CSE/MAT371 Q1 SOLUTIONS Fall 2022
(25pts)

Please write carefully your solutions. NO PARTIAL CREDIT. Formulas must be fully correct for credit.

QUESTION 1 (6pts)

Write the following natural language statement:

From the fact that there is a blue bird we deduce that: it is not necessary that all natural numbers are even OR, if it is possible that it is not true that all natural numbers are even, then it is not true that there is a blue bird.

in the following two ways.

1. (3pts) As a formula $A_1 \in \mathcal{F}_1$ of a language $\mathcal{L}_{\{\neg,\, \cap,\, \cup,\, \Rightarrow\}}$

Solution Propositional Variables are: a, b, c
a denotes statement: *there is a blue bird* ,
b denotes statement: *it is necessary that all natural numbers are even*,
c denotes statement: *possible that it is not true that all natural numbers are even*

Formula $A_1 \in \mathcal{F}_1$ is:

$$(a \Rightarrow (\neg b \cup (c \Rightarrow \neg a)))$$

2. (3pts) As a formula $A_2 \in \mathcal{F}_2$ of a language $\mathcal{L}_{\{\neg,\, \Box,\, \Diamond,\, \cap,\, \cup,\, \Rightarrow\}}$

Solution Propositional Variables are: a, b
a denotes statement: *there is a blue bird*,
b denotes statement: *all natural numbers are even*

Formula $A_2 \in \mathcal{F}_2$ is:

$$(a \Rightarrow (\neg \Box b \cup (\Diamond \neg b \Rightarrow \neg a)))$$

QUESTION 2 (6pts)

1. (3pts) **Circle** formulas that **are** propositional **tautologies**

$$\mathcal{S}_1 = \{\ ((\neg c \cap c) \Rightarrow (\neg b \Rightarrow (d \cap e))),\quad ((a \Rightarrow b) \cup (a \cap \neg b)),\quad ((a \cap \neg b) \cup ((a \cap \neg b) \Rightarrow (\neg d \cup e))),\quad (a \cup \neg b)\ \}$$

Solution $\not\models (a \cup \neg b)$, all other formulas are tautologies

2. (3pts) **Circle** formulas that **are** predicate **tautologies**

$$\mathcal{S}_2 = \{\ (\exists x\, A(x) \Rightarrow \neg \forall x \neg\, A(x)),\quad (\forall x\, (P(x, y) \cap Q(y)) \Rightarrow \neg \exists x\, \neg (P(x, y) \cap Q(y))),$$

$$((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x\, (A(x) \cap B(x))),\quad (\forall x (A(x) \Rightarrow B)\ \Rightarrow\ (\exists x A(x) \Rightarrow B))\ \}$$

Solution $\not\models ((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x\, (A(x) \cap B(x)))$, all other formulas are tautologies

QUESTION 3 (6pts)

Please write carefully your solutions. NO PARTIAL CREDIT. Formulas must be **well formed formulas** for credit

Here is a mathematical statement **S**:

For each integer $m \in Z$ the following holds: If $m > 5$, then there is a natural number $n \in N$, such that $m + n > 5$.

1. (2pts) Re-write **S** as a symbolic mathematical statement **SM** that only uses mathematical and logical symbols.

Solution **S** becomes a symbolic mathematical statement

$$\mathbf{SM}: \quad \forall_{m \in Z}(m > 5 \Rightarrow \exists_{n \in N}\, m + n > 0)$$

2. (2pts) Translate the symbolic statement **SM** into to a corresponding formula with **restricted quantifiers**. Explain your choice of symbols.

Solution We write Z(x) for $x \in Z$, N(y) for $y \in N$, a constant c for the number 5. We use $G \in \mathbf{P}$ to denote the relation $>$, we use $f \in \mathbf{F}$ to denote the function $+$.

The statement $m > 5$ becomes an **atomic formula** G(x, c). The statement $m + n > 5$ becomes an **atomic formula** G(f(x,y), c).

The symbolic mathematical statement **SM** becomes a **restricted quantifiers** formula

$$\forall_{Z(x)}(G(x, c) \Rightarrow \exists_{N(y)} G(f(x, y), c))$$

3. (2pts) Translate your **restricted domain** quantifiers formula into a correct formula A of the predicate language $\mathcal{L}$

Solution We apply now the **transformation rules** and get a corresponding formula $A \in \mathcal{F}$:

$$\forall x(Z(x) \Rightarrow (G(x, c) \Rightarrow \exists y(N(y) \cap G(f(x, y), c))))$$

QUESTION 4 (6pts)

Given a formula A : $\forall x \exists y\, P(f(x, y), c)$ of the predicate language $\mathcal{L}$, and two **model structures**

$$\mathbf{M_1} = (Z, I_1), \quad \text{and} \quad \mathbf{M_2} = (N, I_2)$$

with the **interpretations** defined as follows.

$$P_{I_1} := , \quad f_{I_1} : + , \quad c_{I_1} : 0 \quad \text{and} \quad P_{I_2} : > , \quad f_{I_2} : \cdot , \quad c_{I_2} : 0$$

1. (3pts) Show that $\mathbf{M_1} \models A$

Solution

$\mathbf{M_1} \models A$ because $A_{I_1} : \forall_{x \in Z} \exists_{y \in Z}\, x + y = 0$ is a **true** mathematical statement as we have that each $x \in Z$ exists $y = -x$ and $-x \in Z$ and $x - x = 0$

2. (3pts) Show that $\mathbf{M_2} \not\models A$

Solution

$\mathbf{M_2} \not\models A$ because $A_{I_2} : \forall_{x \in N} \exists_{y \in N}\, x \cdot y > 0$ is a **false** statement for $x = 0$.