NAME: __________________________ ID: __________________________ Math/CS

Points assigned to questions are as if it were a real midterm examination. You don’t get them NOW; you can get 0 – 5 points (extra credit) for all of the practice midterm.

QUESTION 1 (15pts)

(a) (5pts) Describe shortly a difference between logical and semantical paradoxes.

(b) (5pts) Give an example (by name ) of a logical paradox.

(c) (5pts) Describe shortly a difference between classical and intuitionistic logic.

QUESTION 2 (35pts)

(1) (10pts) For the sentence below write its corresponding formula \( A \). Explain your solution.

If from the fact that all sides of a triangle ABC are equal we can deduce that all angles of the triangle ABC are equal and all angles of the triangle ABC are not equal, then all sides of a triangle ABC are equal.

(2) (5pts) Define a formal language to which the formula \( A \) belongs.

(3) (5pts) Determine the degree of \( A \) and write down all its sub-formulas of the degree 2.

(4) (10pts) Determine the following: \( A \in T, A \in C \).

You can use the shorthand notation for your work, but you have to write final answer using proper definitions. This question is about how well you understand and know how to use formal definitions.
(5) (5pts) Determine the following: $A \in \mathbf{LT}, A \in \mathbf{HT}$. Use a shorthand notation.

**QUESTION 3 (10pts)** Write the formula $A$ from Question 2 as a formula of the language $\mathcal{L}_{\neg, \cup}$, i.e. as a formula $B$ of $\mathcal{L}_{\neg, \cup}$, such that $A \equiv B$. Write down all logical equivalences you need while solving this problem.

**QUESTION 4 (15pts)**

$S$ is the following proof system:

$$S = (\mathcal{L}_{\Rightarrow, \cup, \neg}, \mathcal{F}, A1, (r1), (r2))$$

**Axiom**

$A1$ $ (A \Rightarrow (A \cup B)),$

**Rules** of inference:

$$(r1) \frac{A; B}{(A \cup \neg B)},$$

$$(r2) \frac{A; (A \cup B)}{B},$$

1. Verify whether $S$ is sound/not sound under classical semantics.
2. Find a formal proof of $\neg (A \Rightarrow (A \cup B))$ in $S$, ie. show that 

$\vdash_S \neg (A \Rightarrow (A \cup B))$.

3. Does above point 2. prove that $\models \neg (A \Rightarrow (A \cup B))$?

QUESTION 5 (10pts)

$H$ is the following proof system:

$$H = (\mathcal{L}_{\Rightarrow, \neg}, ~ A_1, A_2, A_3, ~ MP)$$

A1 $~~ (A \Rightarrow (B \Rightarrow A))$,

A2 $~~ ((A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C)))$,

A3 $~~ ((\neg B \Rightarrow \neg A) \Rightarrow ((\neg B \Rightarrow A) \Rightarrow B))$

MP Rule of inference:

$$\frac{A; (A \Rightarrow B)}{B}$$

(1) (5pts) Prove that $H$ is SOUND under classical semantics.

(2) (5pts) Prove that $H$ is not sound under $K$ semantics.

(3) (5pts) Is $H$ COMPLETE with respect to classical semantics?
QUESTION 6 (15pts)  Give an example of a sound (classical semantics) proof system $S$ based on $\mathcal{L}_{\{\Rightarrow, \neg\}}$ with 2 axioms different that axioms from QUESTIONS 5 and one two premisses rule of inference that is not MP.

QUESTION 7 (10 extra pts)

Here are consecutive steps $B_1, ..., B_9$ in a proof of $((A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A))$ in $H$ in $H$ from Question 5.

Complete the proof $B_1, ..., B_9$ above by providing comments how each step of the proof was obtained. Write your comments in the space between the steps. Use the next page as work space, if needed.

$B_1 = (A \Rightarrow B)$

$B_2 = (\neg\neg A \Rightarrow A)$

$B_3 = (\neg\neg A \Rightarrow B)$

$B_4 = (B \Rightarrow \neg\neg B)$

$B_5 = (\neg\neg A \Rightarrow \neg\neg B)$
$B_6 = ((\neg A \Rightarrow \neg B) \Rightarrow (\neg B \Rightarrow \neg A))$

$B_7 = (\neg B \Rightarrow \neg A)$

$B_8 = (A \Rightarrow B) \vdash (\neg B \Rightarrow \neg A)$

$B_9 = ((A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A))$
Use it as work space.