A constraint confining both sides (L and R).
    - sum(LHS) < min(RHS) ^ max(RHS) < 5* sum(LHS)

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Why Is the Big Pie Still There?

- More on constraint-based mining of associations
  - **Boolean vs. quantitative associations**
    - Association on discrete vs. continuous data
  - **From association to correlation and causal structure analysis.**
    - Association does not necessarily imply correlation or causal relationships
  - **From intra-trasanction association to inter-transaction associations**
    - E.g., break the barriers of transactions (Lu, et al. TOIS' 99).
  - **From association analysis to classification and clustering analysis**
    - E.g, clustering association rules

Summary

- **Association rule mining**
  - probably the **most significant contribution** from the database community in KDD
  - A large number of papers have been published
- Many interesting issues have been explored
- An interesting research direction:
  - **Association analysis in other types of data**: spatial data, multimedia data, time series data, etc.