CSE/MAT371 QUIZ 1 SOLUTIONS Fall 2016

Problem 1 (3pts)

1. Write the **definition** of a **semantic** paradox.

Semantic paradoxes are are paradoxes concerning **the notion of truth**.

2. Give an example (by name) of a **logical** paradox

Here are 3 of them: Russel Paradox, 1902, Cantor Paradox, 1899, Burali-Forti Paradox, 1897.

3. Write the **definition** of a non-monotonic inference.

A non-monotonic reasoning is a reasoning in which the introduction of a new information can **invalidate** old facts.

Problem 2 (5pts)

Write the following natural language statement as a formula A of our propositional language $\mathcal{L}$.

Don't forget to write down which propositional variables denote which basic statements.

" **From the fact** that it is **not** necessary that a red flower is not a bird **we deduce that:**

red flower is a bird **or**, **if** it is necessary that the red flower is not a bird, **then** the bird flies."

Solution

Propositional Variables: a, b , c, where

a denotes statement: *it is necessary that a red flower is not a bird* ,

b denotes statement: *red flower is a bird*

c denotes statement: *the bird flies*

Translation The formula A_1 is: $(\neg a \Rightarrow (b \cup (a \Rightarrow c))$

Problem 3 (4pts)

Consider a following set of formulas

$$\mathcal{S} = \{((A \cap B) \Rightarrow A), \ (((a \Rightarrow b) \cap (a \Rightarrow c)) \Rightarrow (a \Rightarrow b)), \ (A \cup \neg A), \ (\neg A \Rightarrow (A \Rightarrow B))\}$$

Circle formulas that are propositional **tautologies**. Do NOT verify.

Solution: all formulas is $\mathcal{S}$ are propositional tautologies.

Problem 4 (8pts)

Given a mathematical statement **S** written with logical symbols

$$(\exists_{x \in N} \ x \leq 5 \ \cap \ \forall_{y \in Z} \ y = 0)$$

1. Translate **S** it into a proper logical formula that **uses** the restricted domain quantifiers.

2. Translate your restricted quantifiers formula into a correct formula **without** restricted domain quantifiers.

Write a **short** solution.

Solution

(2pts) The corresponding **atomic formulas** of $\mathcal{L}$ are:

$N(x)$, $L(x, c_1)$, $Z(y)$, $E(y, c_2)$, for $n \in N$, $x \leq 5$, $y \in Z$, $y = 0$, respectively.

(3pts)The statement **S** becomes **restricted quantifiers** formula

$\exists_{N(x)} L(x, c_1) \cap \forall_{Z(y)} E(y, c_2))$

(3pts) By the **transformation rules** we get $A \in \mathcal{F}$:

$(\exists x(N(x) \cap L(x, c_1)) \cap \forall y(Z(y) \Rightarrow E(y, c_2)))$.