CONCEPTUALIZATION DEFINITION  Conceptualization is step one of formalization of knowledge in declarative form $C = (U, F, P, R)$, where $U$ is a non empty finite set of objects called universe of discourse, $F$ a finite set of functions defined on $U$, $R$ is a finite set of relations defined on $U$.

QUESTION 1

Conceptualize the following situation

In a room there are 3 girls, 2 boys, and 2 cars one red and one blue.

The following properties must be true.

1. Each girl likes exactly one boy.
2. Some boys like some girls.
3. Two boys like a red car.
4. One girl likes a blue car.

Use as the universe a set $U = \{ o_1, o_2, o_3, o_4, o_5, o_6, o_7 \}$

Use as the relations: $R = \{ GIRL, BOY, CAR, RCAR, BCAR, LIKE \}$

Use the intended interpretation

SOLUTION

These are MY definitions- you can have different sets of elements defining relations. relations

$GIRL = \{ o_1, o_2, o_3 \}$, $BOY = \{ o_4, o_5 \}$, $CAR = \{ o_6, o_7 \}$, $RCAR = \{ o_6 \}$, $BCAR = \{ o_7 \}$

Observe that $RCAR \subseteq CAR$ and $BCAR \subseteq CAR$

$LIKE = LIKE_1 \cup LIKE_2 \cup LIKE_3 \cup LIKE_4$, where

LIKE 1 makes Each girl likes exactly one boy TRUE and is defined as
LIKE 1 = \{(o1, o4), (o2, o4), (o3, o5)\}

LIKE 2 makes Some boys like some girls TRUE and is defined as
LIKE 2 = \{(o4, o1)\}

LIKE 3 makes Two boys like a red car TRUE and is defined as
LIKE 3 = \{(o4, o6), (o5, o6)\}

LIKE 4 makes One girl likes a blue car TRUE and is defined as
LIKE 4 = \{(o2, o7)\}

QUESTION 2

Here is a small set of RULES proposed for a simple rule-based system $S$ for dealing with cars

R1 IF car is broken AND old AND income is small THEN repair it
R2 IF car is broken AND old AND do not repair AND income is medium THEN buy a used car
R3 IF car is broken AND is not old AND income is large THEN buy a new car
P1. WRITE the rules R1, R2, R3 of the system S in propositional convention 1, i.e. as rules

\[ A_1 \cap A_2 \cap \cdots \cap A_n \Rightarrow C \] 
where \( A_1, A_2, \ldots, A_n, C \) are atomic formulas or negations of atomic formulas

Solution

ATOMIC FORMULAS are: A, B, C, D, E, F, G, H

A - represents "car is broken"
B - represents "car is old"
C - represents "income is small"
D - represents "repair car"
E - represents "income is medium"
F - represents "buy a used car"
G - represents "income is large"
H - represents "buy a new car"

Negations of ATOMIC FORMULAS are

\( \neg B \) - represents "car is not old"
\( \neg D \) - represents "do not repair"

RULES are

R1 \[ A \cap B \cap C \Rightarrow D \]
R2 \[ A \cap B \cap \neg D \cap E \Rightarrow F \]
R3 \[ A \cap \neg B \cap G \Rightarrow H \]

P2. WRITE the rules R1, R2, R3 in propositional convention 2 as rules

\[ A_1 \cap A_2 \cap \cdots \cap A_n \Rightarrow C \] 
where \( A_1, A_2, \ldots, A_n, C \) are atomic formulas

ATOMIC FORMULAS are: A, B, C, D, E, F, G, H, J, K

A - represents "car is broken"
B - represents "car is old"
C - represents "income is small"
D - represents "repair car"
E - represents "income is medium"
F - represents "buy a used car"
G - represents "income is large"
H - represents "buy a new car"
J - represents " car is NOT old "  
K - represents " do not repair car"

RULES are

R1  \( A \cap B \cap C \Rightarrow D \)  
R2  \( A \cap B \cap K \cap E \Rightarrow F \)  
R3  \( A \cap J \cap G \Rightarrow H \)  

QUESTION 3

WRITE the the system 8 rules:

R1  IF car is broken AND old AND income is small THEN repair it  
R2  IF car is broken AND old AND do not repair AND income is medium THEN buy a used car  
R3  IF car is broken AND is not old AND income is large THEN buy a new car  

in the PREDICATE convention using predicates

\[
\text{attribute}(x, \text{value of attribute}), \quad \text{attribute}(\text{object, value of attribute}) 
\]

WRITE a database TABLE with an example of any 4 records needed for solution in this case.

Solution

I use the intended interpretation names for ATTRIBUTES - you can use your own names

The ATTRIBUTES and their VALUES are:

CarBroken with values yes, no  
CarBroken\((x, \text{yes})\) with values yes, no  
Income with values small, medium, large  
CarRepair with values yes, no  
CarBuy with values new, used

Example of Data Table with 4 records is

<table>
<thead>
<tr>
<th>Obj</th>
<th>CarBroken</th>
<th>CarOld</th>
<th>Income</th>
<th>CarRepair</th>
<th>CarBuy</th>
<th>CarRepair</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>yes</td>
<td>yes</td>
<td>small</td>
<td>no</td>
<td>new</td>
<td>new</td>
</tr>
<tr>
<td>02</td>
<td>no</td>
<td>no</td>
<td>large</td>
<td>yes</td>
<td>used</td>
<td></td>
</tr>
<tr>
<td>03</td>
<td>yes</td>
<td>yes</td>
<td>medium</td>
<td>no</td>
<td>new</td>
<td></td>
</tr>
<tr>
<td>04</td>
<td>yes</td>
<td>no</td>
<td>small</td>
<td>no</td>
<td>used</td>
<td></td>
</tr>
</tbody>
</table>

RULES ARE:

R1  \( Car\text{Broken}(x, \text{yes}) \cap Car\text{Old}(x, \text{yes}) \cap Income(x, \text{small}) \Rightarrow Car\text{Repair}(x, \text{yes}) \)  
R2  \( Car\text{Broken}(x, \text{yes}) \cap Car\text{Old}(x, \text{yes}) \cap Car\text{Repair}(x, \text{no}) \cap Income(x, \text{medium}) \Rightarrow Car\text{Buy}(x, \text{used}) \)  
R3  \( Car\text{Broken}(x, \text{yes}) \cap Car\text{Old}(x, \text{no}) \cap Income(x, \text{large}) \Rightarrow Car\text{Buy}(x, \text{new}) \)
QUESTION 4

P1 Given a set of clauses
\[ \Delta = \{ \{ a, \neg b \}, \{ a, b, c \}, \{ \neg a, c \}, \{ \neg c, \neg b \} \} \]

Find all possible complementary pairs and all their possible resolvents

P2. Use Resolution Deduction to decide whether the set \( \Delta \) of clauses is unsatisfiable or satisfiable.
\[ \Delta = \{ \{ \neg a \}, \{ \neg b \}, \{ a \} \} \]

Solution
Consider Resolution Deduction as follows
1. \( \Delta = \{ \{ \neg a \}, \{ \neg b \}, \{ a \} \} \)
2. \{b\} Resolution application on \( \{ \neg a \}, \{ a \} \)
3. {} Resolution application on \( \{ b \}, \{ \neg b \} \)

\( \Delta \) is UNSATISFIABLE

P3. Use the Deletion Strategies "PURE LITERAL", "TAUTOLOGY" where applicable to decide whether the set \( \Delta \) is unsatisfiable or satisfiable.
\[ \Delta = \{ \{ \neg a, a, b, \neg c \}, \{ a, \neg b, c \}, \{ \neg a, \neg c \} \} \]

Solution
1. Transform \( \Delta \) into \( \Delta' \) by deleting TAUTOLOGY \( \{ \neg a, a, b, \neg c \} \)
2. \( \Delta' = \{ \{ a, \neg b, c \}, \{ \neg a, \neg c \} \} \)
   The literal \( \neg b \) is PURE
3. Transform \( \Delta' \) into \( \Delta'' \) by deleting the pure literal clause \( \{ a, \neg b, c \} \)

\( \Delta'' = \{ \{ \neg a, \neg c \} \} \) STOP

\( \Delta \) is SATISFIABLE by the Completeness of TAUTOLOGY and PURE LITERAL Deletion Strategies

QUESTION 5

FIND the SET of CLAUSES that you need to decide whether the ARGUMENT
\[ ((A_1 \cap A_2) \Rightarrow B) \]

is valid/not valid, where
\[ A_1 : (((a \Rightarrow \neg b) \Rightarrow (b \Rightarrow \neg a)) \cup ((a \Rightarrow \neg c) \cup b)) \]
\[ A_2 : ((a \Rightarrow b) \Rightarrow a) \]
\[ B : (\neg (a \Rightarrow (\neg b \cap c)) \Rightarrow (a \cap (b \cap \neg c))) \]

Solution RESOLUTION HOMEWORK