ONE PROBLEM (2pts)

Part 1 (1pts) Write the following natural language statement:

*From the fact that there is a bird that does not fly and 4 + 4 = 4, we deduce the following: it is not possible that all birds fly OR it is not necessary that 4 + 4 = 4.*

in the TWO WAYS:

WAY 1 (0.5pts) As a formula $A_1 \in \mathcal{T}_1$ of a language $\mathcal{L}(\neg, \Box, \circ, \land, \lor)$

SOLUTION Use Propositional Variables $a, b, c$ where

$a$ denotes statement: *there is a bird that does not fly*

$b$ denotes statement: $4 + 4 = 4$

c denotes statement: *all birds fly*

The formula $A_1 \in \mathcal{T}_1$ is:

$((a \cap b) \Rightarrow (\neg c \lor \Box b))$

WAY 2 (0.5pts) As a formula $A_2 \in \mathcal{T}_2$ of a PREDICATE LANGUAGE language $\mathcal{L}(P, F, V)$ with the set $\{\neg, \Box, \Diamond, \land, \lor, \Rightarrow\}$ of propositional connectives.

Use the following Predicates, Functions and Constants

$B(x)$ for $x$ is a bird, $F(x)$ for $x$ can fly, $E(x, y)$ for $x = y$, $f(x, y)$ for $+$, and $c$ for 4.

(0.2pts) Restricted domain formula is:

$((\exists x B(X) \cap \neg F(x)) \cap E(f(c, c), c)) \Rightarrow (\neg \forall x f(B) \Rightarrow F(x)) \cup \neg \Box E(f(c, c), c))$

(0.3pts) Formula $A_2 \in \mathcal{T}_2$ is:

$((\exists x (B(X) \cap \neg F(x)) \cap E(f(c, c), c)) \Rightarrow (\neg \forall x (B(X) \Rightarrow F(x)) \cup \neg \Box E(f(c, c), c)))$

Part 2 (1pts)

(0.5pts) Circle formulas that are propositional tautologies

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$S_1 = \{(\neg c \cap c) \Rightarrow (\neg b \Rightarrow (d \cap e)), (a \Rightarrow b) \cup (\neg (a \Rightarrow b)), ((a \cap \neg b) \Rightarrow ((a \cap \neg b) \Rightarrow (\neg d \cup e))), (\neg a \Rightarrow (\neg a \cup b))\}$

Solution $\not\models ((a \cap \neg b) \Rightarrow ((a \cap \neg b) \Rightarrow (\neg d \cup e)))$, all other formulas are tautologies

(0.5pts) Circle formulas that are predicate tautologies

$S_2 = \{ (\exists x A(x) \Rightarrow \neg \forall x \neg A(x)), (\forall x (P(x, y) \cap Q(y)) \Rightarrow \neg \exists x (P(x, y) \cap Q(y))),

((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x (A(x) \cap B(x))), (\forall x(A(x) \Rightarrow B) \Rightarrow (\exists x A(x) \Rightarrow B)) \}$

Solution $\not\models ((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x (A(x) \cap B(x)))$, all other formulas are tautologies