CSE/MAT371 QUIZ 1 SOLUTIONS Fall 2017

Problem 1

Write the following natural language statement:

From the fact that each natural number is greater than zero we **deduce** *that: it is* **not possible** *that Anne is a boy* **or**, **if** *it is* **possible** *that Anne is* **not** *a boy,* **then** *it is* **necessary** *that it is* **not** *true that each natural number is greater than zero*

in the following two ways.

1. As a formula $A_1 \in \mathcal{F}_1$ of a language $\mathcal{L}_{\{\neg, \Box, \Diamond, \cap, \cup, \Rightarrow\}}$

Solution

Propositional Variables: a, b , where

a denotes statement: *each natural number is greater than zero*,

b denotes statement: *Anne is a boy*

Propositional Modal Connectives: $\Box$, $\Diamond$

$\Diamond$ denotes statement: **it is possible that**, $\Box$ denotes statement: **it is necessary that**

Translation The formula A_1 is

$$(a \Rightarrow (\neg\Diamond b \cup (\Diamond\neg b \Rightarrow \Box\neg a)))$$

2. As a formula $A_2 \in \mathcal{F}_2$ of a language $\mathcal{L}_{\{\neg, \cap, \cup, \Rightarrow\}}$

Solution

Propositional Variables: a, b , c, d where

a denotes statement: *each natural number is greater than zero*,

b denotes statement: *possible that Anne is a boy*

c denotes statement: *possible that Anne is not a boy*

d denotes statement: *necessary that it is not true that each natural number is greater than zero*

Formula A_2 is

$$(a \Rightarrow (\neg b \cup (c \Rightarrow d)))$$

Problem 2

Circle formulas that are propositional/ predicate **tautologies**

$\mathcal{S}_1 = \{(A \Rightarrow (A \cup B)),\ (((a \Rightarrow b) \cap (a \Rightarrow c)) \Rightarrow (a \Rightarrow b)),\ (A \cup \neg A),\ (A \cup (A \Rightarrow B)),\ (a \cup \neg b)\}$

$\mathcal{S}_2 = \{(\forall x\, A(x) \Rightarrow \exists x\, A(x)),\quad (\forall x\, P(x,y) \Rightarrow \exists x\, P(x,y)),\quad ((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x\, (A(x) \cap B(x)))$

Solutions

$$\not\models (a \cup \neg b), \qquad \not\models ((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x\, (A(x) \cap B(x)))$$

Problem 3

Given a formula $A = (\neg a \Rightarrow (\neg b \cup (b \Rightarrow \neg c)))$

and its **restricted model** $v_A : \{a, b, c\} \longrightarrow \{T, F\}$, $v_A(a) = T$, $v_A(b) = T$, $v_A(c) = F$

Extend v_A to the set of all propositional variables VAR to obtain 2 different, non restricted **models** for A

Solution

Model w_1 is a function

$w_1 : VAR \longrightarrow \{T, F\}$ such that

$w_1(a) = v_A(a) = T$, $w_1(b) = v_A(b) = T$,

$w_1(c) = v_A(c) = F$, and $w_1(x) = T$, for all $x \in VAR - \{a, b, c\}$

Model w_2 is defined by a formula

$w_2(a) = v_A(a) = T$, $w_2(b) = v_A(b) = T$,

$w_2(c) = v_A(c) = F$, and $w_2(x) = F$, for all $x \in VAR - \{a, b, c\}$

Problem 4

1. Give an example of an **infinite** set of formulas of $\mathcal{L}_{\{\neg, \cup\}}$, different from the set **T** of tautologies that **consistent**. JUSTIFY your answer.

Reminder: a set $\mathcal{G} \subseteq \mathcal{F}$ of **formulas** is called **consistent** if and only if $\mathcal{G}$ **has a model**

Solution

There plenty of examples; here is the simplest one: $\mathcal{G} = \mathcal{VAR}$

$v : VAR \rightarrow \{T, F\}$, such that $v(x) = T$ for all $x \in VAR$ is obviously a **model** for each formula in $\mathcal{G}$ and hence by definition is a **model** for $\mathcal{G}$.

MORE examples in chapter 3 and corresponding Lectures.

2. Give an example of an **infinite** set of formulas of $\mathcal{L}_{\{\neg, \cup\}}$, different from the set **C** of contradictions that is **inconsistent**

Reminder: a set $\mathcal{G} \subseteq \mathcal{F}$ is called **inconsistent** if and only if $\mathcal{G}$ **does not have a model**

Solution

There plenty of examples; here is the simplest one:

Let c be any variable, i.e. $c \in VAR$, we take

$$\mathcal{G} = VAR \cup \{c, \neg c\}$$

Obviously, the finite set $\{c, \neg c\}$ does not have a model, and hence the **infinite set** $VAR \cup \{c, \neg c\}$ **does not have a model** and hence, by definition is **inconsistent**.

MORE examples in chapter 3 and corresponding Lectures.

Reminder: we define H semantics operations $\cup$ and $\cap$ as $x \cup y = max\{x, y\}$, $x \cap y = min\{x, y\}$

the implication and negation are defined as

$$x \Rightarrow y = \begin{cases} T & \text{if } x \leq y \\ y & \text{otherwise} \end{cases}$$

$$\neg x = x \Rightarrow F$$

Problem 5

We know that $v : VAR \longrightarrow \{F, \perp, T\}$ is such that $v^*((a \cap b) \Rightarrow (a \Rightarrow c)) = \perp$ under H semantics.

Evaluate $v^*(((b \Rightarrow a) \Rightarrow (a \Rightarrow \neg c)) \cup (a \Rightarrow b))$.
You can use SHORTHAND notation.

Solution Look at Lecture 3b.