CSE371 PRACTICE MIDTERM 1 Fall 2017
(10 extra pts)

NAME **ID:**

MY POINTS ARE:

TAKE test as a practice - and **correct it yourself** to see how much points you would get.

Solutions to all problems and questions are somewhere on our webpage! so you CAN correct yourself- but do it **ONLY AFTER you complete it all by yourself**.

This is the **goal** of the PRACTICE TEST!

PLEASE SUBMIT your SOLUTIONS that have been CORRECTED BY YOU - Write corrections in RED. You WILL GET 10 points for THAT! even if all problems you solved were first wrong- and then CORRECTED!

Write a sum of POINTS you give yourself for your solutions -after you check your answers for corrections.

The **real midterm will have less problems**; I will make sure you will be able to complete it within 1 hour and 15 minutes.

BRING YOUR solved-corrected TEST to class on Monday, October 23

PART ONE: DEFINITIONS (10pts)

All Definitions are for language $\mathcal{L} = \mathcal{L}_{\{\neg, \Rightarrow, \cup, \cap\}}$ and **classical semantics**

Write carefully the following DEFINITIONS

D1. Extentional Connectves

D2. Given the truth assignment $v : VAR \longrightarrow \{T, F\}$. Write the definition of its **extension** v^* to the set $\mathcal{F}$ of all formulas of $\mathcal{L}$

D3. Restricted MODEL for a given formula $A \in \mathcal{F}$

D4. Proof System S

D5. Formal proof from Γ in a system S

D6. Sound rule of inference in a system S

D7. Sound proof system S

D8. Soundness and Completeness Theorem for S (classical semantics)

D9. A non-empty set $\mathcal{G} \subseteq \mathcal{F}$ **consistent** (classical semantics)

D10. A non-empty set $\mathcal{G} \subseteq \mathcal{F}$ **inconsistent** (classical semantics)

PART TWO: PROBLEMS (65pts)

Problem 1 (5pts), all other problems (10pts))

Problem 1

Given a mathematical statement **S** written with logical symbols

$$(\exists_{x\in N}\ x \leq 5\ \cap\ \forall_{y\in Z}\ y = 0)$$

1. Translate **S** it into a proper logical formula that **uses** the restricted domain quantifiers.

2. Translate your restricted quantifiers formula into a correct formula **without** restricted domain quantifiers.

Write a **short** solution.

Problem 2

Given a formula A : $\forall x \exists y\ P(f(x,y),c)$ of the predicate language $\mathcal{L}$, and

two **model structures** $\mathbf{M_1} = (Z, I_1)$, $\mathbf{M_1} = (N, I_2)$ with the interpretations defined as follows.

$P_{I_1} := ,\quad f_{I_1}:\ +,\quad c_{I_1}:\ 0$ and $P_{I_2} : >,\quad f_{I_2}:\ \cdot,\quad c_{I_2}:\ 0.$

1. Show that $\mathbf{M_1} \models A$

2. Show that $\mathbf{M_2} \not\models A$

Problem 3

S is the following proof system:

$$S = (\ \mathcal{L}_{\{\Rightarrow,\cup,\neg\}},\ \ \mathcal{F},\ \ LA = \{(A \Rightarrow (A \cup B))\}\ \ (r1),\ (r2)\ )$$

Rules of inference:

$$(r1)\ \frac{A\ ;B}{(A \cup \neg B)},\qquad (r2)\ \frac{A\ ;(A \cup B)}{B}$$

1. Verify whether S is sound/not sound under classical semantics.

2. Verify whether S is sound/not sound under K semantics.

3. Find a formal proof of $\neg(A \Rightarrow (A \cup B))$ in S, i. e. show that $\vdash_S \neg(A \Rightarrow (A \cup B))$

4. Does above point **3.** prove that $\models \neg(A \Rightarrow (A \cup B))$?

Problem 4

1. Given a formula $A = ((a \cap \neg c) \Rightarrow (\neg a \cup b))$ of a language $\mathcal{L}_{\{\neg, \cap, \cup, \Rightarrow\}}$.

Find a formula B of a language $\mathcal{L}_{\{\neg, \Rightarrow\}}$, such that $A \equiv B$. **List** all proper logical equivalences used at at each step.

2. Prove that $\mathcal{L}_{\{\neg,\cap,\cup,\Rightarrow\}} \equiv \mathcal{L}_{\{\neg,\Rightarrow\}}$.

Problem 5

Consider the Hilbert system $H_1 == (\ \mathcal{L}_{\{\Rightarrow\}},\ \ \mathcal{F},\ \ \{A1, A2\},\ \ (MP)\ \frac{A\ ;\ (A\Rightarrow B)}{B}\ )$ where

$A1;\ \ (A \Rightarrow (B \Rightarrow A)),\qquad A2:\ \ ((A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C)))$ and A, B are any formulas from $\mathcal{F}$.

Use **Deduction Theorem** to prove $(A \Rightarrow B), (B \Rightarrow C) \vdash_{H_1} (A \Rightarrow C)$.

Write comments how each step was obtained.

Problem 6

Complete the steps $B_1, ..., B_5$ of the formal proof in H_2 of $(B \Rightarrow \neg\neg B)$ by writing all details for each step of the proof.

You can use the following already proved facts:

F1 $(A \Rightarrow B), (B \Rightarrow C) \vdash_{H_2} (A \Rightarrow C)$

F2 $\vdash_{H_2} (\neg\neg B \Rightarrow B)$

Here are the steps

$B_1 = ((\neg\neg\neg B \Rightarrow \neg B) \Rightarrow ((\neg\neg\neg B \Rightarrow B) \Rightarrow \neg\neg B))$

$B_2 = (\neg\neg\neg B \Rightarrow \neg B)$

$B_3 = ((\neg\neg\neg B \Rightarrow B) \Rightarrow \neg\neg B)$

$B_4 = (B \Rightarrow (\neg\neg\neg B \Rightarrow B))$

$B_5 = (B \Rightarrow \neg\neg B)$

Problem 7 We define, for $A, b_1, b_2, ..., b_n$ and truth assignment v a corresponding formulas A', $B_1, B_2, ..., B_n$ as follows:

$A' = \begin{cases} A & \text{if} \quad v^*(A) = T \\ \neg A & \text{if} \quad v^*(A) = F \end{cases} \qquad B_i = \begin{cases} b_i & \text{if} \quad v(b_i) = T \\ \neg b_i & \text{if} \quad v(b_i) = F \end{cases}$

We proved the following **Main Lemma**: For any formula $A = A(b_1, b_2, ..., b_n)$ and any truth assignment v,

if A', B_1 , B_2, ..., B_n are corresponding formulas defined above, then $B_1, B_2, ..., B_n \vdash A'$.

Let A be a formula $((\neg a \Rightarrow \neg b) \Rightarrow (b \Rightarrow a))$, and let v be such that v(a)= T, v(b) =F.

Write what **Main Lemma** asserts for the formula A.