CSE352 Q1 SOLUTIONS Spring 2022

15 pts + 5extra pts
NO partial Credits

CONCEPTUALIZATION DEFINITION Conceptualization is step one of formalization of knowledge in declarative form $\mathbf{C} = (\mathbf{U},\ \mathbf{F},\ \mathbf{P}, \mathbf{R})$, where $\mathbf{U}$ is a non empty finite set of objects called **universe** of discourse, $\mathbf{F}$ a finite set of functions defined on $\mathbf{U}$, $\mathbf{R}$ is a finite set of relations defined on $\mathbf{U}$.

QUESTION 1 (5pts)

Conceptualize the following situation

In a room there are 3 girls, 2 boys, and 2 cars one red and one blue.

The following properties must be true. **1.** *Each girl likes exactly one boy.* **2.** *Some boys like some girls.*

3. *Two boys like a red car.* **4.** *One girl likes a blue car.*

Use as the the universe a set **U = { o1, o2, o3, o4, o5, o6, o7}** and the set relations:

R = { GIRL, BOY, CAR, RCAR, BCAR, LIKE }. Use the **intended interpretation**

Folllow the steps below to write your solution.

0. (1pt) Define relations GIRL, BOY, CAR, RCAR, BCAR describing the property *In a room there are 3 girls, 2 boys, and 2 cars one red and one blue.*

Solution

These are MY definitions - you can have different sets of elements defining the relations.

GIRL = $\{o1, o2, o3\}$, BOY = $\{o4, o5\}$, CAR= $\{o6, o7\}$, RCAR= $\{o6\}$, BCAR= $\{o7\}$

Observe that RCAR $\subseteq$ CAR and BCAR $\subseteq$ CAR

1. (1pt) Define relation LIKE 1 that makes property *Each girl likes exactly one boy* TRUE

Solution

LIKE 1 = $\{(o1, o4),\ (o2, o4),\ (o3, o5)\}$

2. (1pt) Define relation LIKE 2 that makes property *Some boys like some girls* TRUE

Solution

LIKE 2 = $\{(o4, o1)\}$

3. (1pt) Define relation LIKE 3 that makes property *Two boys like a red car* TRUE

Solution

LIKE 3 = $\{(o4, o6),\ (o5, o6)\}$

4. (1pt) Define relation LIKE 4 that makes property *One girl likes a blue car* TRUE

Solution

LIKE 4 = $\{(o2, o7)\}$

Write LIKE = LIKE1 $\cup$ LIKE2 $\cup$ LIKE3 $\cup$ LIKE 4

Solution

LIKE = $\{(o1, o4),\ (o2, o4),\ (o3, o5),\ (o4, o1),\ (o4, o6),\ (o5, o6),\ (o2, o7)\}$

RULE BASED SYSTEMS

QUESTION 2 (5pts)

Here is a small set of RULES proposed for a simple rule-based system **S** for dealing with cars

R1 IF car is broken AND old AND income is small THEN repair it

R2 IF car is broken AND old AND do not repair AND income is medium THEN buy a used car

R3 IF car is broken AND is not old AND income is large THEN buy a new car

P1. (1pts) WRITE the rules **R1, R2, R3** of the system **S** in **propositional convention 1**, i.e. as rules

$$A_1 \cap A_2 \cap \cdots \cap A_n \Rightarrow C \quad \text{where } A_1, A_2, \ldots A_n, C \text{ are } \textbf{atomic} \text{ or } \textbf{negations} \text{ of atomic formulas}$$

Follow the steps below to write your solution.

1. (0.5 pt) Specify your choice of **atomic** formulas and **negations** of atomic formulas needed to represent the rules

Solution

ATOMIC FORMULAS are: A, B, C, D, E, F, G, H

A - represents " car is broken "

B - represents " car is old "

C - represents " income is small"

D - represents " repair car"

E - represents " income is medium "

F - represents "buy a used car "

G - represents "income is large "

H - represents "buy a new car "

Negations of ATOMIC FORMULAS are

$\neg B$ - represents " car is not old"

$\neg D$ - represents " do not repair"

2. (0.5 pt) Specify the rules

Solution

RULES are

R1 $A \cap B \cap C \Rightarrow D$

R2 $A \cap B \cap \neg D \cap E \Rightarrow F$

R3 $A \cap \neg B \cap G \Rightarrow H$

P2. (1pt) WRITE the rules **R1, R2, R3** in **propositional convention 2** as rules

$A_1 \cap A_2 \cap \cdots \cap A_n \Rightarrow C$ where $A_1, A_2, \ldots A_n, C$ are **atomic** formulas

Follow the steps below to write your solution.

1. (0.5 pt) Specify your choice of **atomic** formulas needed to represent the rules

Solution

ATOMIC FORMULAS are: A, B, C, D, E, F, G, H, J, K

A - represents " car is broken "

B - represents " car is old "

C - represents " income is small"

D - represents " repair car"

E - represents " income is medium "

F - represents "buy a used car "

G - represents "income is large "

H - represents "buy a new car "

J - represents " car is NOT old "

K - represents " do not repair car"

2. (0.5 pt) Specify the rules

Solution

RULES are

R1 $A \cap B \cap C \;\Rightarrow\; D$

R2 $A \cap B \cap K \cap E \Rightarrow F$

R3 $A \cap J \cap G \Rightarrow H$

P3. (3pts) Here is an expert system **S** with rules:

R1 IF car is broken AND old AND income is small THEN repair it

R2 IF car is broken AND old AND do not repair AND income is medium THEN buy a used car

R3 IF car is broken AND is not old AND income is large THEN buy a new car.

Follow the steps below to write the rules of **S** in the **predicate convention**: *attribute(x, attribute value).*

1. (1pt) DEFINE all needed ATTRIBUTES and their values.

USE the intended interpretation NAMES for the ATTRIBUTES

Solution

I use the intended interpretation names for ATTRIBUTES - you can use your own names

The ATTRIBUTES and their VALUES are:

CarBroken with values yes, no

CarOld with values yes, no

Income with values small, medium, large

CarRepair with values yes, no

CarBuy with values new, used

2. (1pt) WRITE the RULES

Solution

RULES ARE:

R1 $CarBroken(x, yes) \cap CarOld(x, yes) \cap Income(x, small) \Rightarrow CarRepair(x, yes)$

R2 $CarBroken(x, yes) \cap CarOld(x, yes) \cap CarRepair(x, no) \cap Income(x, medium) \Rightarrow CarBuy(x, used)$

R3 $CarBroken(x, yes) \cap CarOld(x, no) \cap Income(x, large) \Rightarrow CarBuy(x, new)$

3. (1pt) WRITE a database TABLE with your own example of any **4 records** describing some facts in **S**

Solution

There are my records, yours can be different!

Obj	CarBroken	CarOld	Income	CarRepair	CarBuy
o_1	yes	yes	small	no	new
o_2	no	no	large	yes	used
o_3	yes	yes	medium	no	new
0_4	yes	no	small	no	used

RESOLUTION PART

QUESTION 3 (5pts)

P1. (1pt) Given a set of clauses

$$\Delta = \{\{a, \neg b\}, \quad \{a, b, c\}, \quad \{\neg a, c\}, \ \{\neg c, \neg b\}\}$$

Write all possible **complementary pairs** and all their possible **resolvents**

Solution RESOLUTION HOMEWORK

P2. (1pt) Use Resolution Deduction to decide whether the set Δ of clauses is unsatisfiable or satisfiable.

$$\Delta = \{\{\ \neg a,\ b\ \}, \quad \{\neg b\}, \quad \{a,\ b\}\}$$

Solution

Consider Resolution Deduction as follows

1 $\Delta = \{\{ \neg a,\ b \}, \quad \{\neg b\}, \quad \{a,\ b\}\}$

2 $\{b\}$ Resolution application on $\{ \neg a,\ b \},\ \{a,\ b\}$

3 $\{\}$ Resolution application on $\{b\},\ \{\neg b\}$

Δ is UNSATISFIABLE

P3. (3pts) Use the **Tautology Deletion Strategy** to decide whether the set **C** is unsatisfiable or satisfiable.

$$\mathbf{C} = \{\{ \neg a,\ a,\ b,\ \neg c\}, \quad \{a,\ \neg b,\ c,\ b\}, \quad \{\neg a,\ b, c\}\}$$

Tautology Clause is any clause containing a PAIR of complementary literals. i.e containing any variable a and its negation $\neg a$. Let **C'** be obtained from **C** by **removing** all **tautology clauses** from **C**.

Tautology Deletion Strategy Theorem **C** is unsatisfiable (satisfiable) iff **C'** is unsatisfiable (satisfiable)

Solution

Given $\mathbf{C} = \{\{ \neg a,\ a,\ b,\ \neg c\}, \quad \{a,\ \neg b,\ c,\ b\}, \quad \{\neg a,\ b, c\}\}$.

We remove tautologies $\{ \neg a,\ a,\ b,\ \neg c\}$ snd $\{a,\ \neg b,\ c,\ b\}$ and we get an **satisfiable** set of clauses

$$\mathbf{C}' = \{\{\neg a,\ b, c\}\}$$

By Tautology Deletion Strategy Theorem the set of clauses **C** is also **satisfiable**.

QUESTION 4 Extra 5pts

Use the Propositional Resolution to prove that

$$\models \ (\neg(a \Rightarrow b) \Rightarrow (a \cap \neg b))$$

Write down and explain carefully each steps in the procedure.

Solution in Lecture 6 RESOLUTION page 58