Multi-Dimensional Association
Classification by Association

Cse352

ARTIFICIAL INTELLIGENCE

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Mining Multi-Dimensional Association

- Single-dimensional rules:

  buys(X, “milk”) ⇒ buys(X, “bread”)

- Multi-dimensional rules: ≥ 2 dimensions or predicates
  Inter-dimension assoc. rules (*no repeated predicates*)

age(X,”19-25”) ∧ occupation(X,“student”) ⇒ buys(X, “coke”)

Hybrid-dimension assoc. rules (*repeated predicates*)

age(X,”19-25”) ∧ buys(X, “popcorn”) ⇒ buys(X, “coke”)

Example: Relational Data

Goal:

create multidimensional association rules

Student	Grade	Income	Buys
CS	High	Low	Milk
CS	High	High	Bread
Math	Low	Low	Bread
CS	Medium	High	Milk
Math	Low	Low	Bread

STEP 1: Data Conversion to Transaction and its count

Converted Data

Student = CS (I1)	Student =math (I2)	Grade = high (I3)	Grade =medium (I4)	Grade =low (I5)	Income =high (I6)	Income =low (I7)	Buys =milk (I8)	Buys =bread (I9)
+	-	+	-	-	-	+	+	-
+	-	+	-	-	+	-	-	+
-	+	-	-	+	-	+	-	+
+	-	-	+	-	+	-	+	-
-	+	-	-	+	-	+	-	+
3	2	2	1	2	2	3	2	3

Step 2: Apriori Algorithm

Generating 1-itemset Frequent Pattern

Scan D for support count of each candidate →

Item Set	Support Count
{I1}	3
{I2}	2
{I3}	2
{I4}	1
{I5}	2
{I6}	2
{I7}	3
{I8}	2
{I9}	3

C1

choose (no need for **pruning** here)candidates with support count >=minimum support count →

Item Set	Support Count
{I1}	3
{I2}	2
{I3}	2
{I5}	2
{I6,}	2
{I7}	3
{I8}	2
{I9}	3

L1

Let, the **minimum support count be 2**

Since we have 5 records => **minimum Support** = 2/5 = **40%**

Let, **minimum confidence** required is **70%**

Generating 2-itemset Frequent Pattern

Generate C2 **candidates** from L1

Item Set
{I1,I2}
{I1,I3}
{I1,I4}
{I1,I5}
{I1,I6}
{I1,I7}
{I1,I8}
{I1,I9}
{I2,I3}
{I2,I4}
{I2,I5}
{I2,I6}
{I2,I7}
{I2,I8}
{I2,I9}
{I3,I4}
{I3,I5}
{I3,I6}
{I3,I7}
{I3,I8}
{I3,I9}
{I4,I5}
{I4,I6}
{I4,I7}
{I4,I8}
{I4,I9}
{I5,I6}
{I5,I7}
{I5,I8}
{I5,I9}
{I6,I7}
{I6,I8}
{I6,I9}
{I7,I8}
{I7,I9}
{I8,I9}

C2

No need of **pruning** here-Scan D for count of each **candidate**

Item Set	Support Count
{I1,I2}	0
{I1,I3}	2
{I1,I4}	1
{I1,I5}	0
{I1,I6}	2
{I1,I7}	1
{I1,I8}	2
{I1,I9}	1
{I2,I3}	0
{I2,I4}	0
{I2,I5}	2
{I2,I6}	0
{I2,I7}	2
{I2,I8}	0
{I2,I9}	2
{I3,I4}	0
{I3,I5}	0
{I3,I6}	1
{I3,I7}	1
{I3,I8}	1
{I3,I9}	1
{I4,I5}	0
{I4,I6}	1
{I4,I7}	0
{I4,I8}	1
{I4,I9}	0
{I5,I6}	0
{I5,I7}	2
{I5,I8}	0
{I5,I9}	2
{I6,I7}	0
{I6,I8}	1
{I6,I9}	0
{I7,I8}	1
{I7,I9}	2
{I8,I9}	0

C2

choose **candidates** with support count >= **minimum support count**

Item Set	Support Count
{I1,I3}	2
{I1,I6}	2
{I1,I8}	2
{I2,I5}	2
{I2,I7}	2
{I2,I9}	2
{I5,I7}	2
{I5,I9}	2
{I7,I9}	2

L2

Generating Candidates: C_k

- Join Step: C_k is generated by **joining** L_{k-1} with itself

- Prune Step: Any (k-1)-item set that is not frequent **cannot** be a subset of a frequent k-item set

Example: Joining and Pruning

1. **The join step:** To find C_k, a set of candidate k-itemsets is generated by **joining** L_{k-1} with **itself**.

L_k **– Itemsets** C_k **– Candidates**

For example in our case:

Considering {I2,I5} , {I7,I9} from **L2** to arrive at **C3** we **Join L2*L2**

and we obtain for example {I2,I5,I7}, {I2,I5,I9} as resultant **candidates** in **C3** generated from **L2**

Considering {I1,I3} , {I1,I6} from **L2** we generate a **candidate** {I1,I3,I6} in **C3**

Example: Joining and Pruning

2. The prune step:

C_k is a superset of L_k, that is, its members may or may not be frequent

Ck however, **can be huge** and we **prune it** applying Apriori Principle

"if A is a frequent item set, then each of its subsets is a frequent item set"

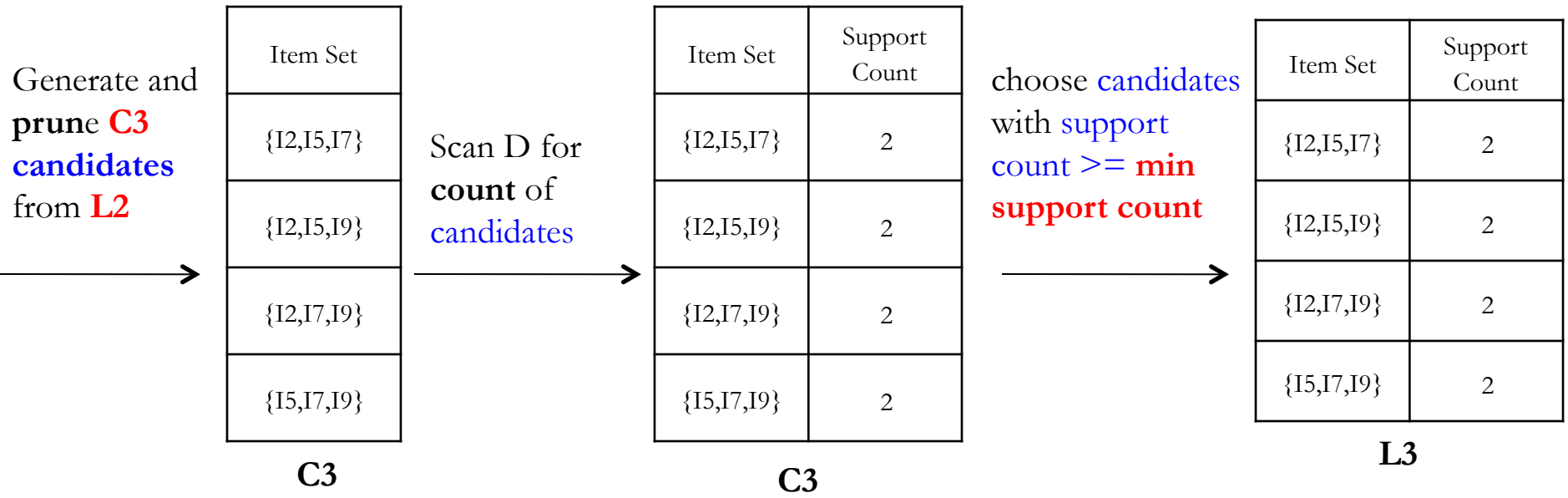
It is expressed by formulation of the

Prune Step: Any (k-1)-item set that is **not frequent cannot** be a subset of a frequent k-item set

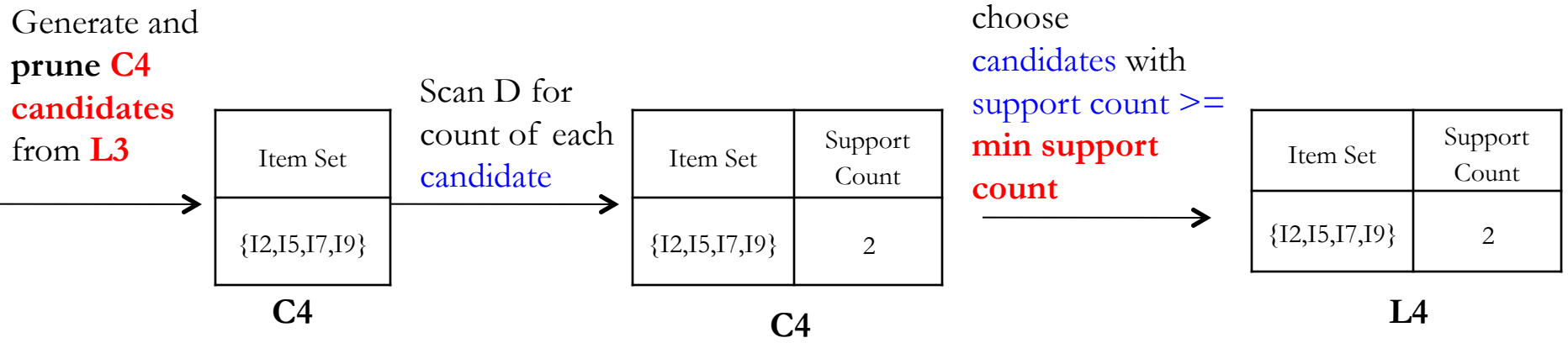
Thus, {I2,I5,I7}, {I2,I5,I9} **from join step** are considered since all their subsets are frequent

but {I1,I3,I6} is **discarded** since it subset {I3,I6} is **not frequent**, i.e. was not in **L2**

Generating 3-itemset Frequent Pattern



Generating 4-itemset Frequent Pattern



Generating Multidimensional Association Rules

Let **minimum confidence** required be **70%**

- **For example**, let's consider 4-item frequent set
- **I={I2,I5,I7,I9}**
- Its **nonempty subsets** needed to create **rules**
- (we write {2} instead of {I2} .. etc) are:
- **{2}, {5}, {7}, {9},**
- **{2,5}, {2,7}, {2,9}, {5,7}, {5,9}, {7,9},**
- **{2,5,7}, {2,5,9}, {2,7,9}, {5,7,9}**

We create **for example** some **association rules** as follows

R1 : 2 ^5 ^ 7 → 9 **R2 : 2 ^5 ^ 9 → 7** **R3 : 5 ^ 7 → 2^9**

Multidimensional Association Rules

- R1 : **2 ^5 ^ 7 → 9**

student(x, math) ∧ grade(X, low) ∧ income(x, low)

⇒ buys(X, bread)

- R2 : **2 ^5 ^ 9 → 7**

student(x, math) ∧ grade(X, low) ∧ buys(X, bread)

⇒ income(x, low)

- R3 : **5 ^ 7 → 2^9**

grade(X, low) ∧ income(x, low) ⇒ student(x, math) ∧ buys(X, bread)

Example: Classification Data

Classification by Association

Student	Grade	Income	Buys
CS	High	Low	Milk
CS	High	High	Bread
Math	Low	Low	Bread
CS	Medium	High	Milk
Math	Low	Low	Bread

Converted Data

Student = CS (I1)	Student =math (I2)	Grade = high (I3)	Grade =medium (I4)	Grade =low (I5)	Income =high (I6)	Income =low (I7)	Buys =milk (I8)	Buys =bread (I9)
+	-	+	-	-	-	+	+	-
+	-	+	-	-	+	-	-	+
-	+	-	-	+	-	+	-	+
+	-	-	+	-	+	-	+	-
-	+	-	-	+	-	+	-	+
3	2	2	1	2	2	3	2	3

Generating **Classification Rules** by **Association**

When mining **association rules** for use in **classification** we are **only interested** in **association rules** of the form

$i1 \;\&\; i2 \;\&\; \ldots \;\&\; ik \rightarrow ic$

where ic is an item associated with a **class label c**

- The process of finding such rules is called
- **Classification by Association**

Classification by Association

- When generating **classification by association rules**
- we are **only interested** in **association rules** of the form

- **(p1 ^ p2 ^ . . . ^ pl) → class = C**

- where the rule antecedent is a conjunction of items
- p1, p2, : : : , pl **associated** with a **class label C**

- In our **example class is** either **I8** or **I9**
- as we want to **predict** whether a **student with given characteristics buys Milk** or **buys Bread**

Generating **Classification Rules** by Association

Let **minimum confidence** required be **70%**

We run **Appriori Algorithm** as before and

- **For example**, let's consider 4-item frequent set
- **I={I2,I5,I7,I9}** where **I9** represents **buys-Bread**
- Its **nonempty subsets** needed to create **association rules**
- (we write {2} instead of {I2} .. etc) are:
- **{2}, {5}, {7}, {9},**
- **{2,5}, {2,7}, {2,9}, {5,7}, {5,9}, {7,9},**
- **{2,5,7}, {2,5,9}, {2,7,9}, {5,7,9}**
- To create **classification rules** we consider **only subsets** that contain the **class item 9**

Generating **Classification Rules** by Association

Consider 3- itemset Frequent Sets {2,5,9}, {2,7,9}, {5,7,9}

We create **classification** by association rules as follows

R1 : 5 ^ 7 → 9 [40%,100%]

- **Confidence** = sc{I5,I7,I9}/ sc{I5,I7} = 2/2 = 100%
- **R2** is **selected**
- **R3 : 2 ^ 7 → 9** [40%,100%]
- **Confidence** = sc{I2,I7,I9}/ sc{I2,I7} = 2/2 = 100%
- **R3** is **selected**
- **R4 : 2 ^ 5 → 9** [40%,100%]
- **Confidence** = sc{I2,I7,I9}/ sc{I2,I7} = 2/2 = 100%
- **R4** is **selected**

Generating Classification by Association Rules

Consider 2- itemset Frequent Sets {2,9}, {5,9}, {7,9},
and **{1,8}** from **L2**
We create **classification by association rules** as follows

R5 : **5 → 9** [40%,100%]

- **Confidence** = sc{I5,I9}/ sc{I9} = 2/2 = 100%
- **R5** is **Selected**

R6 : **2 → 9** [40%,100%]

- **Confidence** = sc{I2,I9}/ sc{I9} = 2/2 = 100%
- **R6** is **Selected**

R7 : **7 → 9** [40%,100%]

- **Confidence** = sc{I7,I9}/ sc{I9} = 2/2 = 100%
- **R7** is **Selected**

R8 : **1 → 8** [40%, 66%]

- **Confidence** = sc{I1,I8}/ sc{I1} = 2/3 = 66.66%
- **R8** is **Rejected**

List of Selected **Classification by Association** Rules

- **2 ^ 5 ^ 7 → 9** [40%,100%]
- **2 ^ 5 → 9** [40%,100%]
- **2 ^ 7 → 9** [40%,100%]
- **5 ^ 7 → 9** [40%,100%]
- **5 → 9** [40%,100%]
- **7 → 9** [40%,100%]
- **2 → 9** [40%,100%]

- We reduce the **confidence** to **66%** to include **I8**
- **1 → 8** [40%,66%]

Test Data

Student	Grade	Income	Buys
Math	Low	Low	Bread
CS	Low	Low	Milk
Math	Low	Low	Milk
Math	Low	Low	Bread
CS	Medium	High	Bread

- **First Tuple**

  is **correctly classified by the rule**

  I2 & I5 & I7 → I9

  Student=math & grade=low & income=low → buys=bread **[Success]**

- **Second Tuple:**

  There is no rule for class I8: buys=bredI8 [Error]

- **Third Tuple:**

  There is no rule for class I8: buys=bredI8 [Error]

Test Data

Student	Grade	Income	Buys
Math	Low	Low	Bread
CS	Low	Low	Milk
Math	Low	Low	Milk
Math	High	Low	Bread
CS	Medium	High	Bread

- **FourthTuple**

is correctly classify by the rule **I2 ^ I7 → I9** **[Success]**

•Student=Math & Income=Low → Buys=Bread

- **Fifth Tuple**

is correctly classify by the rule **I1 → I9** **[Success]**

Student=CS → Buys=Bread

Hence we have **80% predictive accuracy**

And **20% Error rate**