CSE371 EXTRA MIDTERM Fall 2018
(75pts + 10extra pts)

NAME **ID:** **Math/CS**

THIS IS EXTRA TEST. IF YOU TAKE IT AND SUBMIT IT YOUR MIDTERM POINTS WILL BE THE AVERAGE of the MIDTERM and this TEST.

PROBLEM 1 (25pts)

CREATE YOUR OWN 3 valued extensional semantics **M** for the language $\mathcal{L}_{\{\neg,\ \mathbf{L},\ \cup,\ \Rightarrow\}}$ by **defining the connectives** $\neg$, $\cup$, $\Rightarrow$ on a set $\{F, \perp, T\}$ of logical values.

You must follow the following **assumptions**.

Assumption 1: The third logical value value is **intermediate** between truth and falsity, i.e. the set of logical values is **ordered** and we have the following $F < \perp < T$ and T is the **designated value**.

Assumption 2: The semantics has to **model the situation** in which one "likes" only truth; i.e. in which

$$\mathbf{L}T = T, \quad \mathbf{L}\perp = F, \quad \mathbf{L}F = F$$

Assumption 3: The connectives $\neg$, $\cup$, $\Rightarrow$ can be defined as you wish, but you have to define them in such a way as to make sure that

$$\models_{\mathbf{M}} (\mathbf{L}A \cup \neg\mathbf{L}A)$$

Part 1 Definition of your **M** connectives

Write down definition of your logical connectives by "shorthand" ; i.e. by writing functions defining connectives in form of the following "truth tables"

L Connective

L	F	$\perp$	T

Negation :

$\neg$	F	$\perp$	T

Implication

$\Rightarrow$	F	$\perp$	T
F			
$\perp$			
T			

Disjunction :

$\cup$	F	$\perp$	T
F			
$\perp$			
T			

Part 2

Verify **correctness** of your **M** semantics, i.e. verify whether $\models_{\mathbf{M}} (\mathbf{L}A \cup \neg\mathbf{L}A)$ under your semantics - you can use shorthand notation

Part 3

Verify whether a formula $(a \cup (b \Rightarrow (\neg a \cup c)))$ have a **counter model** under your semantics **M**. You can use **shorthand notation**.

Part 4

Verify whether the following set **G** is **M**-consistent. You can use **shorthand notation**

$$\mathbf{G} = \{\ \mathbf{L}a,\ \ (a \cup \neg\mathbf{L}b),\ \ (a \Rightarrow b),\ b\ \}$$

PROBLEM 2 (20pts)

1. Write the formula

$$A = ((((a \cap \neg c) \Rightarrow \neg b) \cup a) \Rightarrow (c \cup b))$$

as a formula of the language $\mathcal{L}_{\{\neg,\cup\}}$, i.e. as a formula B of $\mathcal{L}_{\{\neg,\cup\}}$, such that $A \equiv B$.

LIST all logical equivalences you need while solving this problem

2. PROVE that $\mathcal{L}_{\{\neg,\cup\}} \equiv \mathcal{L}_{\{\neg,\cap\}}$

PROBLEM 3 (20pts)

S is the following proof system:

$$S = (\ \mathcal{L}_{\{\Rightarrow,\cup,\neg\}},\quad \mathcal{F},\quad LA = \{(A \Rightarrow (A \cup B))\}\ \ (r1),\ (r2)\ )$$

Rules of inference:

$$(r1)\ \frac{A\ ;B}{(A \cup \neg B)},\qquad\qquad (r2)\ \frac{A\ ;(A \cup B)}{B}$$

1. Verify whether S is sound/not sound under classical semantics.

2. Find a formal proof of $\neg(A \Rightarrow (A \cup B))$ in S, i. e. show that $\vdash_S \neg(A \Rightarrow (A \cup B))$

3. Does above point **2.** prove that $\models \neg(A \Rightarrow (A \cup B))$?

PROBLEM 4 (20pts)

1 Use the proof system **RS** and its **Completeness Theorem** to prove that

$$\models ((a \cup b) \Rightarrow \neg a) \cup (\neg a \Rightarrow \neg c))$$

2. Use the proof system **RS** to construct a counter model for the formula

$$(((a \Rightarrow b) \cap \neg c) \cup (a \Rightarrow c)).$$