

SOLUTIONS

PROBLEM 1. (10pts)

Translate to Logic following the statement

“Every man likes all tasty apples “

1. Domain: $X \neq \emptyset$

2. Predicates:

$A(x)$ – x is an Apple, $M(x)$ – x is a man,
 $T(x)$ – x is tasty. $L(x,y)$ – x likes y

4. Connectives: \wedge, \Rightarrow

5. Quantifiers:

$\forall_{M(x)}$ – “**Every** man”,

$\forall(A(y) \wedge T(y))$ – “**All** tasty apples” (**restricted**)

6. RESTRICTED FORMULA:

$\forall_{M(x)} \forall(A(y) \wedge T(y)) L(x,y)$

7. LOGIC FORMULA:

$(\forall_x (M(x) \Rightarrow \forall_y ((A(y) \wedge T(y)) \Rightarrow L(x,y))))$

Write your Logic formula as the Intended Interpretation Formula

$$(\forall_x (\text{Men}(x) \Rightarrow \forall_y ((\text{Apple}(y) \wedge \text{Tasty}(y)) \Rightarrow \text{Likes}(x,y))))$$

PROBLEM 2: RESOLUTION (10pts)

Q1. Find all resolvents of the set $\mathbf{CL} = \{ C1, C2 \}$ of clauses for

$$C1 = \{ a, b, c, \neg d \} \quad \text{and} \quad C2 = \{ \neg a, \neg b, d \}$$

It means locate all clauses in \mathbf{CL} that are Complementary Pairs and Resolve them

$$C1(a) \text{ and } C2(\neg a) \text{ resolves on } \{ b, c, \neg d, \neg b, d \}$$

$$C1(b) \text{ and } C2(\neg b) \text{ resolves on } \{ a, c, \neg d, \neg a, d \}$$

$$C1(\neg d) \text{ and } C2(d) \text{ resolves on } \{ a, b, c, \neg a, \neg b \}$$

Q2. Use the Resolution Completeness to prove that the set

$\mathbf{CL} = \{ \{ a, b \}, \{ \neg a, c \}, \{ \neg b, c \} \}$ **of clauses is SATISFIABLE.**

There are only two possible derivations

$$\begin{aligned} \mathbf{D1:} & \{ a, b \}, \{ \neg a, c \}, \{ \neg b, c \} \\ & \{ b, c \}, \{ \neg b, c \} \\ & \{ c \} \end{aligned}$$

$$\begin{aligned} \mathbf{D1:} & \{ a, b \}, \{ \neg a, c \}, \{ \neg b, c \} \\ & \{ a, c \}, \{ \neg a, c \} \\ & \{ c \} \end{aligned}$$

We can never get a derivation of $\{ \}$ - so by **Completeness** the set \mathbf{CL} is **satisfiable**

PROBLEM 3: RULE BASED SYSTEMS (10pts)

Here is a small set of RULES proposed for a simple **rule-based system** for financial advise.

R1 IF savings are not adequate THEN invest in savings

R2 IF savings are adequate AND income is adequate THEN invest in stock

R3 IF there is no children THEN savings are adequate

R4 IF there is a partner AND partner has a job THEN income is adequate

Q1. (5pts) Conceptualize the rules **R1-R4** in **propositional** convention that **admits negation**. Explain your solution.

We define.

- S = savings_adequate
- V = invest_savings
- I = income_adequate
- K = invest_stocks
- C = has_children
- P = has_partner
- J = partner_has_job

Here are the rules expressed in **propositional logic** conceptualization

- R1: $\neg S \rightarrow V$
- R2: $S \wedge I \rightarrow K$
- R3: $\neg C \rightarrow S$
- R4: $P \wedge J \rightarrow I$

Q2. (5pts)

1. Conceptualize the rules **R1-R4** in **predicate** convention using predicates attribute(x, value of attribute), attribute(object, value of attribute).

We have the following ATTRIBUTES:

Savings

Values: adequate, not adequate

Income

Values: adequate, not adequate

InvestStocks

Values: yes, no

InvestSavings

Values: yes, no

Children

Values: yes, no

Partner

Values: yes, no

PartnerJob

Values: yes, no

RULES:

R1: Savings(x, not adequate) \rightarrow InvestSavings(x, yes)

R2: Savings(x, adequate) \wedge Income(x, adequate) \rightarrow InvestStocks(x, yes)

R3: Children(x, no) \rightarrow Savings(x, adequate)

R4: Partner(x, yes) \wedge PartnerJob(x, yes) \rightarrow Income(x, adequate)

2. Write a format of a database TABLE needed for the conceptualization

DATA TABLE - example of a record

record	Savings	Income	Children	Partner	PartnerJob	InvestSavings	InvestStocks
o	not adequate	adequate	no	yes	no	yes	no

PROBLEM 4: Classification Rules 10pts

Q1 (5pts)

Given a dataset **DB**: **C** – class attribute

Record	a_1	a_2	a_3	a_4	C
o1	1	1	1	0	1
o2	2	1	2	0	2
o3	0	0	0	0	0
o4	0	0	2	1	0
o5	2	1	1	0	1

For the following formulas use the proper definitions to **prove** whether they are or they are not **discriminant or characteristic rules** in the dataset **DB**

1) $a_1 = 1 \ \& \ a_2 = 1 \Rightarrow C = 1$

{o1} is a subset of {o1, o5} so this is a **DISCRIMINANT** rule

2) $C = 1 \Rightarrow a_1 = 0 \ \& \ a_2 = 1 \ \wedge \ a_3 = 1$

{o: $a_1 = 0 \ \& \ a_2 = 1 \ \& \ a_3 = 1$ } is an empty set so this is **not** a **CHARACTERISTIC** rule

3) $a_1 = 1 \Rightarrow C = 1$

$\{o1\}$ is a subset of $\{o1, o5\}$ so this is a DISCRIMINANT rule

4) $C = 1 \Rightarrow a_1 = 1$

$\{o1, o5\}$ intersection with $\{o1\}$ is non-empty

5) $a_1 = 2 \ \& \ a_2 = 1 \ \& \ a_3 = 1 \Rightarrow C = 2$

$\{o5\}$ is not a subset of $\{o2\}$, so this is not a DISCRIMINANT rule

Q2 (5pts)

1. Prove that in any classification DB the **inverse implication** to the discriminant rule is a characteristic rule

By definition, for any database DB :

DESCRIPTION \Rightarrow CLASS

is a discriminant rule **iff**

1. $\{o: \text{DESCRIPTION}\}$ is not empty
2. $\{o: \text{DESCRIPTION}\}$ is included in $\{o: \text{CLASS}\}$

We know that for any non-empty sets A, B, if A is included in B, then their intersection is non-empty.

Hence

$\{o: \text{DESCRIPTION}\}$ intersection with $\{o: \text{CLASS}\}$ is not empty and by Definition then inverse implication

CLASS \Rightarrow DESCRIPTION

is a CHARACTERISITIC RULE

SHORT QUESTIONS (10pts)

Q1: (5pts) Define a Classifier

A classifier is a final product of a process that uses data set and a classification algorithm

The classifier is build i.e. we terminate the process if it has been trained and tested and the predictive accuracy is on an acceptable level

Q2: (5pts) Write down **termination conditions** for Decision Tree Model

1. All records (samples) for the given node belong to the same class
2. There are no remaining attributes on which the samples (records in the data table) may be further partitioned – a LEAF is created with majority vote for training sample
3. There is no records (samples) left – a LEAF is created with **majority vote for training sample**

Majority voting involves converting node N into a leaf and labeling it with the most common class in D which is a set of training tuples and their associated class labels

POBLEM 5: Classification by Decision Tree Algorithm (20pts)
 Given the following DATA

TRAIN

Record	a_1	a_2	C
o_1	1	1	1
o_2	0	0	0
o_3	0	1	0
o_4	0	0	0
o_5	1	1	1
o_6	1	1	0
o_7	0	0	0
o_8	1	0	1

TEST

Record	a_1	a_2	C
o_1	1	1	1
o_2	1	0	0
o_3	0	0	1
o_4	0	0	0

Use the TRAIN data to **build a CLASSIFIER** using the basic **Decision Tree** Algorithm with **a_1** as the ROOT. **Here are the STEPS you must follow**

STEP 1: (5pts) Build the Decision Tree and write Rules in Predicate Form

STEP 2: (10pts)

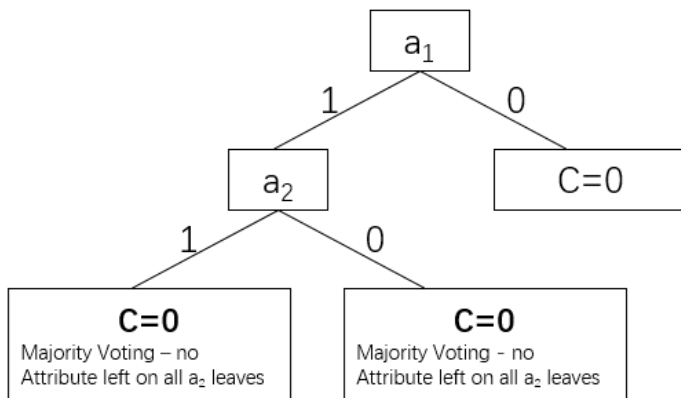
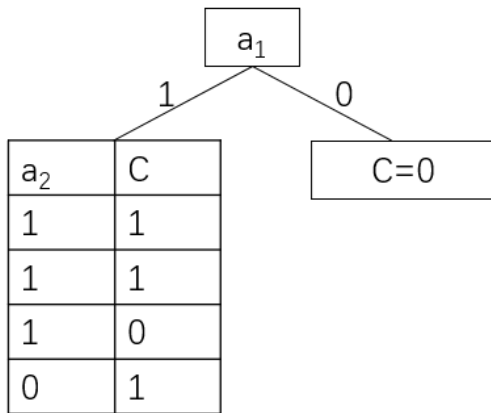
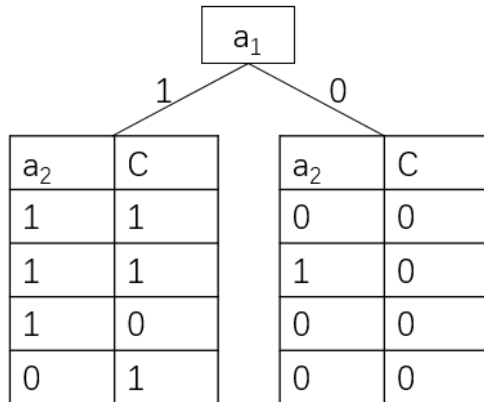
Evaluate: (i) **rules accuracy**, (ii) **predictive accuracy**.

(iii) Write down a **TEST** data that would give a 100% predictive accuracy for your set of rules.

STEP 3: (5pts) Give your answer under a title: **MY CLASSIFIER IS**

Solutions

STEP 1: Build the Decision Tree and write Rules in Predicate Form



$$R1: a1(x,1) \wedge a2(x,1) \rightarrow C(x,0)$$

$$R2: a1(x,1) \wedge a2(x,0) \rightarrow C(x,0)$$

$$R3: a1(x,0) \rightarrow C(x,0)$$

STEP 2:

Evaluate: (i) **rules accuracy**,

o_1 is misclassified

o_2 is classified by R3

o_3 is classified by R3

o_4 is classified by R3

o_5 is misclassified

o_6 is classified by R1

o_7 is classified by R3

o_8 is misclassified

5 Passed/out of 8=62.5% rule accuracy

(ii) **predictive accuracy**.

o_1 is misclassified

o_2 is classified by R2

o_3 is misclassified

o_4 is classified by R3

2 Passed/out of 4=50% predictive accuracy

(iii) Write down a **TEST** data that would give a 100% predictive accuracy for your set of rules.

Record	a_1	a_2	C
o_1	1	1	0
o_2	1	0	0
o_3	0	1	0
o_4	0	0	0

STEP 3: Give your answer under a title: **MY CLASSIFIER IS**

(Answer 1): There is no classifier because predictive accuracy is only 50% and we do not accept the rules.

(Answer 2): I decide to accept predictive accuracy of 50%. **MY CLASSIFIER IS**
R1: $a1(x,1) \wedge a2(x,1) \rightarrow C(x,0)$, R2: $a1(x,1) \wedge a2(x,0) \rightarrow C(x,0)$, R3: $a1(x,0) \rightarrow C(x,0)$