CSE371 Midterm MAKEUP Fall 2018

75pts + 15extra pts

NAME **ID:** **Math/CS**

PART 1 (Extra Credit - 15pts)

DEFINITIONS Write carefully the following defniitions.

D1 Given a language $\mathcal{L}_{\{\Rightarrow, \cup, \cap, \neg\}}$ and a formula A of this language and a truth assignment v,

Write a definition of $v \models A$

D2 A non-empty set $\mathcal{G} \subseteq \mathcal{F}$ **inconsistent** under classical semantics if and only if

D3 Given a proof system $S = (\mathcal{L},\ \mathcal{E},\ LA,\ \mathcal{R})$

Write the definition of $\vdash_S E$

D4 Write the statements of **Soundness Theorem** and of **Completeness Theorem** for a proof system S and semantics **M**

PART 2: PROBLEMS

PROBLEM 1 (15 pts)

We define **H** semantics operations $\cup$ and $\cap$ as follows.

$$a \cup b = max\{a, b\}, \quad a \cap b = min\{a, b\}.$$

The Truth Tables for Implication and Negation are:

H-Implication

$\Rightarrow$	F	$\perp$	T
F	T	T	T
$\perp$	F	T	T
T	F	$\perp$	T

H Negation

$\neg$	F	$\perp$	T
	T	F	F

1. We know that $v : VAR \longrightarrow \{F, \perp, T\}$ is such that

$v^*((a \cap b) \Rightarrow (a \Rightarrow c)) = \perp$ under **H** semantics.

Use this information to **evaluate** $v^*(((b \Rightarrow a) \Rightarrow (a \Rightarrow \neg c)) \cup (a \Rightarrow b))$.

You can use shorthand notation.

2. Verify whether $\models_{\mathbf{H}} ((a \Rightarrow b) \Rightarrow (\neg a \cup b))$

PROBLEM 2 (15pts)

1. Write the formula

$$A = ((((a \cap \neg c) \Rightarrow \neg b) \cup a) \Rightarrow (c \cup b))$$

as a formula of the language $\mathcal{L}_{\{\neg, \cup\}}$, i.e. as a formula B of $\mathcal{L}_{\{\neg, \cup\}}$, such that $A \equiv B$.

LIST all logical equivalences you need while solving this problem

2. PROVE that $\mathcal{L}_{\{\neg, \cup\}} \equiv \mathcal{L}_{\{\neg, \cap\}}$

PROBLEM 3 (25pts)

S is the following proof system:

$$S = (\ \mathcal{L}_{\{\Rightarrow, \cup, \neg\}},\quad \mathcal{F},\quad LA = \{(A \Rightarrow (A \cup B))\}\ \ (r1),\ (r2)\ )$$

Rules of inference:

$$(r1)\ \frac{A\ ;B}{(A \cup \neg B)}, \qquad\qquad (r2)\ \frac{A\ ;(A \cup B)}{B}$$

1. Verify whether S is **sound/not sound** under classical semantics.

2. Find a **formal proof** of $\neg(A \Rightarrow (A \cup B))$ in S, i. e. show that $\vdash_S \neg(A \Rightarrow (A \cup B))$

3. Explain whether the above point **2.** proves that $\models \neg(A \Rightarrow (A \cup B))$?

PROBLEM 4 (20pts)

1 Use the proof system **RS** and its **Completeness Theorem** to prove that

$$\models ((a \cup b) \Rightarrow \neg a) \cup (\neg a \Rightarrow \neg c))$$

2. Use the proof system **RS** to construct a counter model for the formula

$$(((a \Rightarrow b) \cap \neg c) \cup (a \Rightarrow c)).$$