CSE 352 – Artificial Intelligence

RESOLUTION HOMEWORK SOLUTIONS

1. Find all possible resolvents of

A) $\Delta = \{ \{a, \neg b\}, \{a, b, c\}, \{\neg a, c\}, \{\neg c, \neg b\} \}$

C_1 = { a, ~b }
C_2 = { a, b, c }
C_3 = { ~a, c }
C_4 = { ~c, ~b }
CL = { C_1, C_2, C_3, C_4 }

P1: C_1(a), C_3(~a)

{ a, ~b } : { ~a, c }
(C_1-{a} U C_3-{~a}) Resolvent: { ~b, c }

P2: C_2(a), C_3(~a)

{ a, b, c }: { ~a, c }
(C_2-{a} U C_3-{~a}) Resolvent: { b, c }

P3: C_2(b), C_1(~b)

{ a, b, c }: { a, ~b }
(C_2-{b} U C_1-{~b})   Resolvent: { a, c }

P4: C_2(b), C_4(~b)

{ a, b, c }: { ~c, ~b }
(C_2-{b} U C_4-{~b})   Resolvent: { a, c, ~c }

P5: C_2(c), C_4(~c)

{ a, b, c }: { ~c, ~b }
(C_2-{c} U C_4-{~c})   Resolvent: { a, b, ~b }

P6: C_3(c), C_4(~c)

{ ~a, c }: { ~c, ~b }
(C_3-{c} U C_4-{~c})   Resolvent: { ~a, ~b }

B) Δ = { {a, ¬a, ¬b}, {a, b, c}, {¬a, ¬b, ¬c}, {b} }

C_1 = { a, ~a, ~b }
C_2 = { a, b, c }
C_3 = { ~a, ~b, ~c }
C_4 = { b }
Δ = { C_1, C_2, C_3, C_4 }

P1: C_1(a), C_3(~a)

{ a, ~a, ~b }: { ~a, ~b, ~c }
(C_1-{a} U C_3-{~a}) Resolvent: { ~a, ~b, ~c }

P2: C_2(a), C_1(~a)

{ a, b, c }: { a, ~a, ~b }
(C_2-{a} U C_1-{~a}) Resolvent: { b, c, a, ~b }

P3: C_2(a), C_3(~a)

{ a, b, c }: {~a, ~b, ~c }
(C_2-{a} U C_3-{~a}) Resolvent: { b, c, ~b, ~c }

P4: C_2(b), C_1(~b)

{ a, b, c }: { a, ~a, ~b }
(C_2-{b} U C_1-{~b}) Resolvent: { a, c, ~a }

P5: C_2(b), C_3(~b)

{ a, b, c } : {~a, ~b, ~c }
(C_2-{b} U C_3-{~b}) Resolvent: { a, c, ~a, ~c }

P6: C_2(c), C_3(~c)

{ a, b, c } : {~a, ~b, ~c }
(C_2-{c} U C_3-{~c}) Resolvent: { a, b, ~a, ~b }

P7: C_4(b), C_1(~b)

{ b } : { a, ~a, ~b }
(C_4-{b} U C_1-{~b}) Resolvent: { a, ~a }

P8: C_4(b), C_3(~b)

{ b } : {~a, ~b, ~c }
(C_4-{b} U C_3-{~b}) Resolvent: { ~a, ~c }

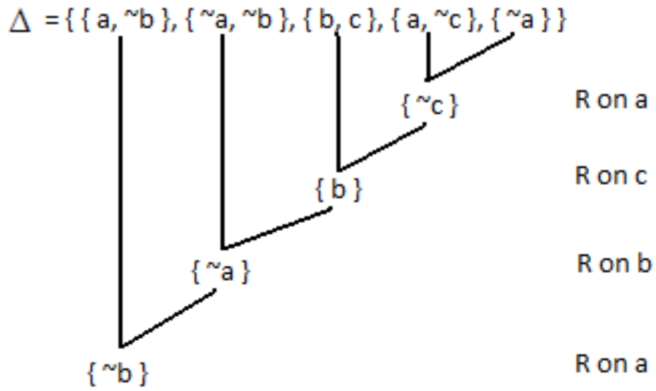
2. Use a proper Resolution strategies do decide whether Δ is unsatisfiable or satisfiable.

A) Δ = { {a, ¬b}, {¬a, ¬b }, {b, c}, {a, ¬c}, {¬a} }

Pure literals: none
Tautologies: none

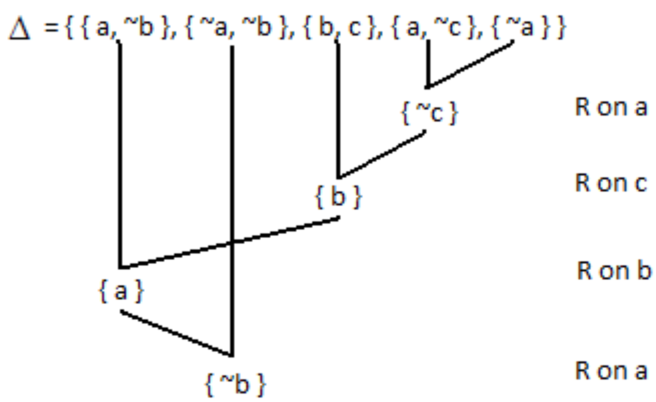
Some Resolution Deduction

Derivation 1:



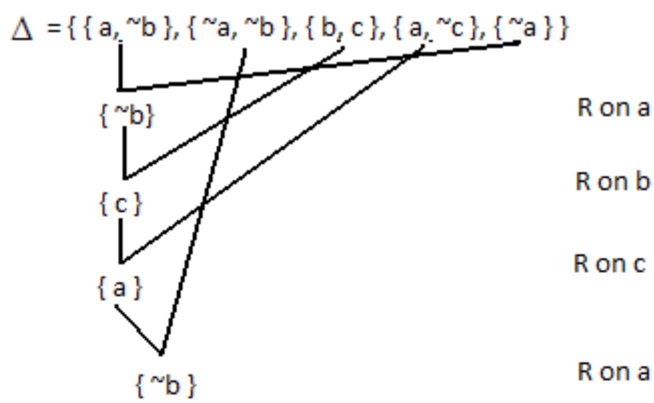
$\Delta \mid\!-_{R} \{\sim b\}$

Derivation 2:



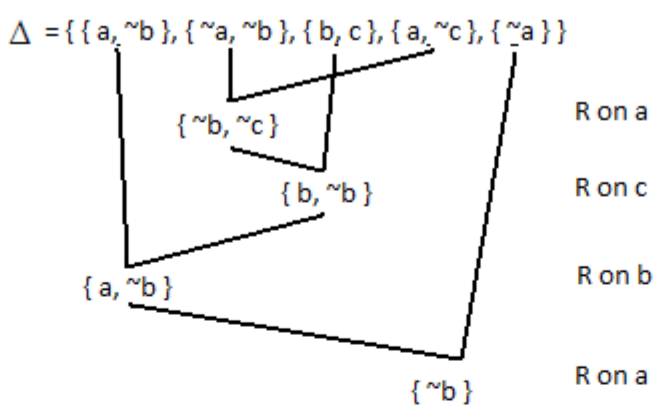
$\Delta \mid\!-_{R} \{\sim b\}$

Derivation 3:



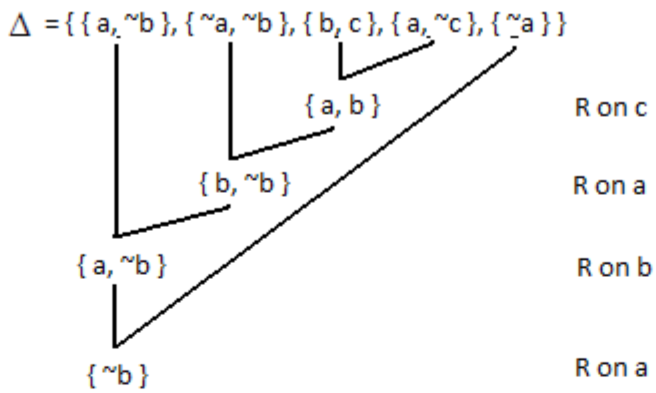
$\Delta \; |\!-_R \{ \sim b \}$

Derivation 4:



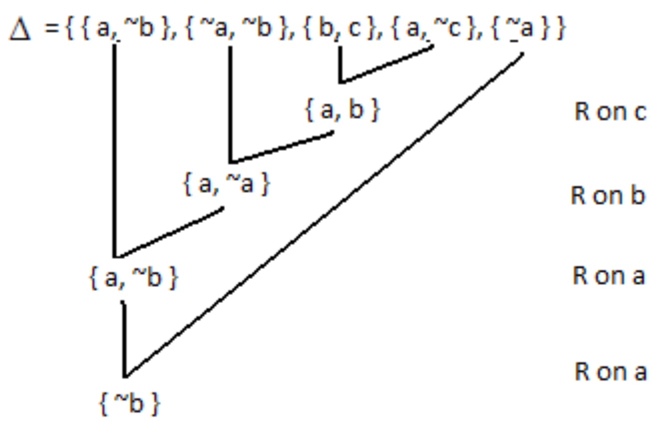
$\Delta \; |\!-_R \{ \sim b \}$

Derivation 5:



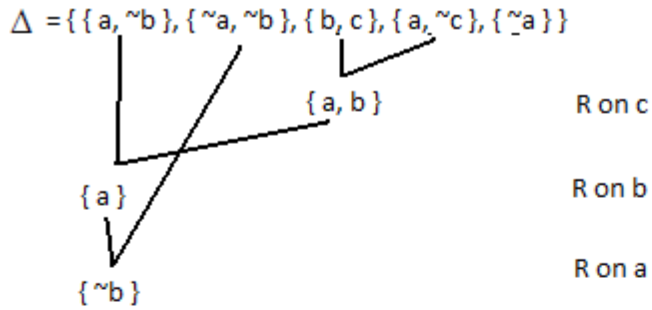
$\Delta \mid\!-_R \{\sim b\}$

Derivation 6:



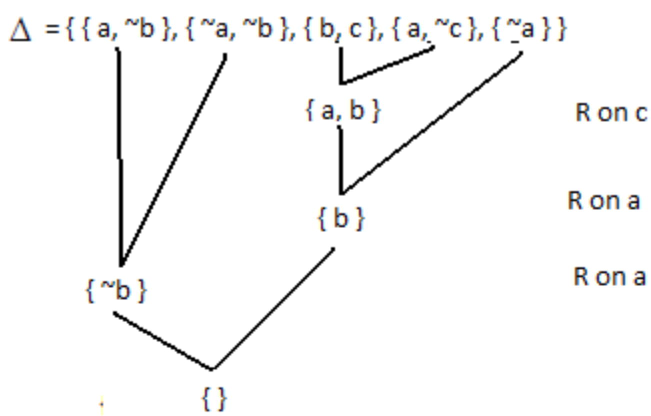
$\Delta \mid\!-_R \{\sim b\}$

Derivation 7:



Δ $\vdash_R$ { ~b }

Derivation 8:



Δ $\vdash_R$ { }

Δ is UNSATISFIABLE!

B) Δ = { {a, ¬b}, {¬a, a, ¬b }, {b, ¬b, c}, {a, c, ¬c} }

Pure literals: none

Tautologies:

{¬a, a, ¬ }, {b, ¬b, c}, {a, c, ¬c}

Delete Tautologies Clauses

Δ' = { {a, ¬b}} SATISFIABLE - so Δ is satisfiable

C) Δ = { {a, b}, {b, ¬c}, {a, ¬c} }

Pure literals: a, b, ¬c

Tautologies: none

Delete clauses with pure literals

Δ' is EMPTY SET of clauses - SATISFIABLE - so Δ is satisfiable

3. Use resolution (proper strategy) to decide VALIDITY of the following argument:

A1: (((a → ¬b) → (b → ¬a)) \/ ((a → ¬c) \/ b))

A2: ((a → b) → a)

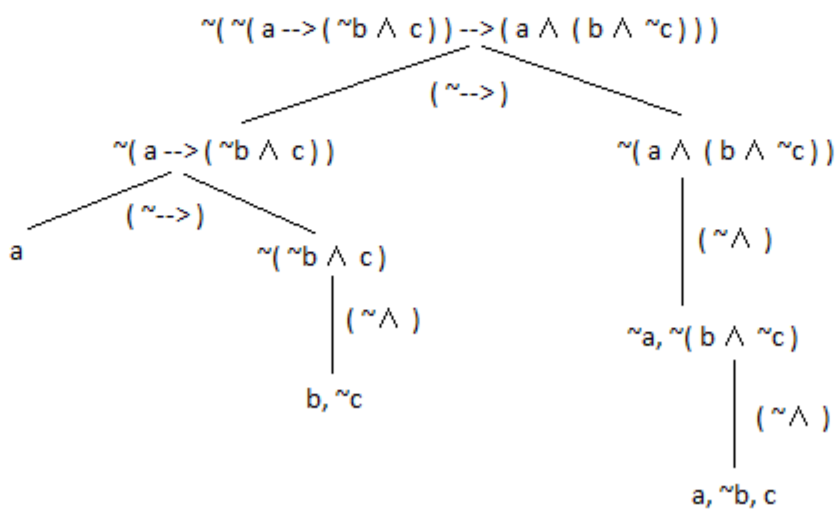
B: (¬(a → (¬b /\ c)) → (a /\ (b /\ ¬c)))

Remark: To verify the VALIDITY of

((A1 /\ A2) → B), we have to test if

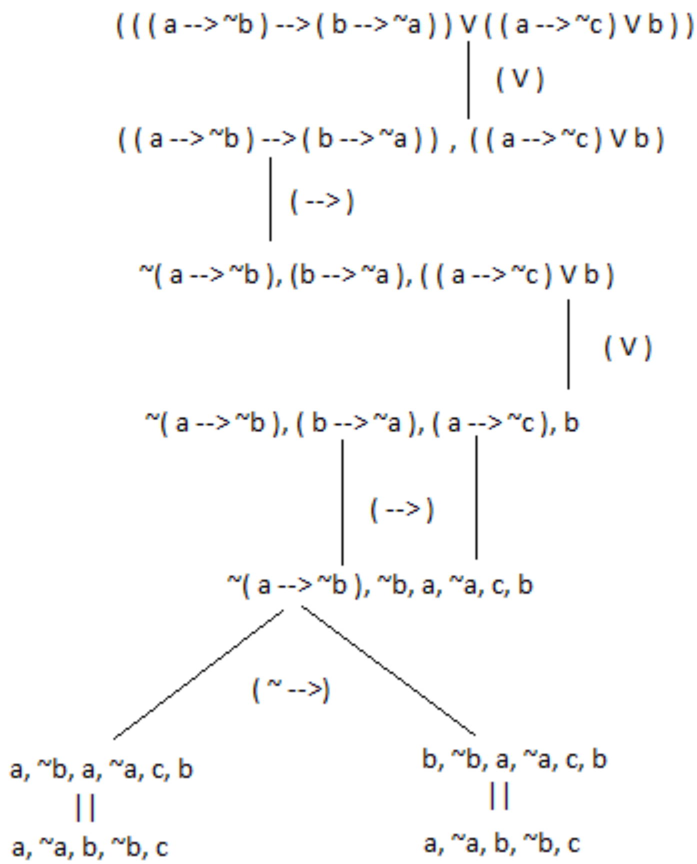
¬ ((A1 /\ A2) → B) is unsatisfiable.

We find the set $\Delta_{\sim B}$ of clauses logically equivalent to ~B:



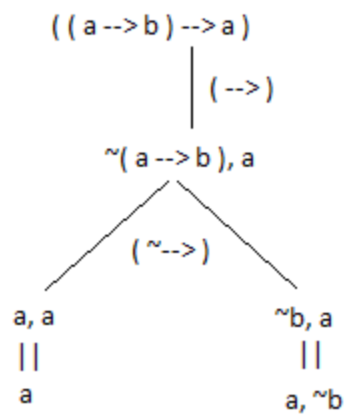
$\Delta_{\sim B}$ = { {a}, {b, ~c}, {a, ~b, c} }

We find the set Δ_{A1} of clauses logically equivalent to A1:



Δ_{A1} = { {a, ~a, b, ~b, c} }

We find the set Δ_{A2} of clauses logically equivalent to A2



Δ_{A2} = { {a}, {a, ~b} }

We write the set of clauses logically equivalent to the argument

$\Delta = \Delta_{A1} \vee \Delta_{A2} \vee \Delta_{\sim B}$ = { {a}, {b, ~c}, {a, ~b, c}, {a, ~a, b, ~b, c}, {a, ~b} }

Tautologies: {a, ~a, b, ~b, c}

Δ' = { {a}, {b, ~c}, {a, ~b, c}, {a, ~b} }

Pure literals: a

Δ'' = { {b, ~c} }

Resolution Deduction:

Cannot resolve any further.

Δ'' is SATISFIABLE

Argument is NOT VALID