# Association Analysis

cse352

Artificial Intelligence

Professor Anita Wasilewska
Computer Science Department
Stony Brook University

# Association Rules Mining
# An Introduction

- This is an intuitive (more or less ) **introduction**
- It contains explanation of the main ideas:
- Frequent item sets, association rules, how we construct the association rules
- How we **judge** the goodness of the rules
- **Example** of an intuitive "run" of the **Appriori Algorithm** and association rules generation
- Discussion of the **relationship** between the Association and Correlation analysis

# What Is Association Mining?

**Association rule mining:**

Finding frequent patterns called associations, among sets of items or objects in transaction databases, relational databases, and other information repositories

- **Applications:**
  - Basket data analysis, cross-marketing, catalog design, loss-leader analysis, clustering, classification, etc.

# Association Rules

- Rule **general** form:

  "Body → Head [support, confidence]"

  Rule **Predicate** form:

  buys(x, "diapers") → buys(x, "beer")

  [0.5%, 60%]

  major(x, "CS") ^ takes(x, "DB") → grade(x, "A") [1%, 75%]

  Rule **Attribute** form:

  Diapers → beer [1%, 75%]

# Association Analysis: Basic Concepts

- **Given: a** database of transactions, where each transaction is a list of items
- **Find:** all rules that associate the presence of one set of items with that of another set of items
- **Example**

  *98% of people who purchase tires and auto accessories also get automotive services done*

# Association Model

- **I** ={i1, i2, ...., in} a set of **items**
- **J** = P(**I** ) set of **all subsets** of the set of items, elements of **J** are called **itemsets**
- **Transaction T: T** is **subset** of set **I** of **items**
- **Data Base:** set of transactions
- **An association rule** is an implication of the form : **X-> Y,** where **X, Y** are disjoint subsets of items **I** (elements of **J** )
- **Problem:** Find rules that have support and confidence greater that user-specified minimum support and minimun confidence

# Apriori Algorithm

- **Apriori Algorithm:**
- **First Step:** we find all **frequent** item-sets
- An item-set is **frequent** if it has a **support** greater or equal a fixed minimum support
- We fix minimum support usually low
- **Rules generation** from the frequent item-sets is a **separate problem** and we will cover it as a part of Association Process

# Apriori Algorithm

- In order **to calculate** efficiently frequent item-sets:
- 1-item-sets (one element item-sets)
- 2-item-sets (two elements item-sets)
- 3-item-sets ( three elements item-sets), etc..
- **we use** a **principle**, called an Apriori Principle (hence the name: Apriori Algorithm):
- **Apriori Principle**
- ANY **SUBSET** OF A **FREQUENT** ITEMSET **IS** A **FREQUENT** ITEMSET

# The Apriori Algorithm (Han Book)

- **Pseudo-code**:

**$C_k$: Candidate itemset of size k**
**$L_k$ : frequent itemset of size k**

$L_1$ = {frequent items};
**for** ($k$ = 1; $L_k$ !=$\emptyset$; $k$++) **do begin**
    **$C_{k+1}$ = candidates generated from $L_k$;**
    **for each** transaction $t$ in database do
        increment the count of all candidates in $C_{k+1}$ that are contained in $t$
    $L_{k+1}$ = candidates in $C_{k+1}$ with min_support
    **end**
**return** $\cup_k L_k$;

# Appriori Process: Rules Generation

- **Appriori Algorithm stops** after the **First Step**
- **Second Step** in the **Appriori Proces** (item-sets generation AND rules generation) is the **rules generation:**
- We calculate, from the frequent item-sets a set of the **strong rules**
- **Strong rules:** rules with at least minimum support (low) and minimum confidence (high)
- **Apriori Process** is then **finished** .

# Apriori Process
## Rules Generation

- The Apriori Process problem is:
- How do we form the association rules (A =>B) from the frequent item sets?

- **Remember:** A, B are disjoint subsets of the set **I** of items in general, and of the set 2- frequent, 3-frequent item sets ….. etc, … as generated by the Apriori Algorithm

# How we find the rules?

- 1-frequent item set: {i1}- no rule
- 2-frequent item set {i1, i2}: there are two rules:
- {i1} => {i2} and {i2} => {i2}
- We write them also as
- i1 => i2 and i2 => i2
- We **decide** which rule we **accept** by calculating its support (greater= minimum support) and confidence (greater= minimum confidence)

# How we find the rules?

- 3-frequent item set: {i1, i2, i3}
- **The rules**, by definition are of the form (A =>B) where A and B are **disjoint subsets** of {i1, i2, i3}, i.e.
- we have to find all subsets A,B of {i1, i2, i3} such that $A \cup B = \{i1, i2, i3\}$ and $A \cap B = \Phi$
- **For example**,
- let A= {i1, i2} and B= {i3}
- **The rule** is
- {i1, i2} => {i3}
- and we write it in a form:

  $i1 \cap i2 \Rightarrow i3$ or milk ∩ bread => vodka

if item i1 is milk, item i2 is bread and item i3 is vodka

# How we find the rules?

- Another choice for A and B is, for example:
- A= {i1} and B= {12, i3}.
- **The rule** is
- {i1} => {i2, i3} and we write it in a form:

  i1 => i2∩ i3 or milk => bread ∩ vodka

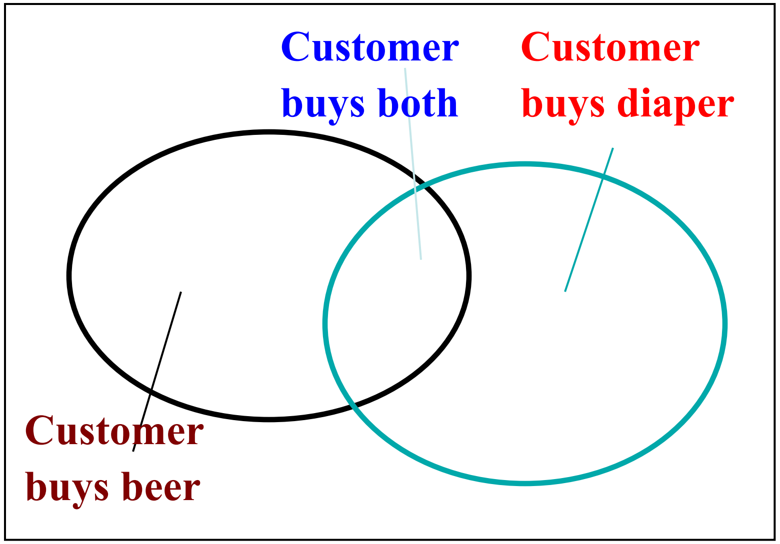
  if item i1 is milk, item i2 is bread and item i3 is vodka
- REMEMBER:
- We have to cover all the choices for A and B!
- **Which rule we accept is being decided by calculating its support (greater = minimum support) and confidence (greater = minimum confidence)**

# Rules Confidence and Support

- **Confidence:**
- the rule **X->Y holds** in the database **D** with confidence c if the c% of the transactions in **D** that contain **X** also contain **Y**

- **Support:** The rule **X->Y** has **support** s in **D** if s% of the transaction contain **XUY**

# Support and Confidence




- **Find** all the rules *X & Y* $\Rightarrow$ *Z* with minimum confidence and support
  - **Support** ***s:*** **probability** that a transaction contains {X, Y, Z}
  - **confidence** ***c:*** **conditional probability** that a transaction containing {X, Y} also contains *Z*

# Support Definition

- **Support** of a rule A=>B in the database **D** of transactions is given by formula (where sc=support count)
- Support( A => B) = P(A U B) =

$$\frac{sc(A \cup B)}{\#D}$$

**Frequent Item sets:** sets of items with a support support >= MINIMAL support
We (user) **fix** MIN support usually **low** and
Min Confidence **high**

# Confidence Definition

- **Confidence** of a rule A=>B in the database **D** of **transactions** is given by formula (where sc=support count)

- **Conf( A => B)** = P(B|A) = $\boxed{\frac{P(A \cup B)}{P(A)}}$

- $= \boxed{\frac{sc(AUB)}{\#D} \text{ divided by } \frac{scA}{\#D}} = \boxed{\frac{sc(AUB)}{scA}}$

# Example

- Let consider a data base **D** ={ T1, T2, …. T9}, where
- T1={ 1,2,5} (we write k for item ik)
- T2= {2, 4}, T3={2, 3}, T4={1, 2, 4}, T5={1, 3}
- T6={2, 3}, T7={1, 3}, T8={1, 2, 3,5}, T9={1,2,3}
- **To find association rules** we follow the **following steps**
- **STEP 1**: **Count** occurrences of items in **D**
- **STEP2:** Fix Minimum support (usually **low**)
- **STEP 3:** Calculate frequent 1-item sets
- **STEP 4:** Calculate frequent 2-item sets
- **STEP 5:** Calculate frequent 3-item sets
- **STOP** when **there is no more** frequent item sets
- This is the **end** of **Apriori Algorithm** phase

# Example

- **How to** generate **all** frequent 3-item sets (in **Step 5**)
- FIRST: use the frequent 2-item sets to **generate** all **3-item set candidates**
- SECOND: use **Apriori Principle** to **prune** the candidates set
- THIRD: **Evaluate** the count of the **pruned set**
- FOUR: **list** the frequent 3-item sets
- **STEP 6:** repeat the procedure for 4-item sets etc (if any)

# Example- Apriori Process

- **Apriori Process Steps:**

- **STEP 7:** Fix the **minimum** confidence (usually **high**)

- **STEP 8:** Generate **strong rules** (support >min support and confidence> min confidence)

- **END** of rules generation phase
- **END** of the **Apriori Process**

# Example- Apriori Pocess

- Lets now **calculate** **all steps** of our **Apriori Process** for a data base
- **D** ={ T1, T2, …. T9}, where
- T1={ 1,2,5} (we write k for item ik)
- T2= {2, 4}, T3={2, 3}, T4={1, 2, 4}, T5={1, 3}
- T6={2, 3}, T7={1, 3}, T8={1, 2, 3,5}, T9={1,2,3}
- **Here is our Step 1**
- We represent our **transactional** data base as **relational** data base (a table) and put the occurrences of items as an extra row, on the bottom

# Example: Step 1

**STEP 1:** items occurrences=sc

| its | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| T1 | + | + | 0 | 0 | + |
| T2 | 0 | + | 0 | + | 0 |
| T3 | 0 | + | + | 0 | 0 |
| T4 | + | + | 0 | + | 0 |
| T5 | + | 0 | + | 0 | 0 |
| T6 | 0 | + | + | 0 | 0 |
| T7 | + | 0 | + | 0 | 0 |
| T8 | + | + | + | 0 | + |
| T9 | + | + | + | 0 | 0 |
| sc | **6** | **7** | **6** | **2** | **2** |

# Example; Step 2

- **STEP 2:** Fix minimal support count, for example
- msc = 2
- **Minimal support** = msc/#D= 2/9=**22%**
- ms=22%

- **Observe:** minimal support of an item set is **determined uniquely** by the minimal support count (msc) and we are going to use only **msc** to choose our frequent k-itemsets

# Example: steps 3, 4

- **STEP 3:** calculate frequent 1-item sets: look at the sc count – we get that all 1-item sets are **frequent**

- **STEP 4:** calculate frequent 2-item sets
- **First** we calculate **2-item sets candidates from frequent 1-item sets.**

- As our all 1-item sets are frequent so all **subsets** of any 2-item set **are frequent** and we have to find **counts** of **all** 2-item sets

# Observation

- If **for example** we set our **msc=6**, i.e we would have only {1}, {2} and {3} as **frequent item** sets

- Then by **Apriori Principle:**
- **"if A is a frequent item set, then each of its subsets is a frequent item set"**

- we would be examining **only those** 2-item sets that have {1}, {2}, {3} as subsets

- **Apriori Principle reduces the complexity of the algorithm**

# Example: Step 4

- **STEP 4 :** All 2-item sets are all 2-element subsets of {1,2,3,4,5}. They are called are **candidates** and we evaluate their sc=support counts (in red). They are called **2-item set candidates:**
- { 1,2}, {1, 3}, {1, 4}, {1, 5},
- {2, 3}, {2, 4}, {2, 5}, {3,4}, {3,5}, {4,5}
- **WE calculate their SUPPORT COUNT from SC TABLE**
- {1,2,} **(4)**, {1,3} **(4)**, {1,4} **(1)**, {1,5} **(2)**,
- {2,3} **(4)**, {2,4} **(2)**, {2,5} **(2) ,**
- {3,4} **(0)**, {3,5} **(1)**,
- {4,5} **(0)**

- **msc=2** and we choose **candidates** with **sc >= 2** and get the following
- **Frequent 2- item sets:**
- {1,2}, {1,3}, {1,5}, {2,3}, {2,4}, {2,5}

# Support Count TABLE

**STEP 1:** items occurrences=sc

| its | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| T1 | + | + | 0 | 0 | + |
| T2 | 0 | + | 0 | + | 0 |
| T3 | 0 | + | + | 0 | 0 |
| T4 | + | + | 0 | + | 0 |
| T5 | + | 0 | + | 0 | 0 |
| T6 | 0 | + | + | 0 | 0 |
| T7 | + | 0 | + | 0 | 0 |
| T8 | + | + | + | 0 | + |
| T9 | + | + | + | 0 | 0 |
| sc | **6** | **7** | **6** | **2** | **2** |

# Example: Step 5

- **STEP 5 :** generate all frequent 3-item sets
- We use **frequent 2- item sets:**
- {1,2}, {1,3}, {1,5}, {2,3}, {2,4}, {2,5}
- and proceed as follows
- FIRST: we calculate from the **frequent** 2- item sets a set of all **3-item set candidates:**
- {1,2,3}, {1,2,4}, {1,2,5}, {1,3,4}, {1,3,5}, {2,3,4}, {2,3,5}, {2,4,5}
- **Observe** that the **candidates**
- {1,3,4}, {1,3,5}, {2,3,4}, {2,3,5}, {2,4,5}
- **do not** follow **Apriori Principle:**
- "if A is a frequent item set, then each of its subsets is a frequent item set"

# Example: Step 5

- **Frequent 2- item sets are:**
- {1,2}, {1,3}, {1,5}, {2,3}, {2,4}, {2,5}
- We reject {1,3,4} as its subset {3,4} **is not** a frequent 2- item set
  We reject {1,3,5} as its subset {3,5} **is not** a frequent 2- item set
  We reject {2,3, 4} as its subset {3,4} **is not** a frequent 2- item set
  We reject {2,3, 5} as its subset {3,5} **is not** a frequent 2- item set
  We reject {2, 4, 5} as its subset {4,5} **is not** a frequent 2- item set

This **rejection process** is called **pruning**

The following form of the Apriori Algorithm is called

- **Prune Step:** Any (k-1)-item set that is not frequent **cannot** be a subset of a frequent k-item set

# Example: Step 5

- SECOND: we perform the **Prune Step** and write the **pruned** frequent 3-item set **candidates:**
- {1,2,3}, {1,2,5}, {1,2, 4}
- THIRD: we calculate the sc=support count for the **pruned** frequent 3-item **candidates**
- {1,2,3} **(2)**, {1,2,5} **(2)**, {1,2, 4} **(1)**

- FOUR:
- **msc=2** and we choose the 3-item **candidates** with **sc >= 2** and get the following list of
- **Frequent 3-item sets:**
- **{1,2,3}, {1,2,5}**

# Support Count TABLE

**STEP 1:** items occurrences=sc

| its | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| T1 | + | + | 0 | 0 | + |
| T2 | 0 | + | 0 | + | 0 |
| T3 | 0 | + | + | 0 | 0 |
| T4 | + | + | 0 | + | 0 |
| T5 | + | 0 | + | 0 | 0 |
| T6 | 0 | + | + | 0 | 0 |
| T7 | + | 0 | + | 0 | 0 |
| T8 | + | + | + | 0 | + |
| T9 | + | + | + | 0 | 0 |
| sc | **6** | **7** | **6** | **2** | **2** |

# Example: Steps 6, 7

- **STEP 6:** there is no 4-item sets
- We **STOP** when **there is no more** frequent item sets

- This is the **end** of **Apriori Algorithm phase**

- **STEP 7:**
- We fix **minimum confidence** (usually high) as

- **min conf = 70%**

- We use the **confidence** to generate **Apriori Rules**

# Example: Step 8
# Association Rules Generation

- **Step 8:** Strong Rules Generation
- We will generate, as **an example** rules only from **one** frequent 2-item set: {1,2}
- Rule generation for other 2-item sets is **similar**
- **Reminder:** conf(A=>B) =

$$\frac{sc(A \cup B)}{scA}$$

- We split {1,2} into disjoint subsets A and B as follows: A={1} and B={2} or A={2} and B={1} and get two possible rules:
- {1}=>{2}     or     {2}=>{1}

# Example: Association Rules Generation

- Conf(1=>2)= $\boxed{\dfrac{sc\{1,2\}}{sc\{1\}}}$ = 4/6 = 66%

The rule is **not accepted** (min conf= 70%)

- Conf(2=>1) = $\boxed{\dfrac{sc\{1,2\}}{sc\{2\}}}$ = 4/7= 57%

The rule is **not accepted**

# Example: Step 8

- Now we use one **frequent 3-item** set
- **{1,2,5}** to show how to generate **strong rules**

- **First** we evaluate all possibilities how to **split** the set {1,2,5} into to disjoint subsets A,B to obtain **all possible** rules A=>B
- For each rule we **evaluate** its confidence and choose only those with conf ≥ 70% (our **minimal** confidence)

- The minimal support condition is **fulfilled** as we **deal only** with frequent items
- The rules such obtained are **strong rules**

# Example: Association Rules Generation

- The rules for {1,2,5} are the following:
- **R1:** {1,2}=>{5}
- conf(R1)=sc{1,2,5}/sc{1,2}= 2/4 = ½ = 50%
- **R1 is rejected**
- **R2:** {1,5} => {2}
- conf(R2)=sc{1,2,5}/sc{1,5}= 2/2 = 100%
- **R2 is a strong rule (keep)**
- **R3:** {2,5} => {1}
- conf(R3)=sc{1,2,5}/sc{2,5}= 2/2 = 100%
- **R3 is a strong rule (keep)**
- **R4:** {1} => {2,5}
- conf(R4)=sc{1,2,5}/sc{1}= 2/6 = 33%
- **R4 is rejected**

# Example: Association Rules Generation

- The next set of rules for {1,2,5} are the following:
- **R5:** {2}=>{1,5}
- conf(R5)=sc{1,2,5}/sc{2}= 2/7 = 27%
- **R5 is rejected**
- **R6:** {5} => {1,2}
- conf(R6)=sc{1,2,5}/sc{5}= 2/2 = 100%
- **R6 is a strong rule (keep)**
- As the last step we evaluate the **exact support** for the **strong rules**
- We **know** already that it is **greater** or **equal** to minimum support, as rules were obtained from the frequent item sets

# Example: Association Rules Generation

- **Exact support** for the **strong rules** is:

- Sup({1,5}=>{2})=sc{1,2,5}/#D=2/9= 22%
- We write:
- 1∩ 5 => 2    [22%, 100%]
- Sup({2,5}=>{1}) =sc{1,2,5}/#D=2/9= 22%
- We write:
- 2∩ 5 => 1    [22%, 100%]
- Sup({5}=>{1,2}) =sc{1,2,5}/#D=2/9= 22%
- We write:
- 5 => 1 ∩ 2   [22%, 100%]

- **THE END of Apriori Process**

# Association and Correlation

- As we can see the support-confidence framework can be misleading;
- it can identify a rule A=>B as interesting (strong) when, in fact the occurrence of A might not imply the occurrence of B
- **Correlation Analysis** provides an alternative framework for finding **interesting relationships**,
- or to **improve** understanding of meaning of some association rules (a lift of an association rule)

# Correlation and Association

- **Definition:** Two item sets A and B are **independent** (the occurrence of A is independent of the occurrence of item set B) iff probability P fulfills the condition
- $P(A \cup B) = P(A) \cdot P(B)$
- Otherwise A and B are **dependent** or **correlated**
- The **measure of correlation**, or **correlation** between A and B is given by the formula:
- $\mathrm{Corr}(A,B) = \boxed{\dfrac{P(A \cup B)}{P(A)P(B)}}$

# Correlation and Association

- **corr(A,B) >1** means that A and B are positively correlated i.e. the occurrence of one implies the occurrence of the other
- **corr(A,B) < 1** means that the occurrence of A is negatively correlated with B
- or **discourages** the occurrence of B
- **corr(A,B) =1** means that A and B are independent

# Correlation and Association

- The correlation formula can be re-written as
- Corr(A,B) = $\boxed{\dfrac{P(B|A)}{P(B)}}$
- Supp(A=>B)= P(AUB)
- Conf(A=>B)= P(B|A), i.e.
- **Conf(A=>B)= corr(A,B) P(B)**
- So correlation, support and confidence are all different, but the correlation provides an **extra information** about the **association rule** (A=>B)
- We say that the correlation corr(A,B) provides the **LIFT** of the **association rule** (A=>B), i.e.
- A is said to **increase** or to **LIFT** the likelihood of B by the factor of the value returned by the formula for corr(A,B)

# Correlation Rule (HAN Book)

- **A correlation rule** is a set of items
- {i1, i2, ….in}, where the items occurrences are **correlated**
- The correlation value is given by the **correlation formula** and we use X square test to determine if correlation is statistically significant
- The X square test can also determine the negative correlation
- We can also form minimal correlated item sets, etc…
- Limitations: X square test is less accurate on the data tables that are sparse and can be misleading for the contingency tables larger then 2x2

# Criticism to Support and Confidence
## (Han book)

- **Example 1:** (Aggarwal & Yu, PODS98)
  - Among 5000 students
    - 3000 play basketball
    - 3750 eat cereal
    - 2000 both play basket ball and eat cereal

*RULE: play basketball* $\Rightarrow$ *eat cereal* [40%, 66.7%] is misleading because the overall percentage of students eating cereal is 75% which is higher than 66.7%.

*RULE: play basketball* $\Rightarrow$ *not eat cereal* [20%, 33.3%] is far more accurate, although with lower support and confidence

| | basketball | not basketball | sum(row) |
|---|---|---|---|
| cereal | 2000 | 1750 | 3750 |
| not cereal | 1000 | 250 | 1250 |
| sum(col.) | 3000 | 2000 | 5000 |

# EXTRTA Slides

- ADDITIONAL MATERIAL
- Read, explore; much of it I already covered in our slides

# Mining Association Rules in Large Databases

Slightly modified HAN Book slides  follow from now

# Mining Association Rules in Large Databases

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
- Mining multilevel association rules from transactional databases
- Mining multidimensional association rules from transactional databases and data warehouse
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

# Association Rule Mining: A Road Map

- **Boolean (Qualitative) vs. quantitative associations** (Based on the types of values handled)

  buys(x, “SQLServer”) ^ income(x, “DMBook”) => buys(x, “DBMiner”) [0.2%, 60%] (**Boolean/Qualitative**)

  age(x, “30..39”) ^ income(x, “42..48K”) => buys(x, “PC”) [1%, 75%] **(quantitative)**

- Single dimension (one predicate) vs. multiple dimensional associations (multiple predicates )

# Association Rule Road Map (c.d)

- **Single level vs. multiple-level analysis**
  - What brands of beers are associated with what brands of diapers – **single level**
  - **Various extensions**

1. Correlation analysis (just discussed)
2. **Association** does not necessarily imply **correlation** or causality
3. Constraints enforced

   Example:

   smallsales (sum < 100) implies bigbuys (sum >1,000)?

# Chapter 5: Mining Association Rules

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
- Mining multilevel association rules from transactional databases
- Mining multidimensional association rules from transactional databases and data warehouse
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

# An Example

| Transaction ID | Items Bought |
|---|---|
| 2000 | A,B,C |
| 1000 | A,C |
| 4000 | A,D |
| 5000 | B,E,F |

Min. support 50%
Min. confidence 50%

| Frequent Itemset | Support |
|---|---|
| {A} | 75% |
| {B} | 50% |
| {C} | 50% |
| {A,C} | 50% |

For rule ***A*** $\Rightarrow$ ***C***:

support = support({*A*, *C*}) = 50%

confidence = sc({*A*, *C*})/sc({*A*}) = 66.6%

**The Apriori principle:**

**Any subset of a frequent itemset must be frequent**

# Mining Frequent Itemsets: the Key Step

- **Find the frequent item sets:** the sets of items that have minimum support
  - A subset of a frequent item set must also be a frequent item set
    - **i.e., if {*A, B*} is a frequent item set, both {*A*} and {*B*} should be a frequent item set**
  - Iteratively find frequent item sets with cardinality from 1 to *k* (*k*-item set)
- **Use the frequent item sets to generate association rules.**

# Apriori Algorithm — Book Example of frequents items sets generation

Database D

| TID | Items |
|---|---|
| 100 | 1 3 4 |
| 200 | 2 3 5 |
| 300 | 1 2 3 5 |
| 400 | 2 5 |

Scan D

$C_1$

| itemset | sup. |
|---|---|
| {1} | 2 |
| {2} | 3 |
| {3} | 3 |
| {4} | 1 |
| {5} | 3 |

$L_1$

| itemset | sup. |
|---|---|
| {1} | 2 |
| {2} | 3 |
| {3} | 3 |
| {5} | 3 |

$C_2$

| itemset |
|---|
| {1 2} |
| {1 3} |
| {1 5} |
| {2 3} |
| {2 5} |
| {3 5} |

Scan D

$C_2$

| itemset | sup |
|---|---|
| {1 2} | 1 |
| {1 3} | 2 |
| {1 5} | 1 |
| {2 3} | 2 |
| {2 5} | 3 |
| {3 5} | 2 |

$L_2$

| itemset | sup |
|---|---|
| {1 3} | 2 |
| {2 3} | 2 |
| {2 5} | 3 |
| {3 5} | 2 |

$C_3$

| itemset |
|---|
| {2 3 5} |

Scan D

$L_3$

| itemset | sup |
|---|---|
| {2 3 5} | 2 |

# Generating Candidates: $C_k$

- Join Step: $C_k$ is generated by **joining** $L_{k-}$ with itself
- Prune Step: Any (k-1)-item set that is not frequent **cannot** be a subset of a frequent k-item set

# Example of Generating Candidates

- $L_3=\{abc, abd, acd, ace, bcd\}$
- We write abc for {a,b,c}, etc…
- **Self-joining**: $L_3 * L_3$
  - *abcd* from *abc* and *abd*
  - *acde* from *acd* and *ace*
- **Pruning**:
  - *acde* is removed because *ade* is **not frequent**: is not in $L_3$
- $C_4=\{abcd\}$

# Appriori Performance Bottlenecks

- The **core** of the Apriori algorithm:
  - Use frequent $(k-1)$-item sets to generate **candidate** frequent $k$-item sets
  - Use database scan and pattern matching to collect counts for the **candidate** item sets
- The **bottleneck** of Apriori: **candidate generation**
  - Huge candidate sets:
    - $10^4$ frequent 1-itemset will generate $10^7$ **candidate** 2-itemsets
    - To discover a frequent pattern of size 100, e.g.,
    - $\{a_1, a_2, \ldots, a_{100}\}$, one needs to generate $2^{100} \approx 10^{30}$ **candidates**
  - Multiple scans of database:
    - Needs $(n+1)$ scans, $n$ is the length of the longest pattern

# How to Count Supports of Candidates?

- **Why counting supports of candidates is a problem?**
  - The **total number** of candidates can be **very huge**
  - **One transaction** may contain **many candidates**
- **Method:**
  - **Candidate itemsets** are stored in a **hash-tree**
  - *Leaf* node of hash-tree contains a list of itemsets and counts
  - *Interior* node contains a hash table
  - *Subset function*: finds all the candidates contained in a transaction

# Methods to Improve Apriori's Efficiency

- Hash-based itemset counting: A *k*-itemset whose corresponding **hashing bucket** count is below the threshold cannot be frequent
- Transaction reduction: A **transaction** that does not contain any frequent k-itemset is **useless** in subsequent scans
- Partitioning: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
- Sampling: mining on a subset of given data, lower support threshold + a method to determine the completeness
- Dynamic itemset counting: add new candidate itemsets only when all of their subsets are estimated to be frequent

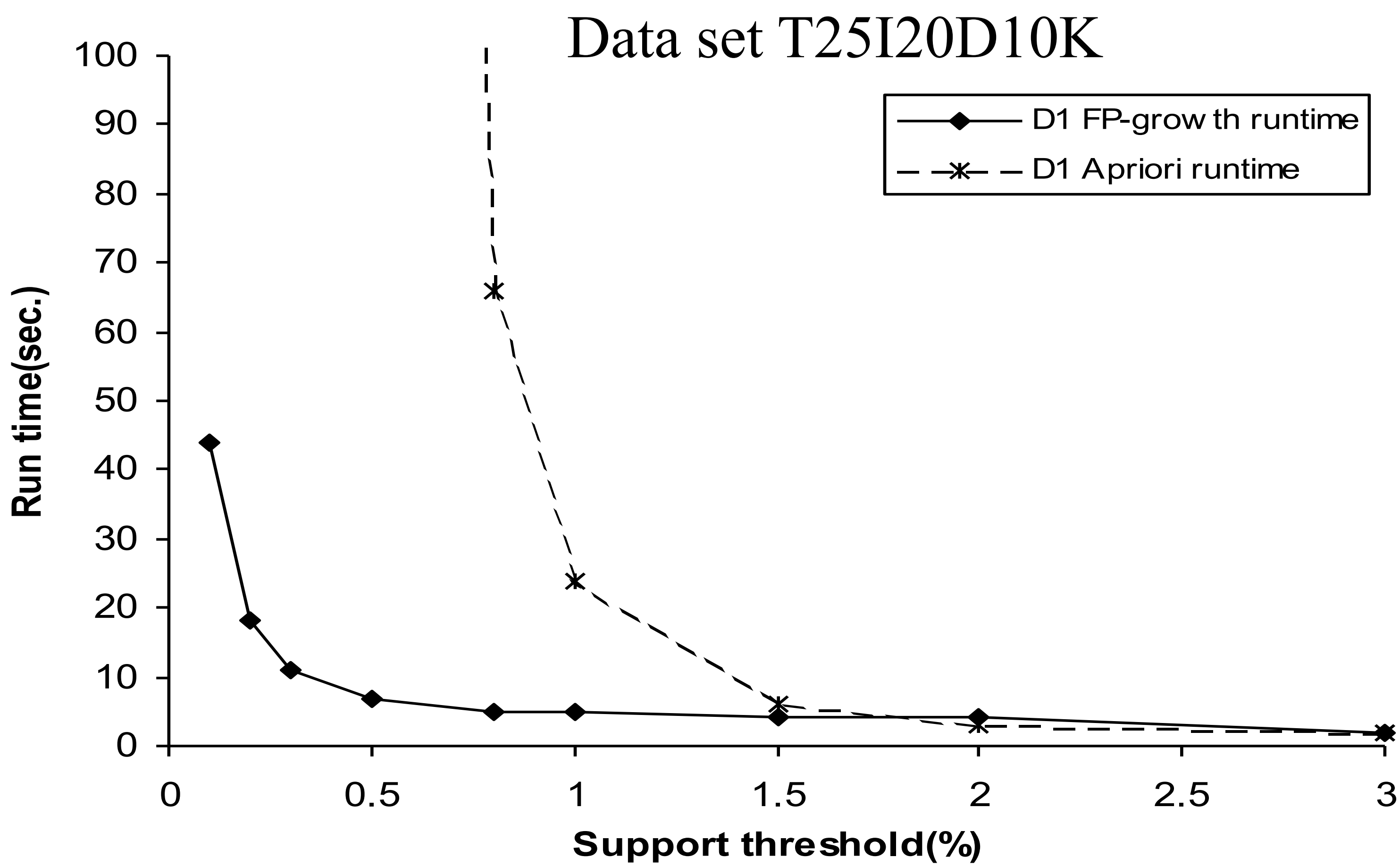
# An Alternative: Mining Frequent Patterns Without Candidate Generation

- **Compress** a large database into a compact,
- **Frequent-Pattern tree (FP-tree)** structure
  - highly condensed, but complete for frequent pattern mining
  - avoid costly database scans
- **Develop** an efficient, FP-tree-based frequent pattern mining method
  - A **divide-and-conquer** methodology: decompose mining tasks into smaller ones
  - **Avoid candidate generation**: sub-database test only!

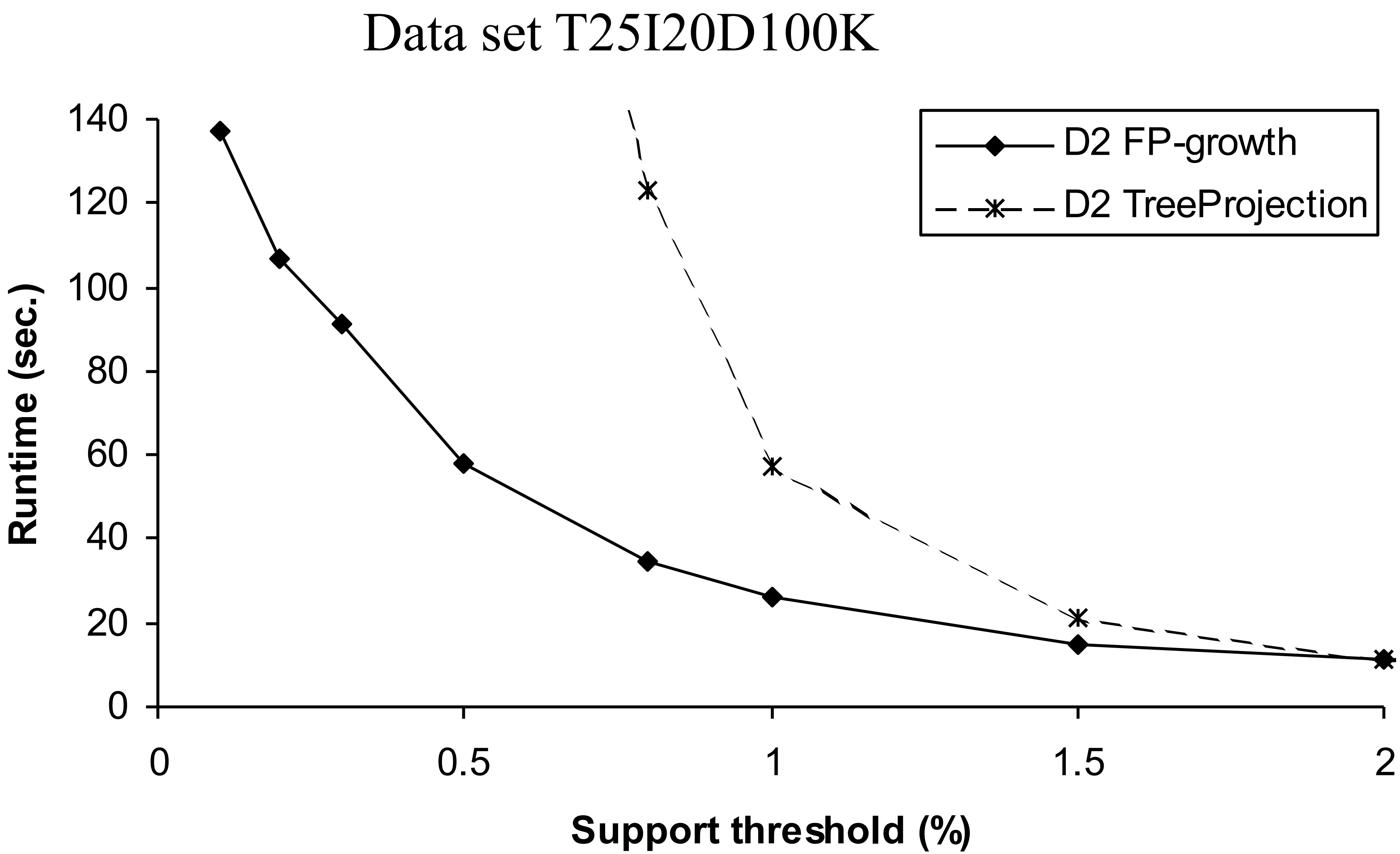
# Why Is Frequent Pattern Growth Fast?

- **Performance study shows**
  - FP-growth is an order of magnitude faster than Apriori, and is also faster than tree-projection
- **Reasoning**
  - No candidate generation, no candidate test
  - Use compact data structure
  - Eliminate repeated database scan
  - Basic operation is counting and FP-tree building

# FP-growth vs. Apriori: Scalability With the Support Threshold



# FP-growth vs. Tree-Projection: Scalability with Support Threshold



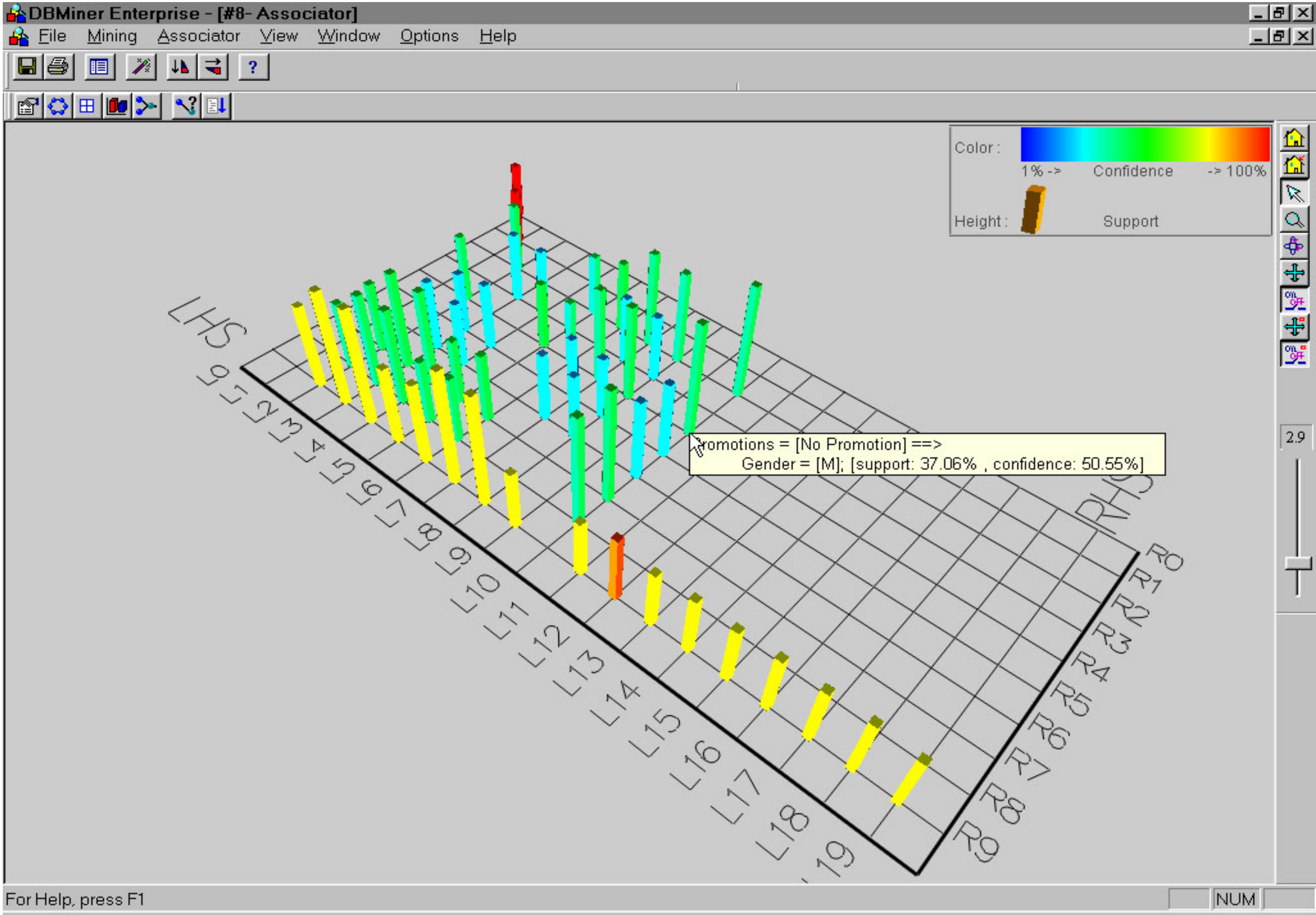
# Presentation of Association Rules

## (Table Form )

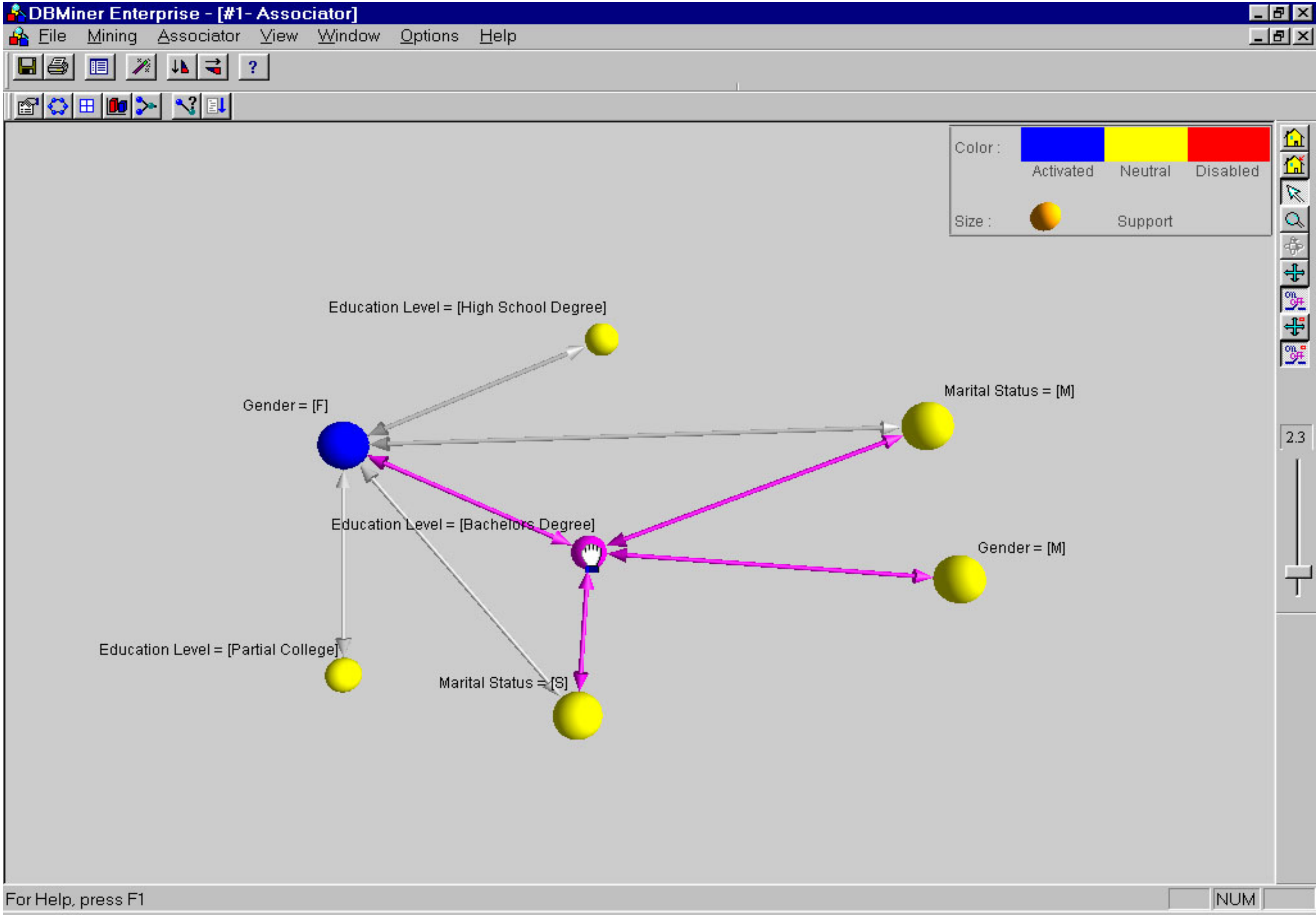
| | Body | Implies | Head | Supp (%) | Conf (%) | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | cost(x) = '0.00~1000.00' | ==> | revenue(x) = '0.00~500.00' | 28.45 | 40.4 | | | | |
| 2 | cost(x) = '0.00~1000.00' | ==> | revenue(x) = '500.00~1000.00' | 20.46 | 29.05 | | | | |
| 3 | cost(x) = '0.00~1000.00' | ==> | order_qty(x) = '0.00~100.00' | 59.17 | 84.04 | | | | |
| 4 | cost(x) = '0.00~1000.00' | ==> | revenue(x) = '1000.00~1500.00' | 10.45 | 14.84 | | | | |
| 5 | cost(x) = '0.00~1000.00' | ==> | region(x) = 'United States' | 22.56 | 32.04 | | | | |
| 6 | cost(x) = '1000.00~2000.00' | ==> | order_qty(x) = '0.00~100.00' | 12.91 | 69.34 | | | | |
| 7 | order_qty(x) = '0.00~100.00' | ==> | revenue(x) = '0.00~500.00' | 28.45 | 34.54 | | | | |
| 8 | order_qty(x) = '0.00~100.00' | ==> | cost(x) = '1000.00~2000.00' | 12.91 | 15.67 | | | | |
| 9 | order_qty(x) = '0.00~100.00' | ==> | region(x) = 'United States' | 25.9 | 31.45 | | | | |
| 10 | order_qty(x) = '0.00~100.00' | ==> | cost(x) = '0.00~1000.00' | 59.17 | 71.86 | | | | |
| 11 | order_qty(x) = '0.00~100.00' | ==> | product_line(x) = 'Tents' | 13.52 | 16.42 | | | | |
| 12 | order_qty(x) = '0.00~100.00' | ==> | revenue(x) = '500.00~1000.00' | 19.67 | 23.88 | | | | |
| 13 | product_line(x) = 'Tents' | ==> | order_qty(x) = '0.00~100.00' | 13.52 | 98.72 | | | | |
| 14 | region(x) = 'United States' | ==> | order_qty(x) = '0.00~100.00' | 25.9 | 81.94 | | | | |
| 15 | region(x) = 'United States' | ==> | cost(x) = '0.00~1000.00' | 22.56 | 71.39 | | | | |
| 16 | revenue(x) = '0.00~500.00' | ==> | cost(x) = '0.00~1000.00' | 28.45 | 100 | | | | |
| 17 | revenue(x) = '0.00~500.00' | ==> | order_qty(x) = '0.00~100.00' | 28.45 | 100 | | | | |
| 18 | revenue(x) = '1000.00~1500.00' | ==> | cost(x) = '0.00~1000.00' | 10.45 | 96.75 | | | | |
| 19 | revenue(x) = '500.00~1000.00' | ==> | cost(x) = '0.00~1000.00' | 20.46 | 100 | | | | |
| 20 | revenue(x) = '500.00~1000.00' | ==> | order_qty(x) = '0.00~100.00' | 19.67 | 96.14 | | | | |
| 21 | | | | | | | | | |
| 22 | | | | | | | | | |
| 23 | cost(x) = '0.00~1000.00' | ==> | revenue(x) = '0.00~500.00' AND order_qty(x) = '0.00~100.00' | 28.45 | 40.4 | | | | |
| 24 | cost(x) = '0.00~1000.00' | ==> | revenue(x) = '0.00~500.00' AND order_qty(x) = '0.00~100.00' | 28.45 | 40.4 | | | | |
| 25 | cost(x) = '0.00~1000.00' | ==> | revenue(x) = '500.00~1000.00' AND order_qty(x) = '0.00~100.00' | 19.67 | 27.93 | | | | |
| 26 | cost(x) = '0.00~1000.00' | ==> | revenue(x) = '500.00~1000.00' AND order_qty(x) = '0.00~100.00' | 19.67 | 27.93 | | | | |
| 27 | cost(x) = '0.00~1000.00' AND order_qty(x) = '0.00~100.00' | ==> | revenue(x) = '500.00~1000.00' | 19.67 | 33.23 | | | | |

Sheet1

# Visualization of Association Rule Using Plane Graph



# Visualization of Association Rule Using Rule Graph



# Iceberg Queries

- Iceberg query: Compute aggregates over one or a set of attributes only for those whose aggregate values is above certain threshold
- Example:

```
select P.custID, P.itemID, sum(P.qty)
from purchase P
group by P.custID, P.itemID
having sum(P.qty) >= 10
```

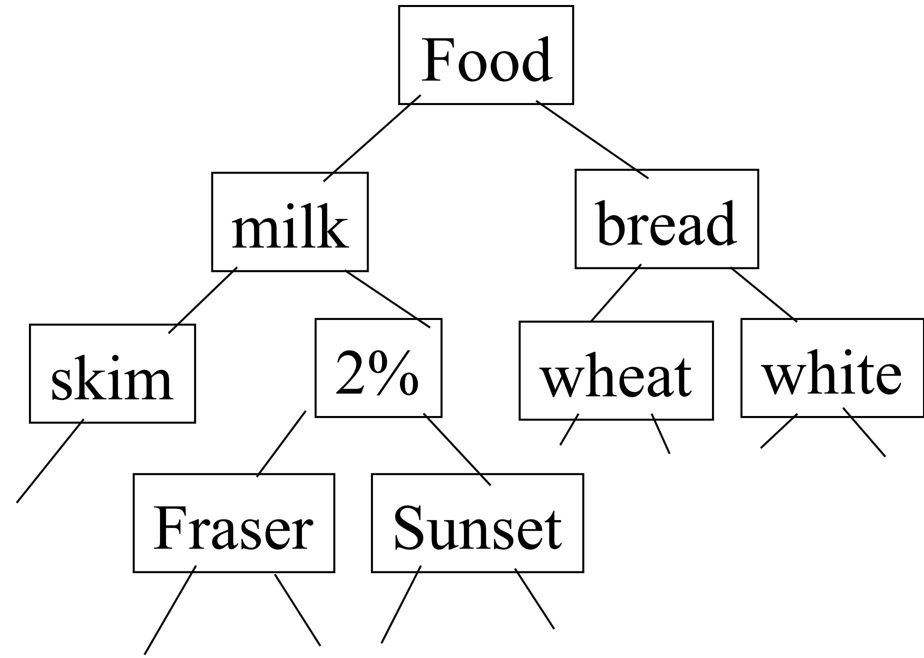
- Compute iceberg queries efficiently by Apriori:
  - First compute lower dimensions
  - Then compute higher dimensions only when all the lower ones are above the threshold

# Chapter 6: Mining Association Rules in Large Databases

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
- Mining multilevel association rules from transactional databases
- Mining multidimensional association rules from transactional databases and data warehouse
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

# Multiple-Level Association Rules

- **Items often form hierarchy**
- Items at the lower level are expected to have lower support.
- Rules regarding itemsets at appropriate levels could be quite useful.
- Transaction database can be encoded based on dimensions and levels
- We can explore shared multi-level mining




| TID | Items |
|---|---|
| T1 | {111, 121, 211, 221} |
| T2 | {111, 211, 222, 323} |
| T3 | {112, 122, 221, 411} |
| T4 | {111, 121} |
| T5 | {111, 122, 211, 221, 413} |

# Mining Multi-Level Associations

- A top_down, progressive deepening approach:
  - **First find high-level strong rules**:

    milk → bread [20%, 60%]
  - **Then find their lower-level "weaker" rules**:

    2% milk → wheat bread [6%, 50%]
- **Variations at mining multiple-level association rules**.
  - **Level-crossed association rules:**

    *2% milk → **Wonder** wheat bread*
  - **Association rules with multiple, alternative hierarchies:**

    *2% milk → **Wonder** bread*

# Chapter 6: Mining Association Rules in Large Databases

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
- Mining multilevel association rules from transactional databases
- Mining multidimensional association rules from transactional databases and data warehouse
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

# Multi-Dimensional Association (1)

- **Single-dimensional rules**:
- buys(X, “milk”) $\Rightarrow$ buys(X, “bread”)
- **Multi-dimensional rules**: Involve 2 or more dimensions or predicates
  - **Inter-dimension association rules (*no repeated predicates*)**
    - age(X,”19-25”) $\wedge$ occupation(X,“student”) $\Rightarrow$ buys(X,“coke”)

# Multi-Dimensional Association

- **Hybrid-dimension association rules (*repeated predicates*)**
  - age(X,"19-25") ∧ buys(X, "popcorn") ⇒ buys(X, "coke")
- **Categorical (qualitative) Attributes**
  - finite number of possible values, no ordering among values
- **Quantitative Attributes**
  - numeric, implicit ordering among values

# Techniques for Mining MD Associations

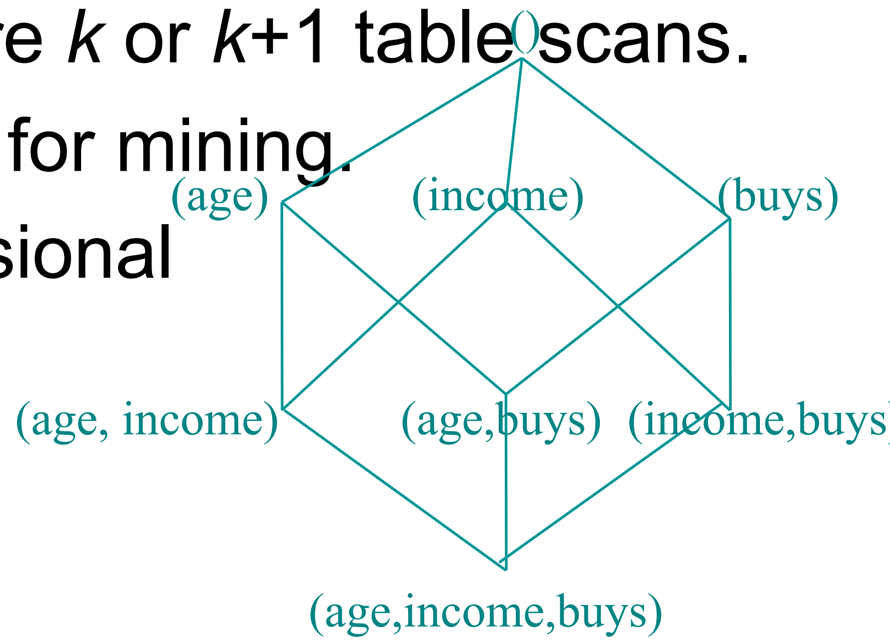
- Search for frequent *k*-predicate set:
  - **Example:**
  - {age, occupation, buys} is a **3-predicate set**.
  - Techniques can be categorized by how age are treated.

1. Using static discretization of quantitative attributes
   - Quantitative attributes are statically discretized by using predefined concept hierarchies.
2. Quantitative association rules
   - Quantitative attributes are dynamically discretized into "bins" based on the distribution of the data.
3. Distance-based association rules
   - This is a dynamic discretization process that considers the distance between data points.

## Static Discretization of Quantitative Attributes

- **Discretized prior** to mining using concept hierarchy.
- Numeric values are replaced by ranges
- In relational database, finding all frequent k-predicate sets will require *k* or *k*+1 table scans.
- Data cube is well suited for mining.
- The cells of an n-dimensional

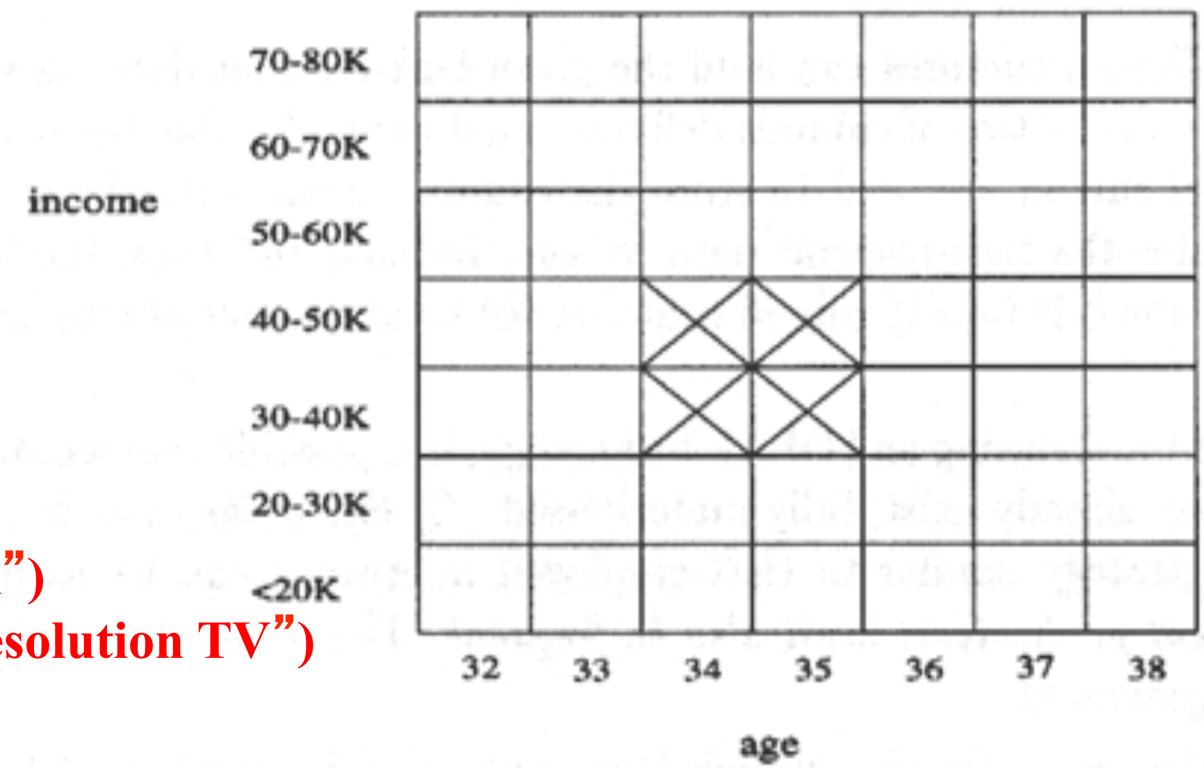
  cuboid correspond to the predicate sets.
- Mining from data cubes can be much faster.



# Quantitative Association Rules

- Numeric attributes are *dynamically* discretized
  - Such that the confidence or compactness of the rules mined is maximized.
- 2-D quantitative association rules: $A_{quan1} \wedge A_{quan2} \Rightarrow A_{cat}$
- Cluster "adjacent"

  association rules to form general rules using a 2-D grid.
- Example:




**age(X,"30-34") ∧**

**income(X,"24K - 48K")**

**⇒ buys(X,"high resolution TV")**

# ARCS (Association Rule Clustering System)

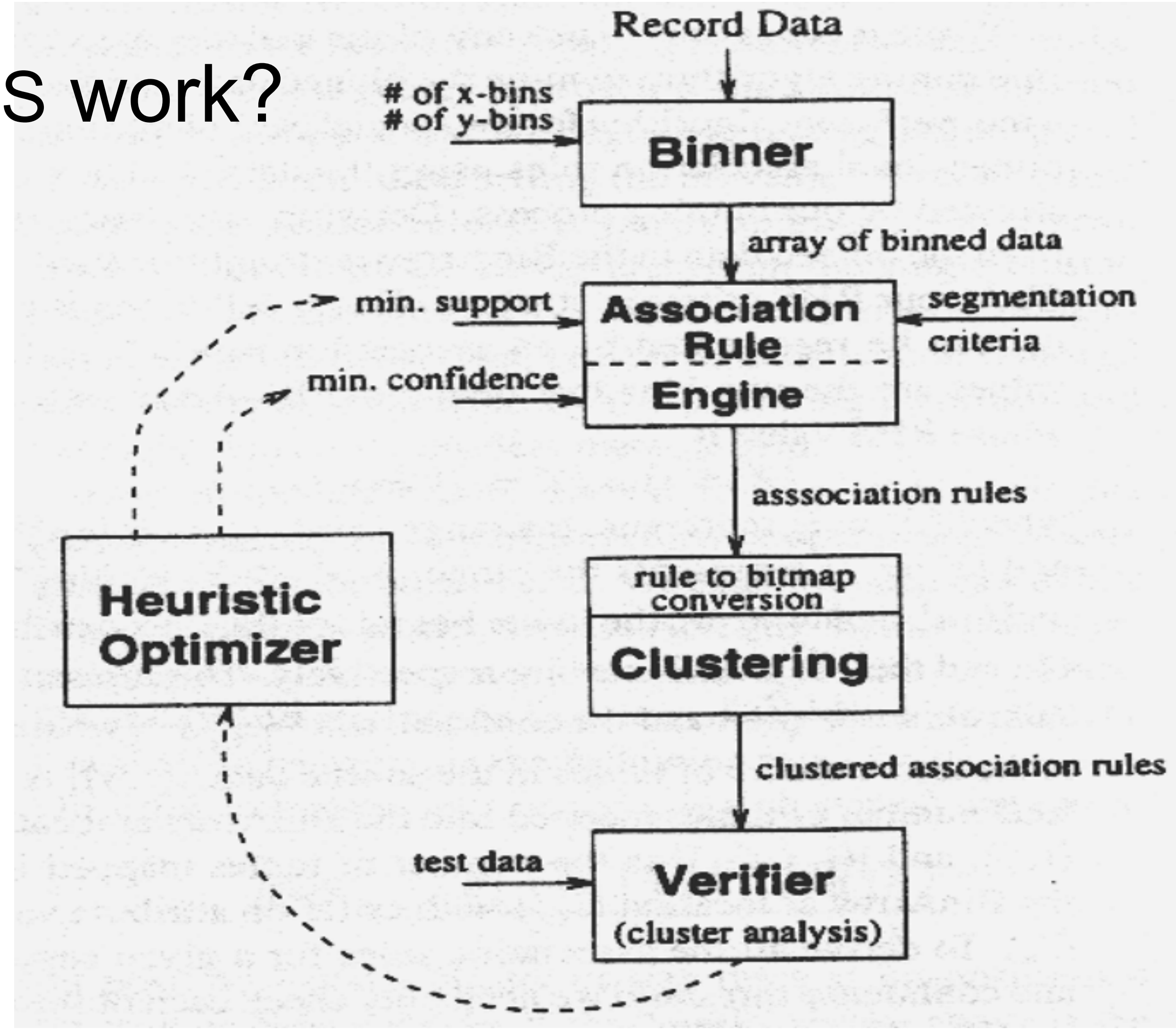
How does ARCS work?

1. Binning

2. Find frequent predicateset

3. Clustering

4. Optimize



# Limitations of ARCS

- Only quantitative attributes on LHS of rules.
- Only 2 attributes on LHS. (2D limitation)
- An alternative to ARCS
  - Non-grid-based
  - equi-depth binning
  - clustering based on a measure of *partial completeness*.
  - "***Mining Quantitative Association Rules in Large Relational Tables***" by R. Srikant and R. Agrawal.

# Clusters and Distance Measurements

- The diameter, *d,* assesses the density of a cluster $C_X$, where

$$d(C_X) \le d_0^{\,X}$$

$$|C_X| \ge s_0$$

- Finding clusters and distance-based rules
  - the density threshold, $d_0$, replaces the notion of support
  - modified version of the BIRCH clustering algorithm

# Mining Distance-based Association Rules

- Binning methods do not capture the semantics of interval data

| Price($) | Equi-width (width $10) | Equi-depth (depth 2) | Distance-based |
|---|---|---|---|
| 7 | [0,10] | [7,20] | [7,7] |
| 20 | [11,20] | [22,50] | [20,22] |
| 22 | [21,30] | [51,53] | [50,53] |
| 50 | [31,40] | | |
| 51 | [41,50] | | |
| 53 | [51,60] | | |

- Distance-based partitioning, more meaningful discretization considering:
  - density/number of points in an interval
  - "closeness" of points in an interval

# Clusters and Distance Measurements

- S[X] is a set of N tuples $t_1, t_2, \ldots, t_N$, projected on the attribute set X
- The diameter of S[X]:

$$d(S[X]) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} dist_X(t_i[X], t_j[X])}{N(N-1)}$$

  – $dist_x$:distance metric, e.g. Euclidean distance or Manhattan

# Chapter 6: Mining Association Rules in Large Databases

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
- Mining multilevel association rules from transactional databases
- Mining multidimensional association rules from transactional databases and data warehouse
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

# Interestingness Measurements

- Objective measures

  Two popular measurements:

  ❶ *support;* and

  ❷ *confidence*

- Subjective measures (Silberschatz & Tuzhilin, KDD95)

  A rule (pattern) is interesting if

  ❶ it is *unexpected* (surprising to the user); and/or

  ❷ *actionable* (the user can do something with it)

# Criticism to Support and Confidence

- Example 2:
  - X and Y: positively correlated,
  - X and Z, negatively related
  - support and confidence of X=>Z dominates

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| X | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| Y | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Z | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| Rule | Support | Confidence |
|---|---|---|
| X=>Y | 25% | 50% |
| X=>Z | 37.50% | 75% |

# Other Interestingness Measures: Interest

- **Interest** $$\frac{P(A \wedge B)}{P(A)P(B)}$$
  - taking both $P(A)$ and $P(B)$ in consideration
  - $P(A \wedge B) = P(B) * P(A)$, if A and B are independent events
  - A and B negatively correlated, if the value is less than 1; otherwise A and B positively correlated.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| X | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| Y | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Z | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| Itemset | Support | Interest |
|---|---|---|
| X,Y | 25% | 2 |
| X,Z | 37.50% | 0.9 |
| Y,Z | 12.50% | 0.57 |

# Chapter 6: Mining Association Rules in Large Databases

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
- Mining multilevel association rules from transactional databases
- Mining multidimensional association rules from transactional databases and data warehouse
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

# Constraint-Based Mining

- Interactive, exploratory mining giga-bytes of data?
  - Could it be real? — Making good use of constraints!
- What kinds of constraints can be used in mining?
  - Knowledge type constraint: classification, association, etc.
  - Data constraint: SQL-like queries
    - Find product pairs sold together in **Vancouver in Dec.' 98**
  - Dimension/level constraints:
    - in relevance to **region, price, brand, customer category**
    - small sales (price < \$10) triggers big sales (sum > \$200).
  - Interestingness constraints:
    - strong rules (min_support ≥ 3%, min_confidence ≥ 60%).

# Rule Constraints in Association Mining

- Two kind of rule constraints:
  - Rule form constraints: meta-rule guided mining.
    - P(x, y) ^ Q(x, w) → takes(x, "database systems").
  - Rule (content) constraint: constraint-based query optimization (Ng, et al., SIGMOD'98).
    - sum(LHS) < 100 ^ min(LHS) > 20 ^ count(LHS) > 3 ^ sum(RHS) > 1000
- 1-variable vs. 2-variable constraints (Lakshmanan, et al. SIGMOD'99):
  - 1-var: A constraint confining only one side (L/R) of the rule, e.g., as shown above.
  - 2-var: A constraint confining both sides (L and R).
    - sum(LHS) < min(RHS) ^ max(RHS) < 5* sum(LHS)

# Chapter 6: Mining Association Rules in Large Databases

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
- Mining multilevel association rules from transactional databases
- Mining multidimensional association rules from transactional databases and data warehouse
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

# Why Is the Big Pie Still There?

- More on constraint-based mining of associations
  - **Boolean vs. quantitative associations**
    - Association on discrete vs. continuous data
  - **From association to correlation and causal structure analysis.**
    - Association does not necessarily imply correlation or causal relationships
  - **From intra-trasanction association to inter-transaction associations**
    - E.g., break the barriers of transactions (Lu, et al. TOIS' 99).
  - **From association analysis to classification and clustering analysis**
    - E.g, clustering association rules

# Summary

- **Association rule mining**
  - probably the **most significant** **contribution** from the database community in KDD
  - A large number of papers have been published
- Many interesting issues have been explored
- An interesting research direction:
  - **Association analysis in other types of data**: spatial data, multimedia data, time series data, etc.