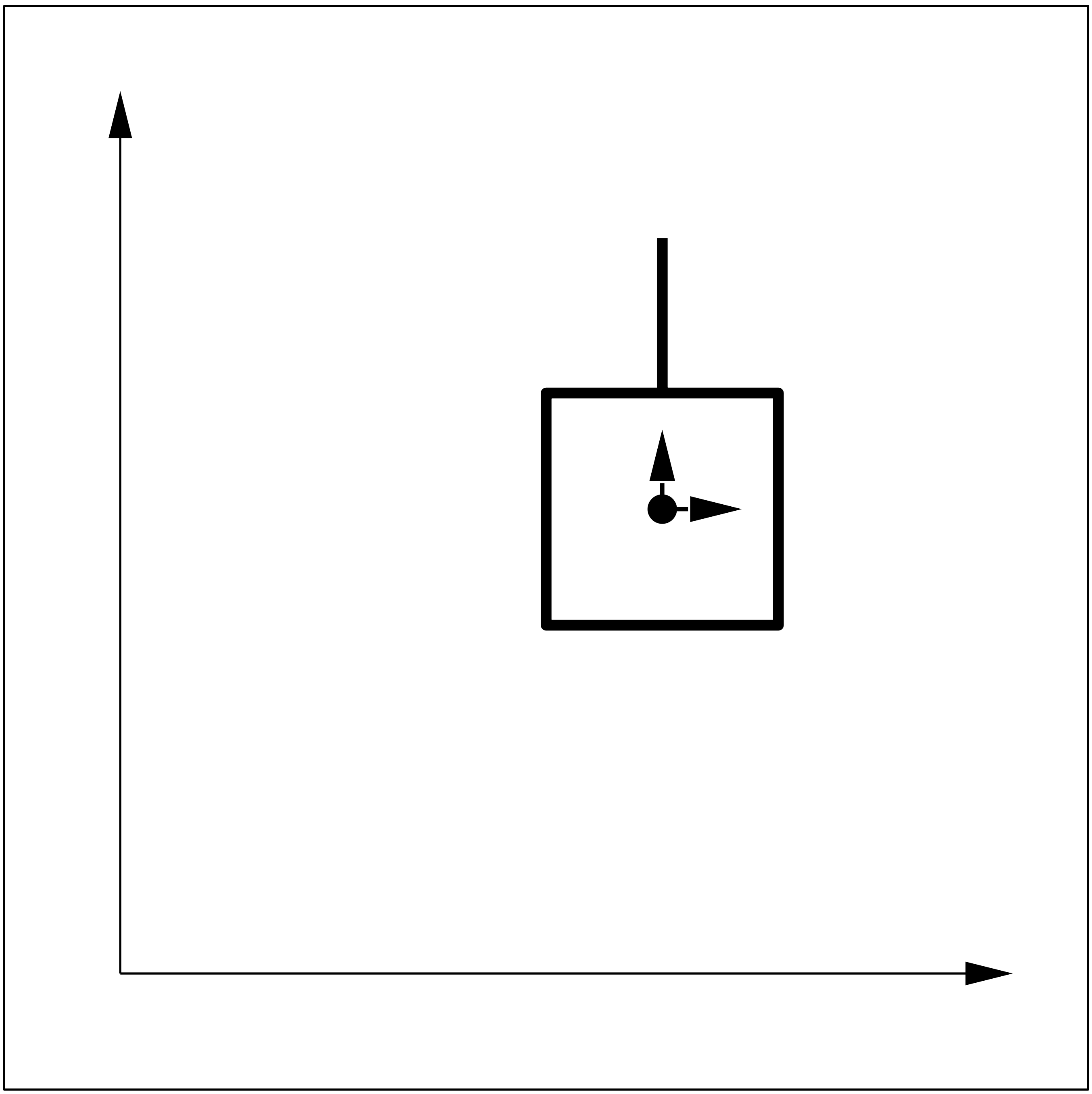
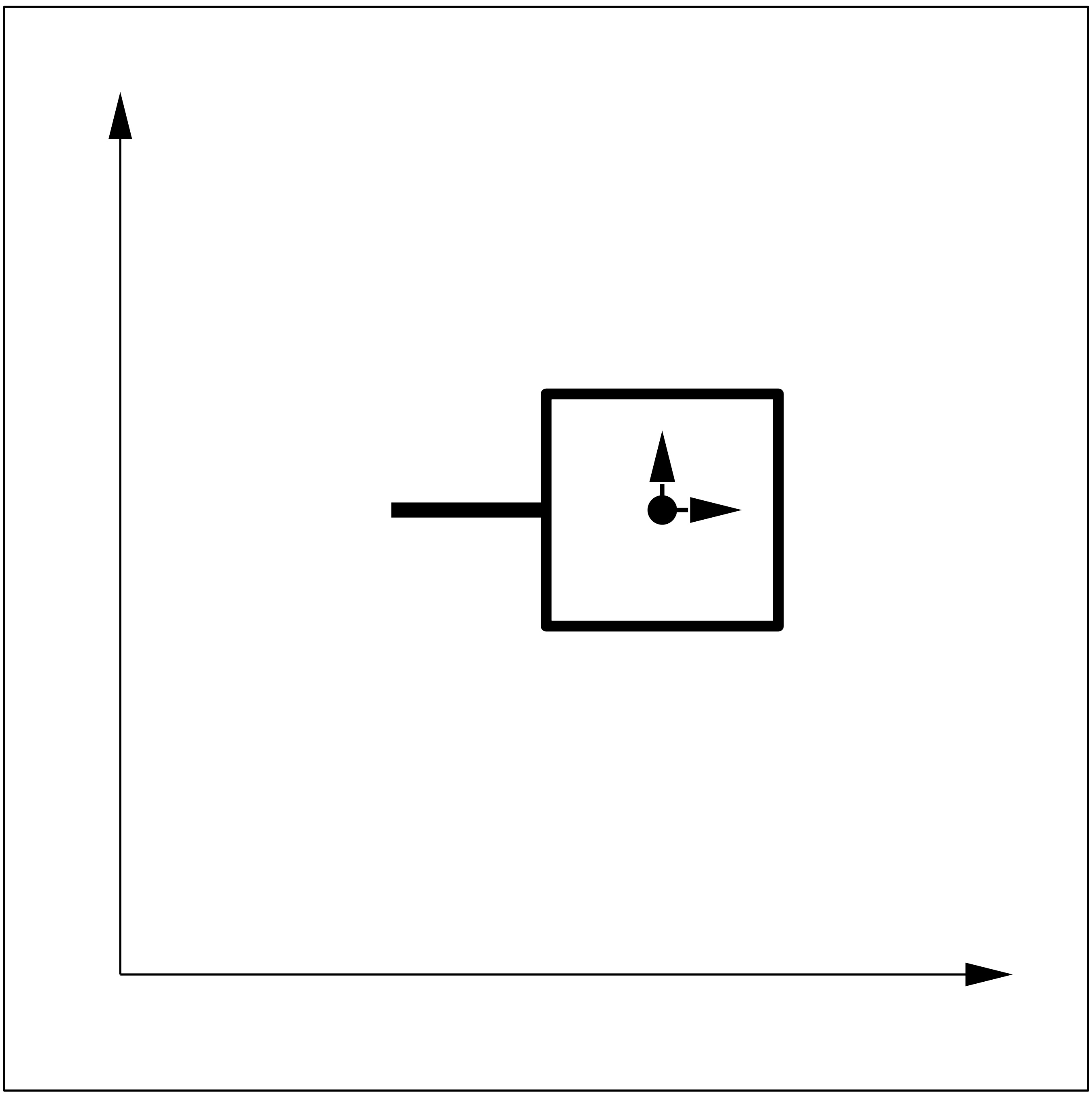
# Arbitrary Rotation



# Arbitrary Rotation



# More Examples

- **More complicated examples**
  **rotation about an arbitrary point**
  **(1) translation**

$$\begin{bmatrix} 1 & 0 & -\mathbf{p}_x \\ 0 & 1 & -\mathbf{p}_y \\ 0 & 0 & 1 \end{bmatrix}$$

**(2) rotation**

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**(3) translation again**

$$\begin{bmatrix} 1 & 0 & \mathbf{p}_x \\ 0 & 1 & \mathbf{p}_y \\ 0 & 0 & 1 \end{bmatrix}$$

# Important Properties

- Two rotations are additive

$$\mathbf{R}(\theta_1) \star \mathbf{R}(\theta_2) = \mathbf{R}(\theta_1 + \theta_2)$$

- Two rotations are commutative

$$\mathbf{R}(\theta_1) \star \mathbf{R}(\theta_2) = \mathbf{R}(\theta_2) \star \mathbf{R}(\theta_1)$$

- Two translations are commutative

$$\mathbf{T}(\delta x_1, \delta y_1) \star \mathbf{T}(\delta x_2, \delta y_2) = \mathbf{T}(\delta x_2, \delta y_2) \star \mathbf{T}(\delta x_1, \delta y_1)$$

- Two scalings are commutative

$$\mathbf{S}(s_{x_1}, s_{y_1}) \star \mathbf{S}(s_{x_2}, s_{y_2}) = \mathbf{S}(s_{x_2}, s_{y_2}) \star \mathbf{S}(s_{x_1}, s_{y_1})$$

- What about
  one rotation and one translation
  one rotation and one scaling
  one translation and one scaling

- What about involving shearing?

- Please verify your results

# Coordinate Systems

- Transformation between two different coordinate systems
  Given objects in one coordinate system
  Figure out their location(s) in the second coordinate system

- Let's consider several simple cases !

- One system is obtained from one translation of the second system
  In coordinate system 1, a point has the following coordinates:

$$\mathrm{v}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

  In coordinate system 2, the SAME point has the following coordinates:

$$\mathrm{v}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

**How to determine the matrix:**

$$\mathrm{M}_{1,2} = \mathrm{T}(\delta x, \delta y)$$

**so that this matrix transforms the coordinates of the SAME point from CS-1 to CS-2:**

$$\mathrm{M}_{1,2}\mathbf{p}_1 = \mathbf{p}_2$$

- **Note that, $\mathrm{M}_{1,2}$ is derived by transforming CS-2 to CS-1 using coordinate values in CS-1 !!!**

$$\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$$

$$\delta x = x_2 - x_1$$

$$\delta y = y_2 - y_1$$

- **One system is obtained from one rotation of the second system**

$$\mathrm{M}_{1,2}\mathbf{p}_1 = \mathbf{p}_2$$

$$\mathrm{M}_{1,2} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- **One translation and one rotation are involved !**

$$\mathbf{M}_{1,2}\mathbf{p}_1 = \mathbf{p}_2$$

$$\mathbf{M}_{1,2} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \delta x \\ 0 & 1 & \delta y \\ 0 & 0 & 1 \end{bmatrix}$$

- **Geometric Meaning of the above formulation ???**

- **First, translation**

$$\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$$

**Please pay attention to the coordinates of** $\mathbf{v}_1$ **and** $\mathbf{v}_2$ **!!!**

$$T(\delta x, \delta y)\mathbf{p}_1 = \mathbf{v}_2$$

**Note that, the value of** $\mathbf{v}_2$
**is NOT the coordinates of** $\mathbf{p}_2$ **!!!**

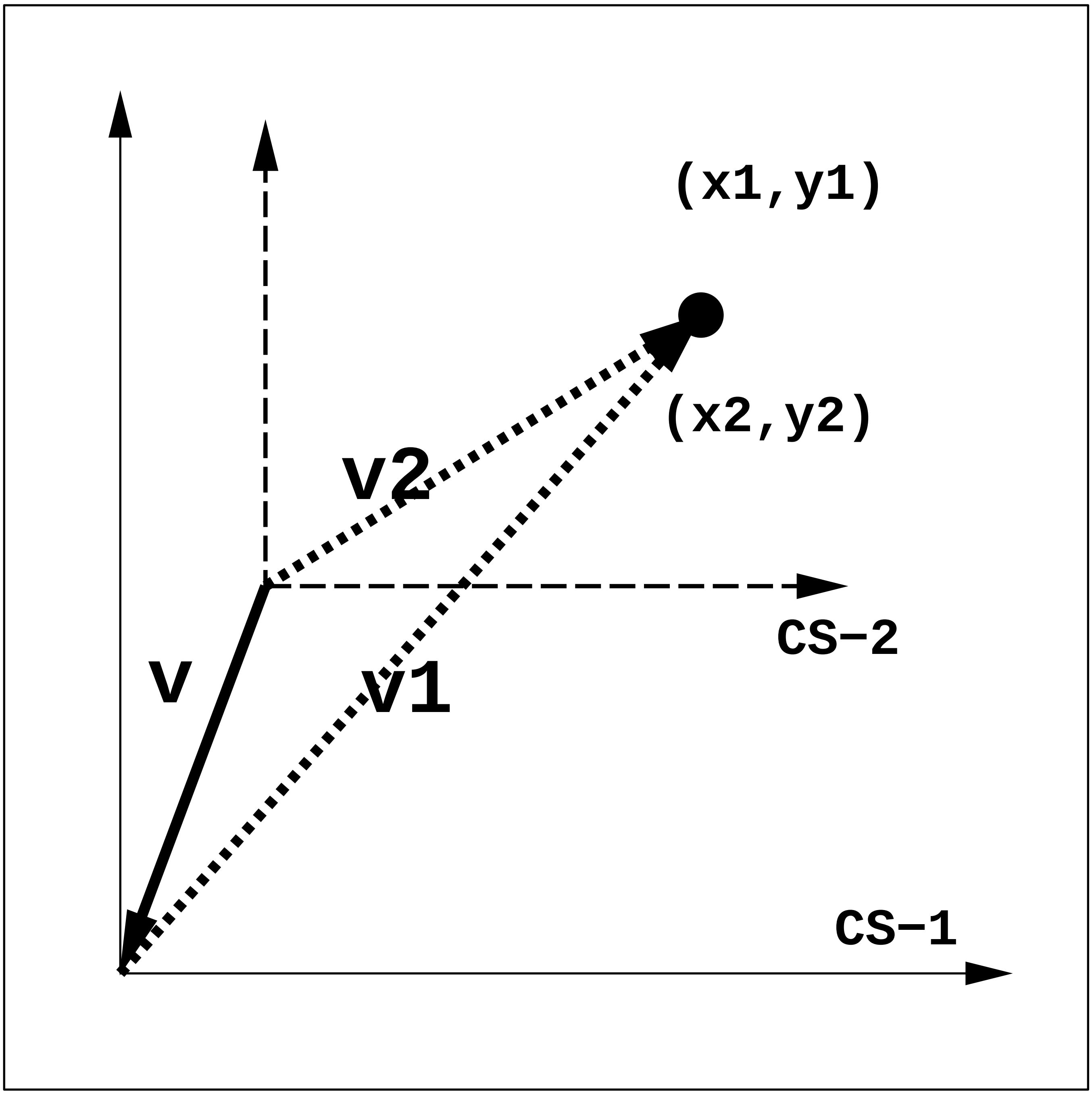
- **Second, rotation**

$$R(-\theta)\mathbf{v}_2 = \mathbf{p}_2$$

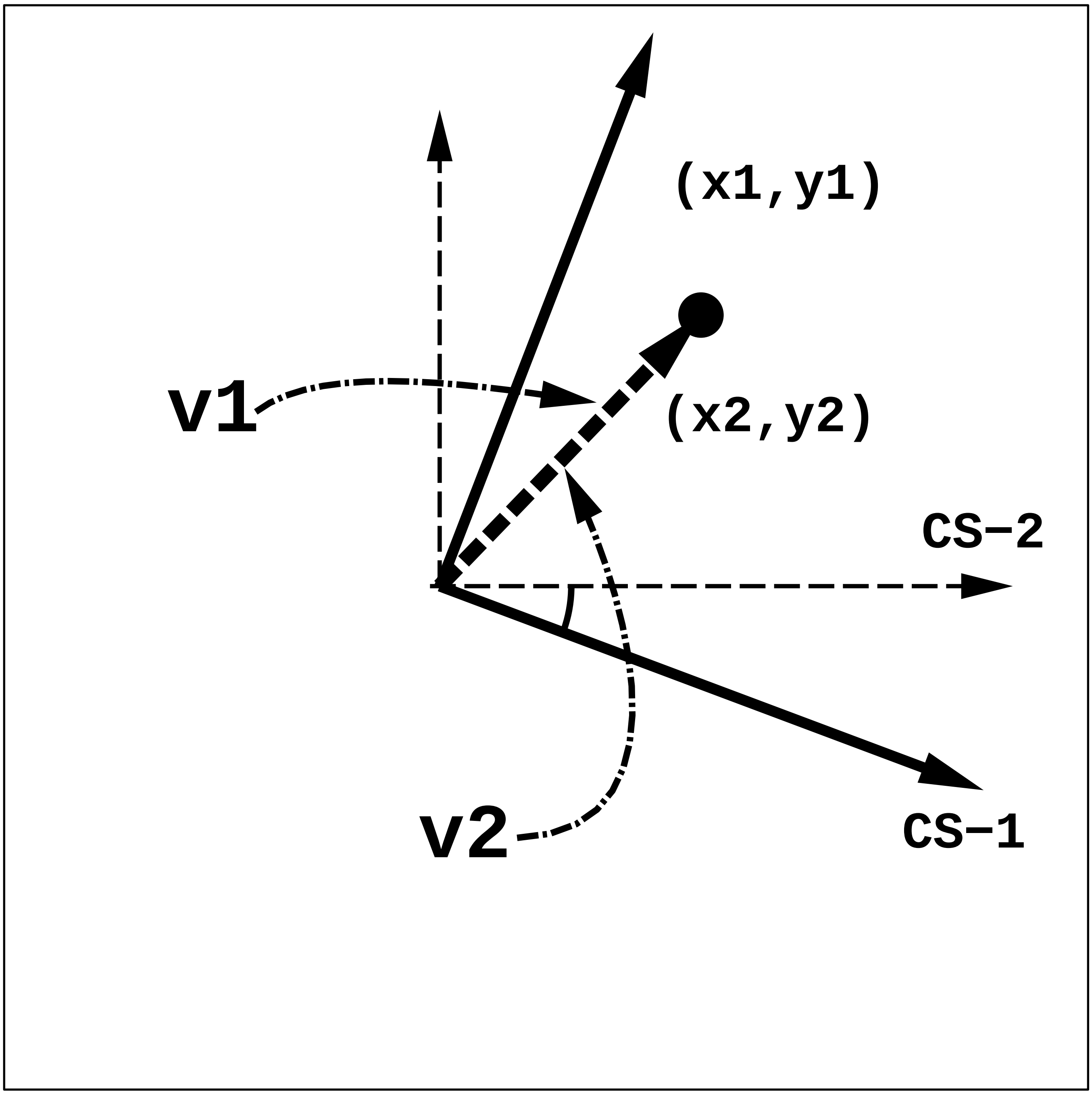
- **Let us put them together**

$$R(-\theta) \star T(\delta x, \delta y) \star \mathbf{p}_1 = \mathbf{p}_2$$

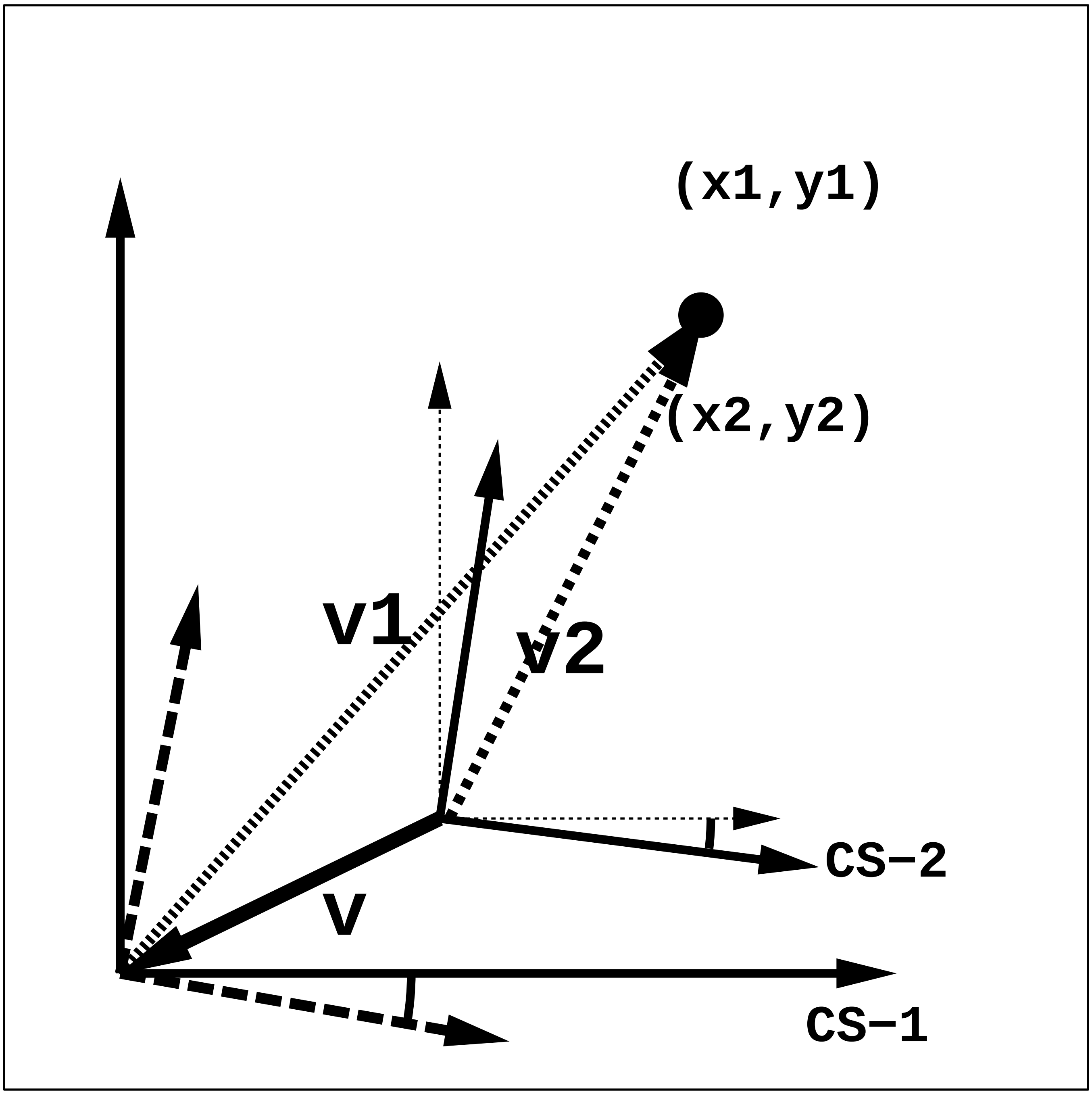
# Coordinate Systems



# Coordinate Systems



# Coordinate Systems



# Summary

- Why transformation
- Basis transformation operations
- Composite transformation operations
- Why homogeneous coordinates
- Transformation matrices using homogeneous coordinates
- Transformation between different coordinate systems