CSE328 Fundamentals of Computer Graphics: Concepts, Theory, Algorithms, and Applications

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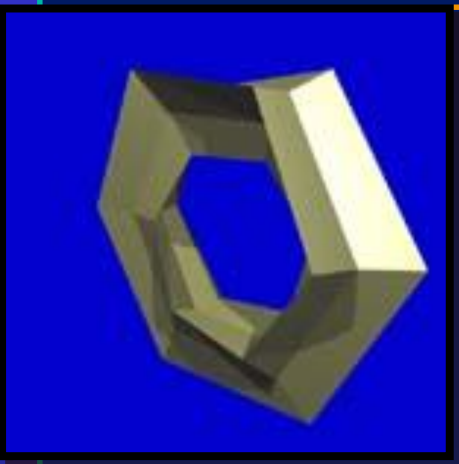
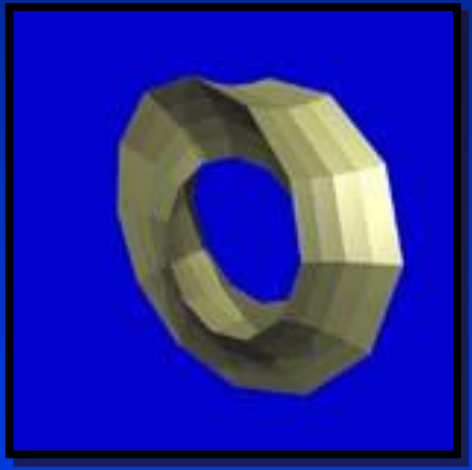
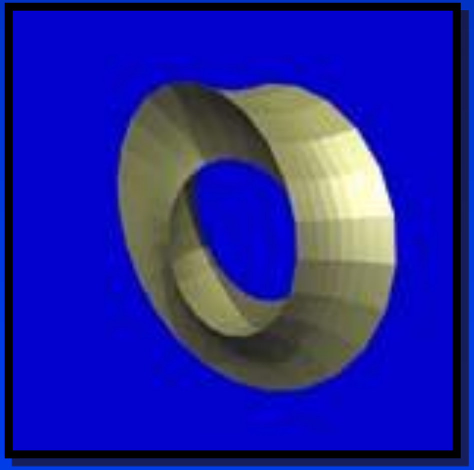
Subdivision Surfaces

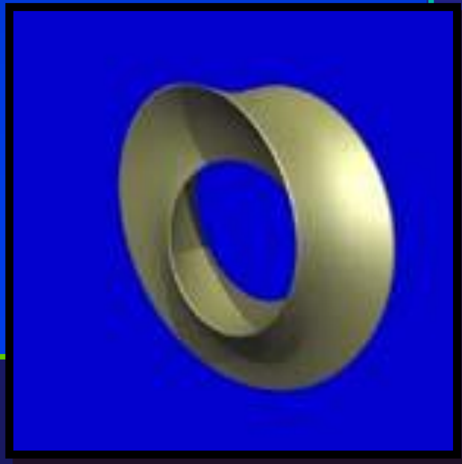


Subdivision surface

(different levels of refinement)





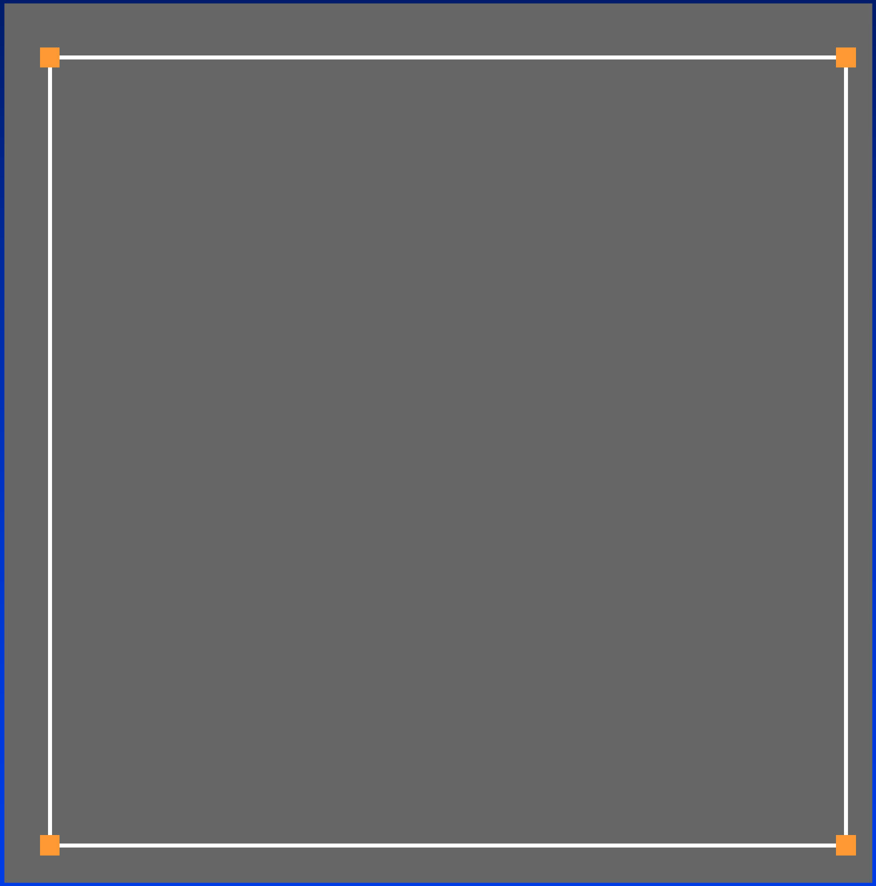


Subdivision Schemes in Interactive Surface Design

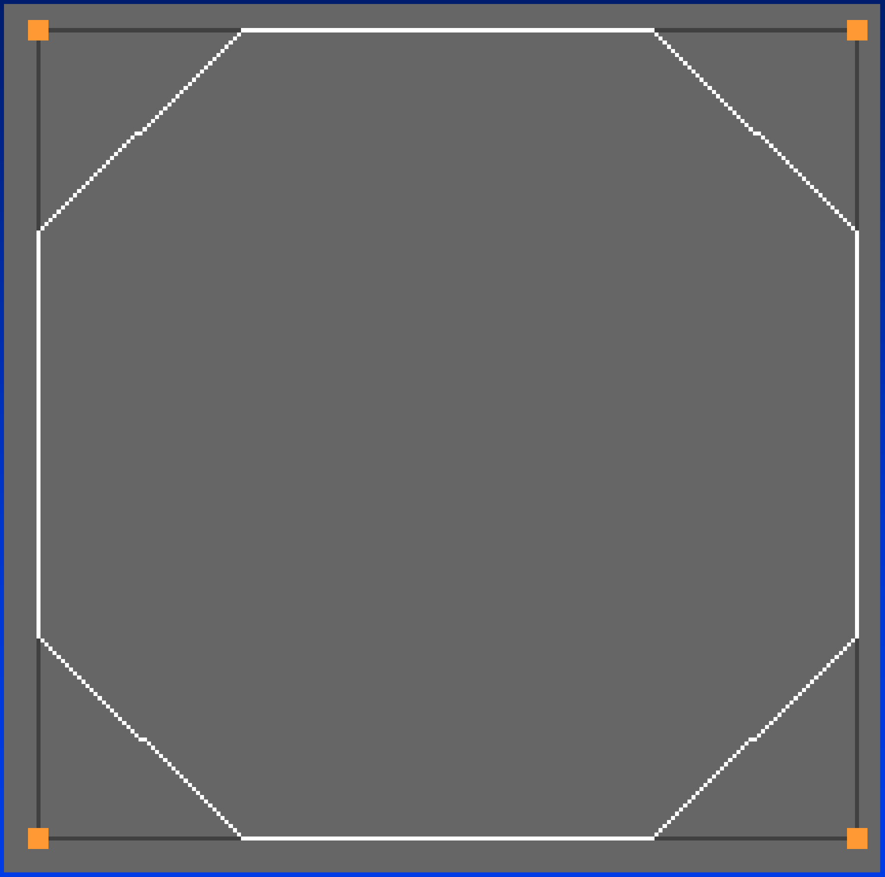
Basic Idea of Subdivision

- Start from an initial control polygon.
- Recursively refine it by some rules.
- A smooth surface (curve) in the limit.

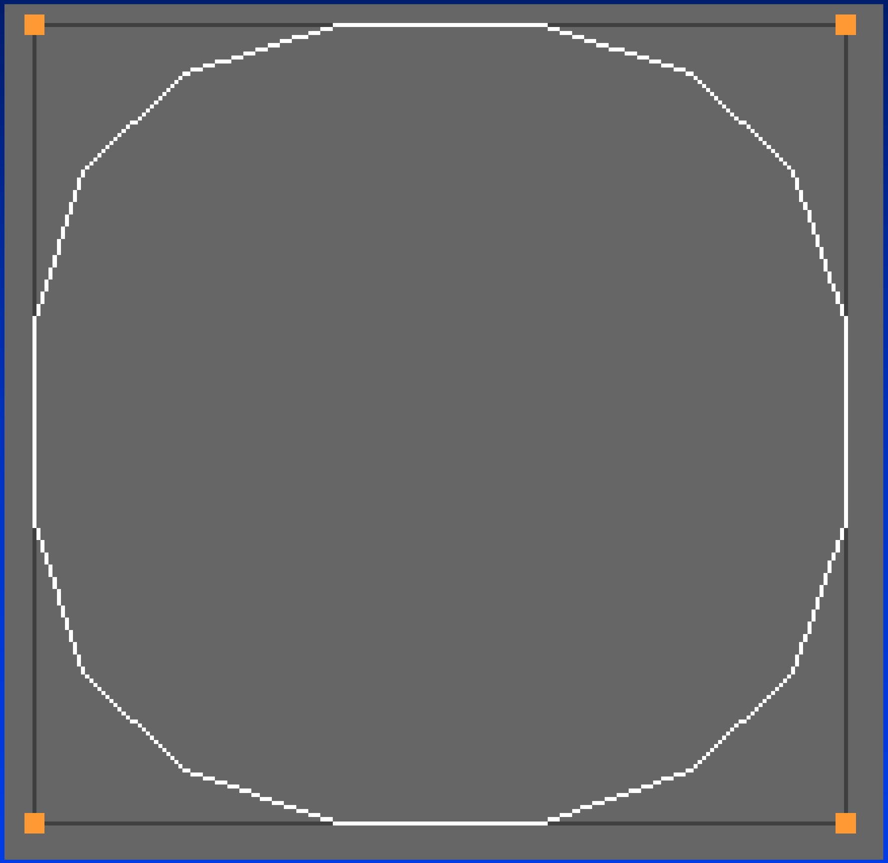
Chaikin's Corner Cutting Scheme



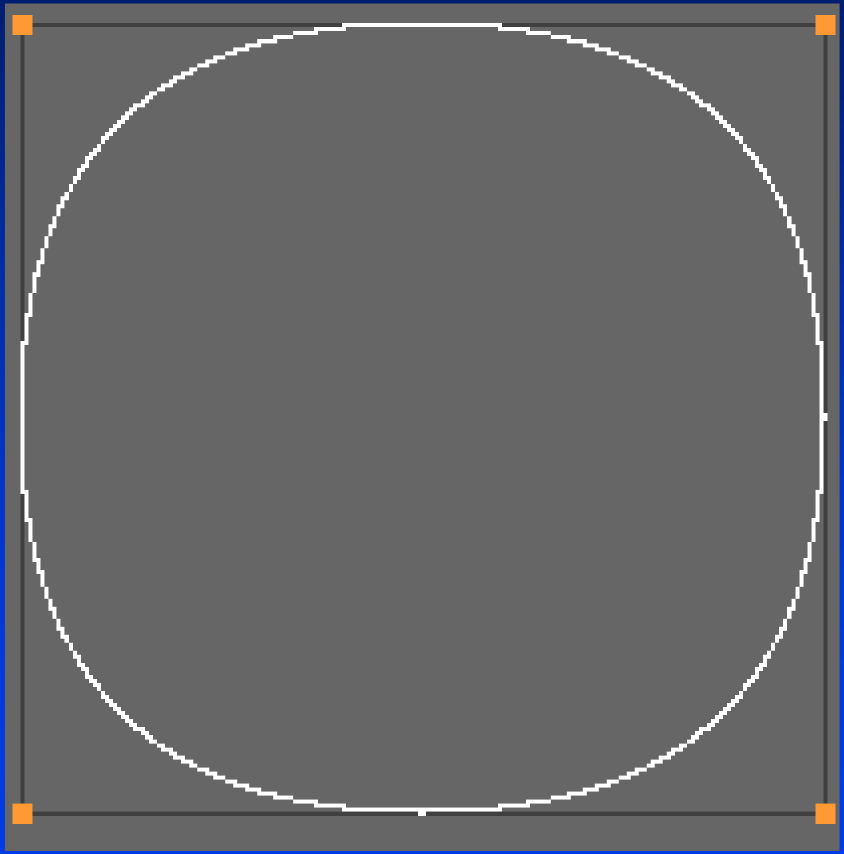
Chaikin's Corner Cutting Scheme



Chaikin’s Corner Cutting Scheme



Chaikin's Corner Cutting Scheme



Chaikin's Algorithm

- A set of control points to define a polygon

$$\mathbf{p}_0^0, \mathbf{p}_1^0, \mathbf{p}_2^0, \ldots, \mathbf{p}_n^0$$

- Subdivision process (more control vertices)
- Rules (corner chopping)

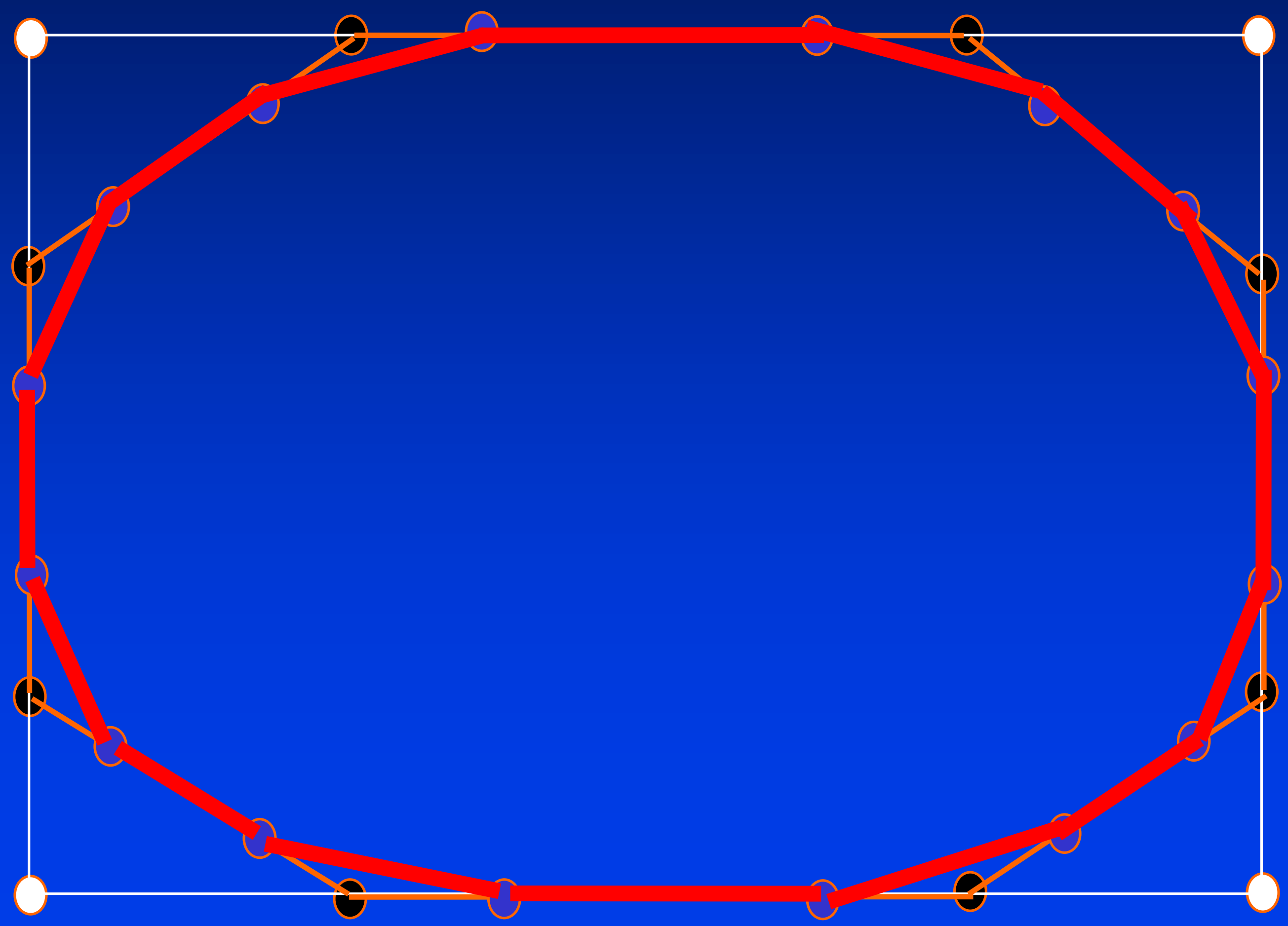
$$\mathbf{p}_0^k, \mathbf{p}_1^k, \mathbf{p}_2^k, \ldots \mathbf{p}_{2^k n}^k$$

$$\mathbf{P}_{2i}^{k+1} = \frac{3}{4}\mathbf{P}_i^k + \frac{1}{4}\mathbf{P}_{i+1}^k$$

$$\mathbf{P}_{2i+1}^{k+1} = \frac{1}{4}\mathbf{P}_i^k + \frac{3}{4}\mathbf{P}_{i+1}^k$$

- Properties:
  - quadratic B-spline curve, C1 continuous, tangent to each edge at its mid-point

Chaikin's Algorithm



Cubic Spline

- Subdivision rules

$$\mathbf{p}_{2i}^{k+1} = \frac{1}{2}\mathbf{p}_i^k + \frac{1}{2}\mathbf{p}_{i+1}^k$$

$$\mathbf{p}_{2i+1}^{k+1} = \frac{1}{4}\left(\frac{1}{2}\mathbf{p}_i^k + \frac{1}{2}\mathbf{p}_{i+2}^k\right) + \frac{3}{4}\mathbf{p}_{i+1}^k$$

- C2 cubic B-spline curve
- Corner-chopping
- No interpolation

Curve Interpolation

- Control points $\mathbf{p}_{-2}^{0}, \mathbf{p}_{-1}^{0}, \mathbf{p}_{0}^{0}, \cdots \mathbf{p}_{n+2}^{0}$
- Rules:

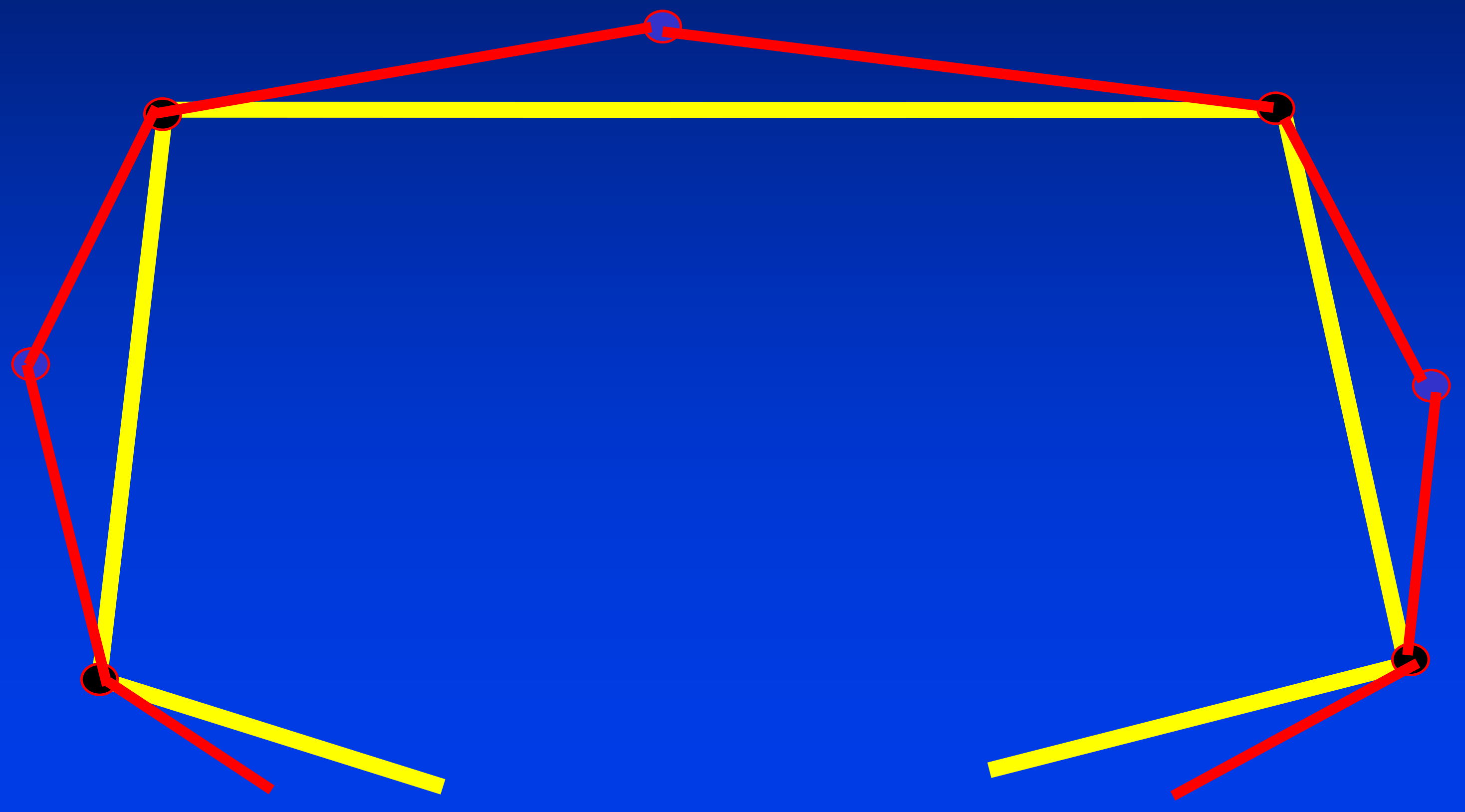
$$\mathbf{p}_{2i}^{k+1} = \mathbf{p}_{i}^{k}, -1 \le i \le 2^{k} n + 1$$

$$\mathbf{p}_{2i+1}^{k+1} = (\frac{1}{2} + w)(\mathbf{p}_{i}^{k} + \mathbf{p}_{i+1}^{k}) - w(\mathbf{p}_{i-1}^{k} + \mathbf{p}_{i+2}^{k}),$$

$$-1 \le i \le 2^{k} n$$

- At each stage, we keep all the OLD points and insert NEW points “in between” the OLD ones
- Interpolation!
- The behaviors and properties of the limit curve depend on the parameter w
- Generalize to SIX-point interpolatory scheme!

Curve Interpolation



Polygonal Meshes

Advantages:

- Very general.
- Can describe very fine detail accurately.
- Direct hardware implementation.

Disadvantages:

- Heavy weight representation.
- A simplification algorithm is always needed.

Subdivision Schemes

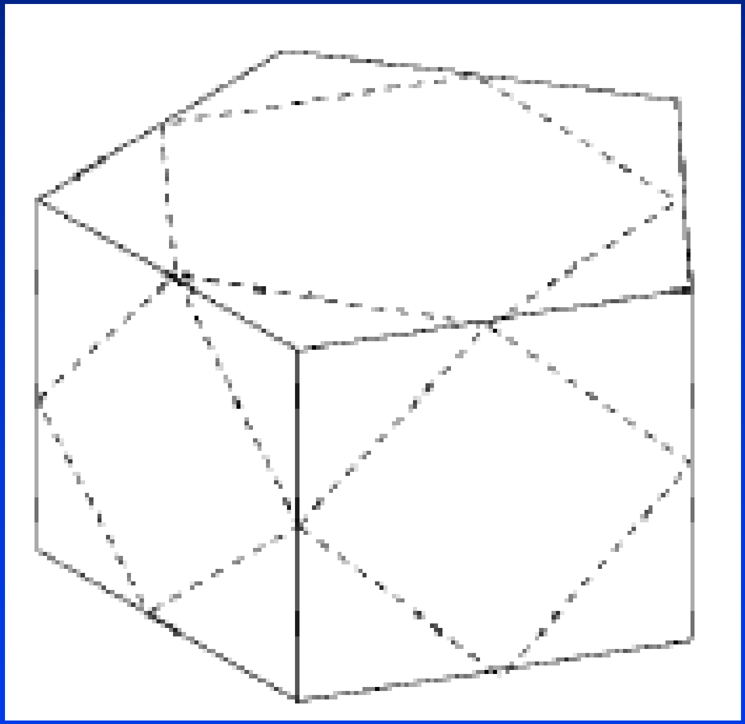
Advantages:

- Arbitrary topology.
- Level of detail.
- Unified representation.

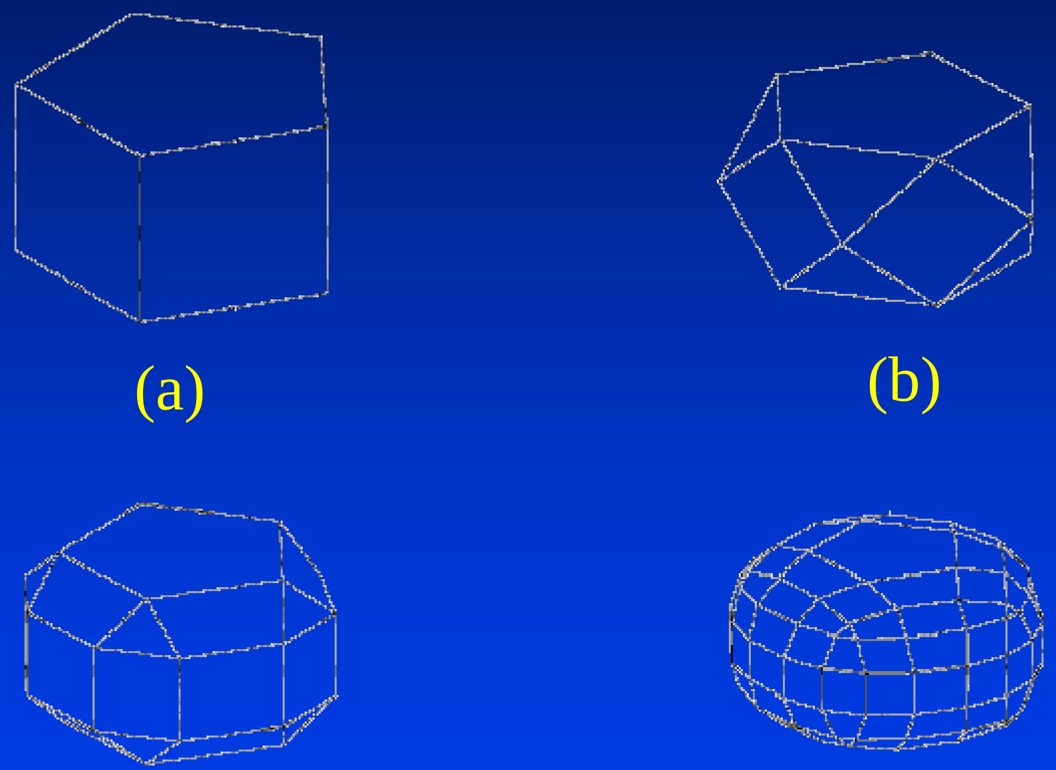
Disadvantages:

- Difficult for analysis of properties like smoothness and continuity.

Midedge Scheme



Midedge Scheme



(a) (b) (c) (d)

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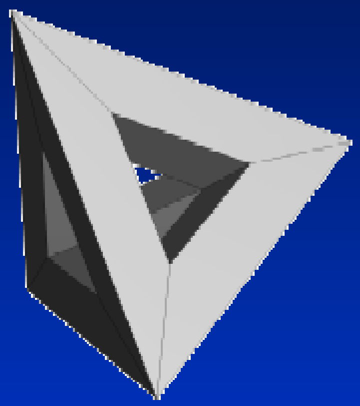
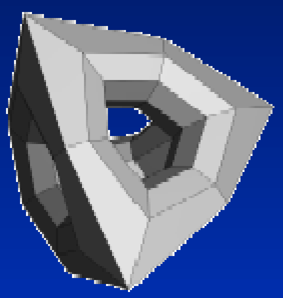
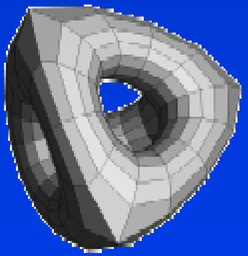
Catmull-Clark Scheme



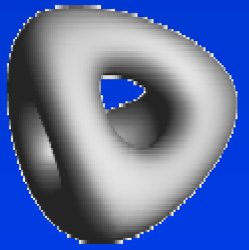
Initial mesh



Step 1

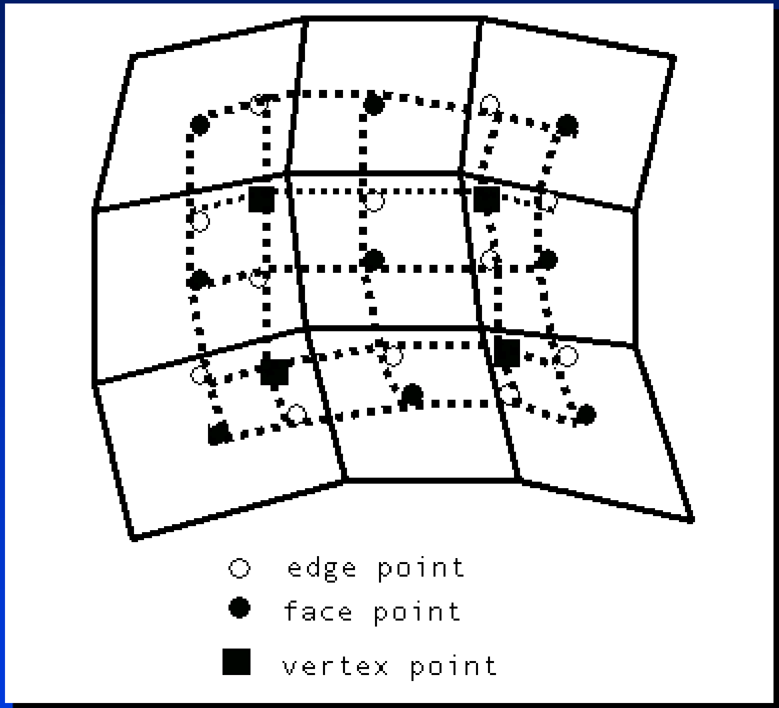


Step 2



Limit surface

Catmull-Clark Scheme



- **Face point:**

  the average of all the points defining the old face.

- **Edge point:**

  the average of two old vertices and two new face points of the faces adjacent to the edge.

- **Vertex point:** $(F + 2E + (n-3)V)/n$

  F: the average of the new face points of all faces adjacent to the old vertex.

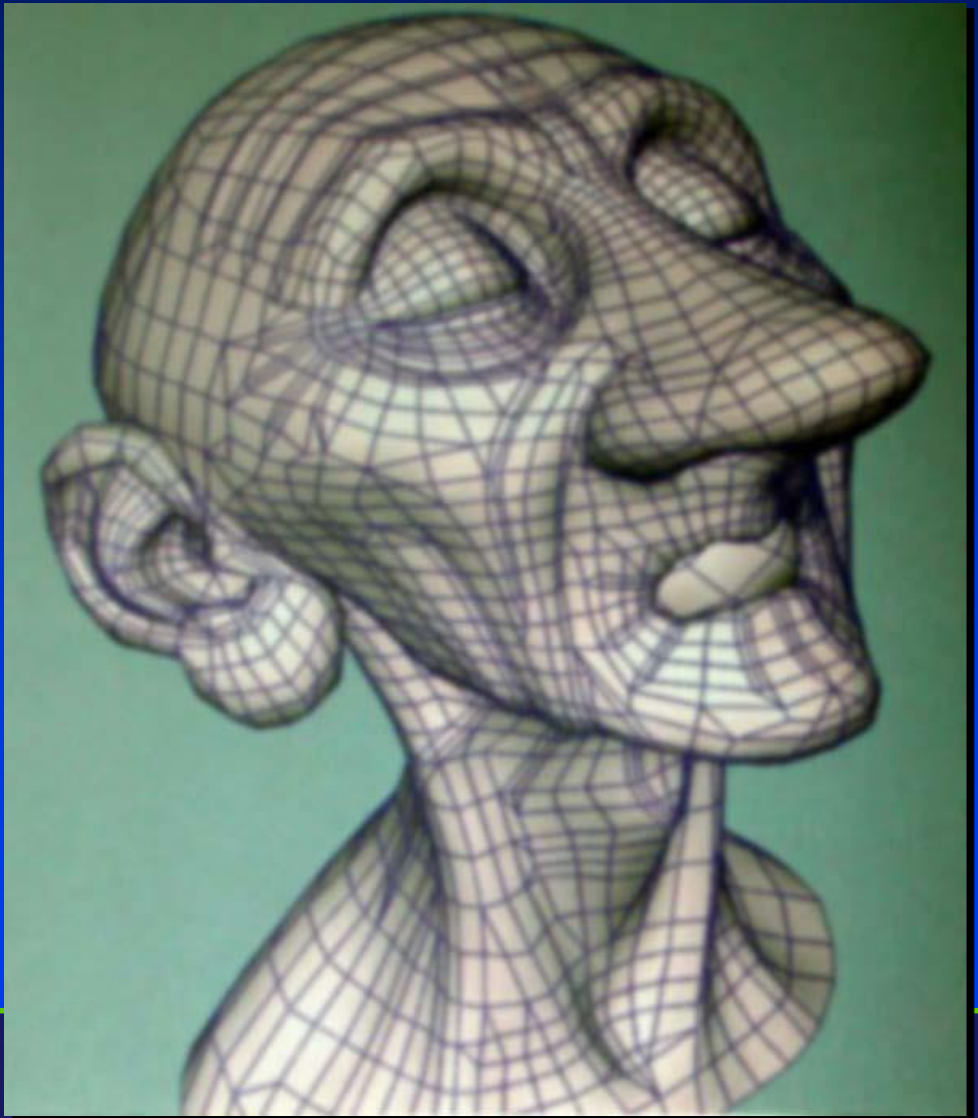
  E: the average of the midpoints of all adjacent edges.

  V: the old vertex.

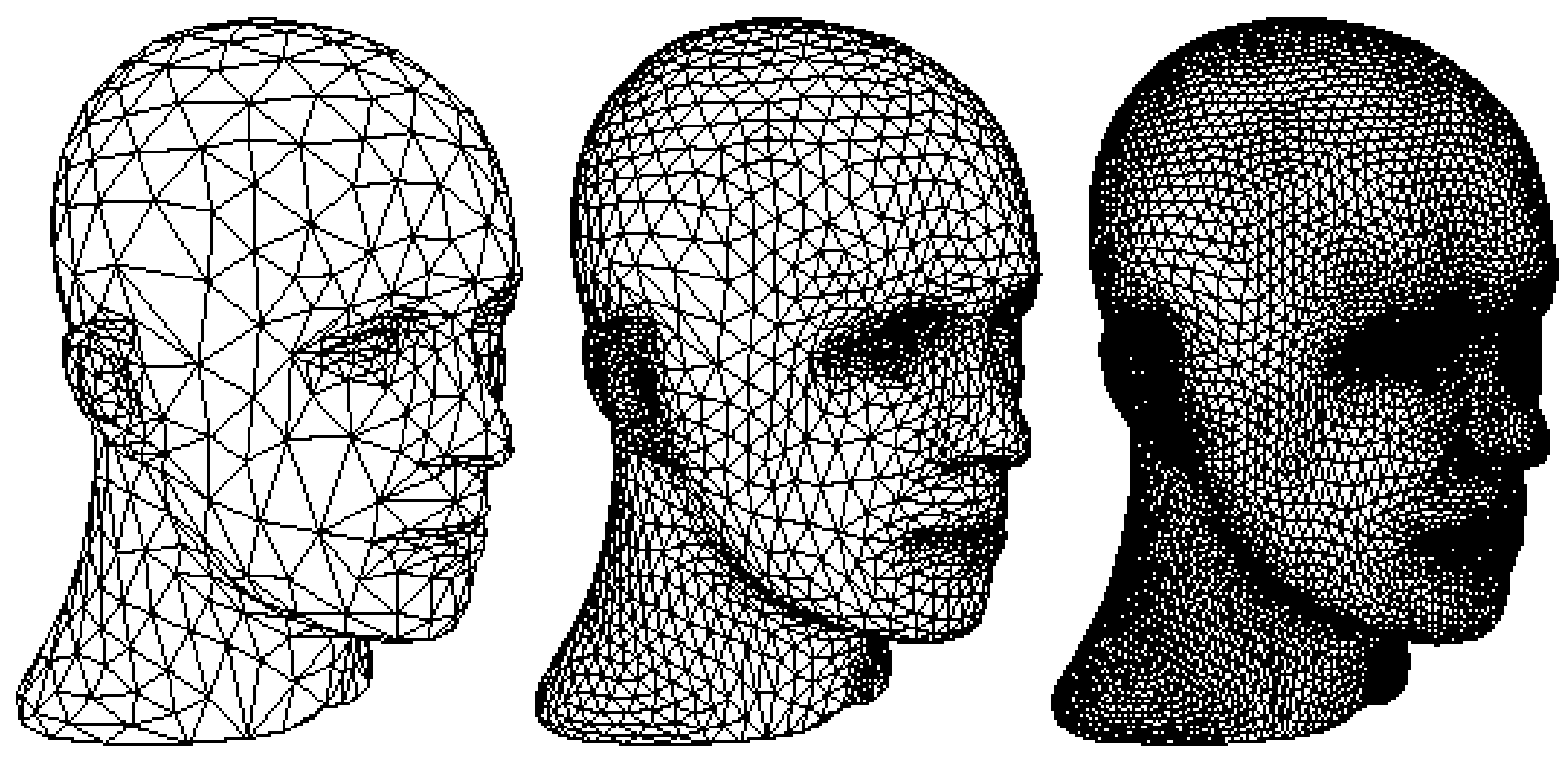
"Geri's Game"

DeRose et al. Siggraph 98

Mesh Structure

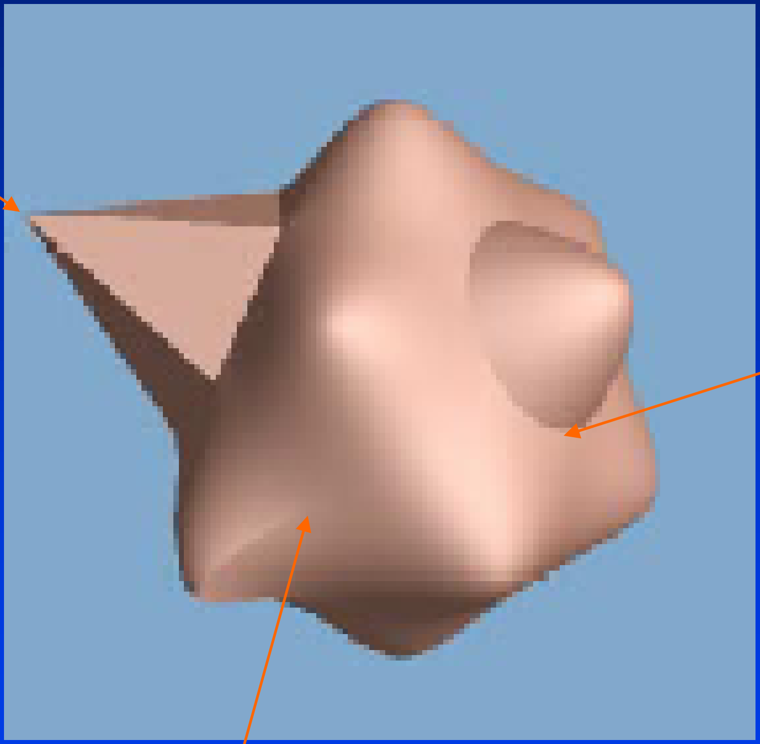


Interpolation Scheme

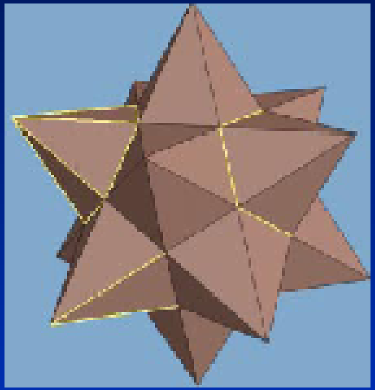


Initial mesh | One refinement step | Two refinement steps

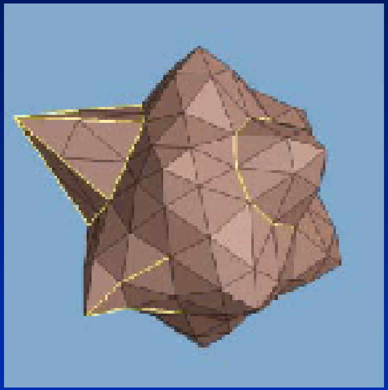
Modeling Sharp Features



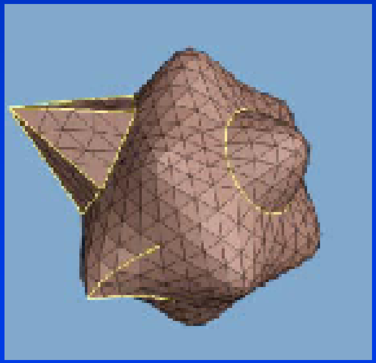
Piecewise Smooth Subdivision



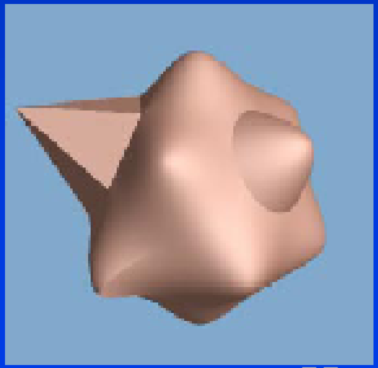
(a)



(b)



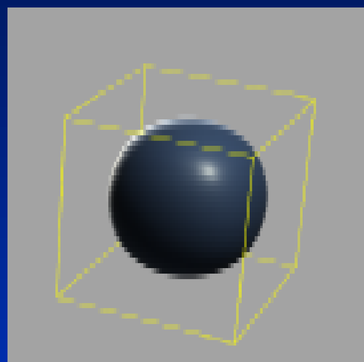
(c)



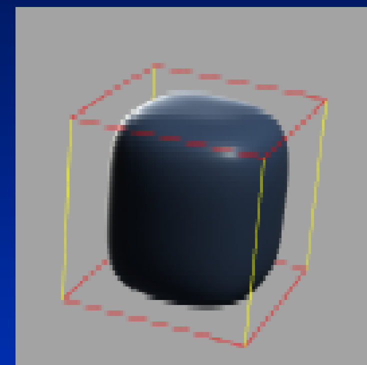
(d)

Hoppe et al. Siggraph 94

Hybrid Subdivision Scheme



(a)



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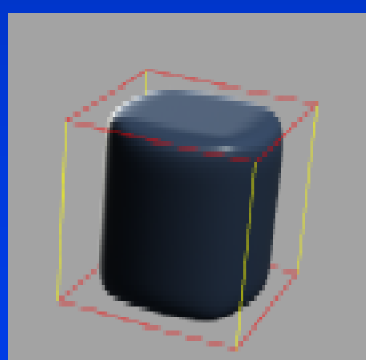
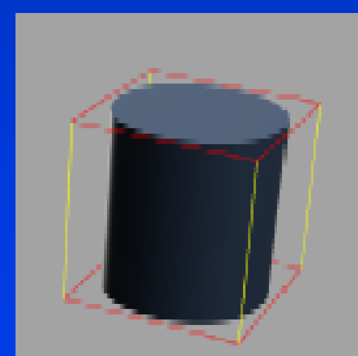


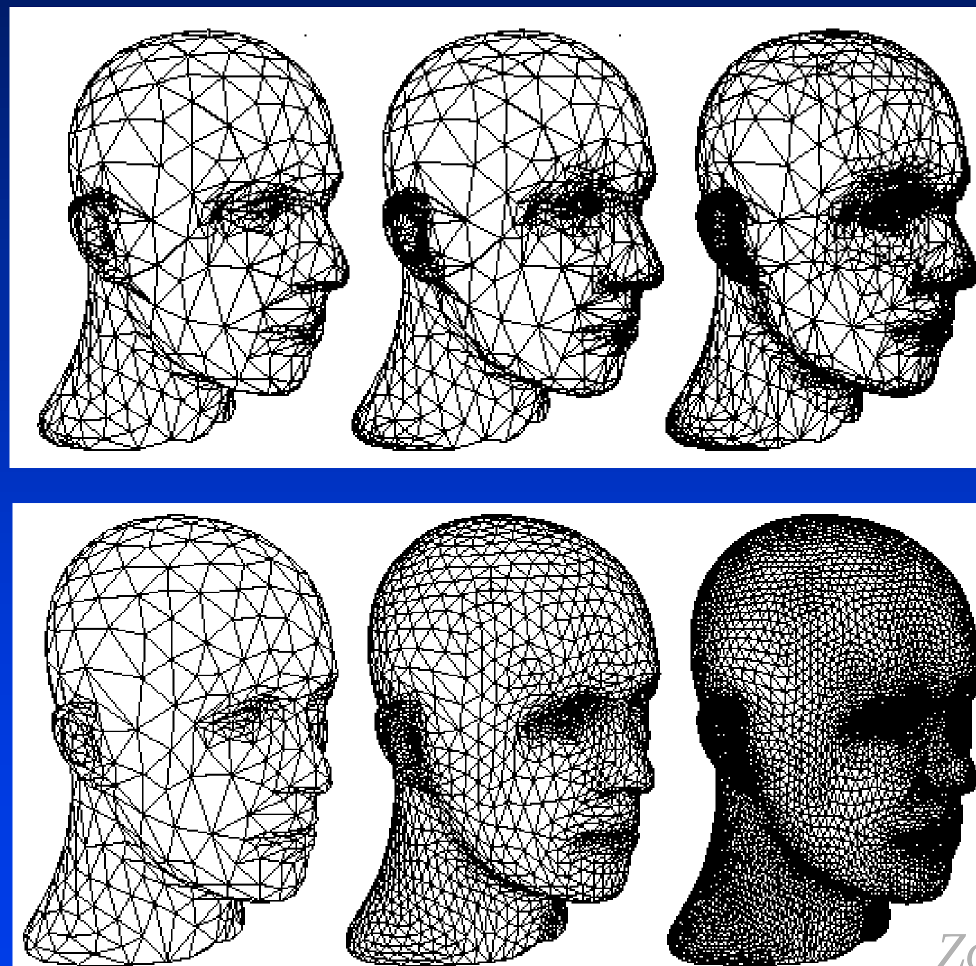
(c)



(d)

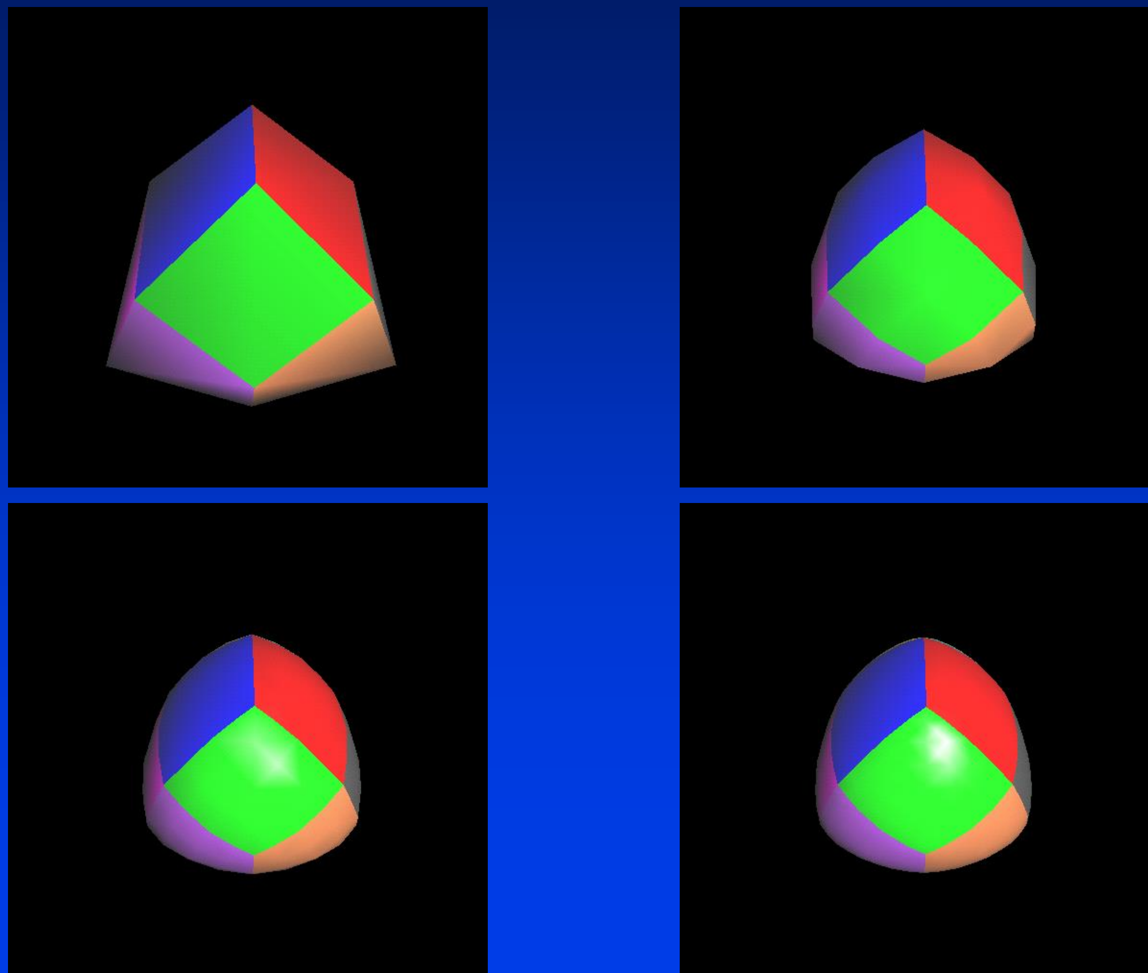
DeRose et al.
Siggraph 98

Hierarchical Editing

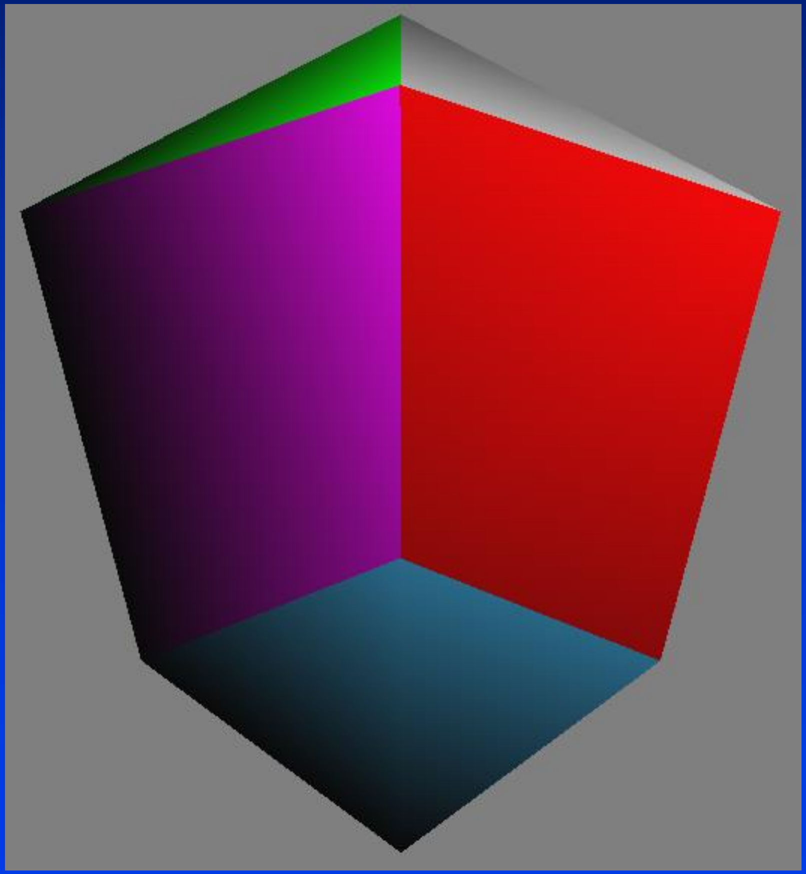


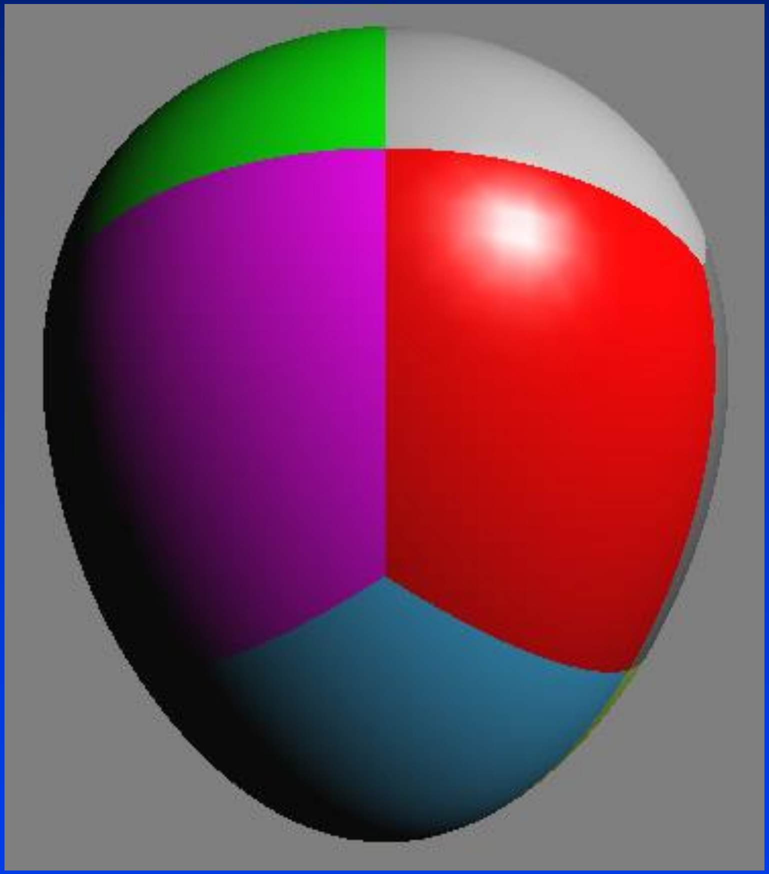
Zorin et al. Siggraph 97

Catmull-Clark Surface Example

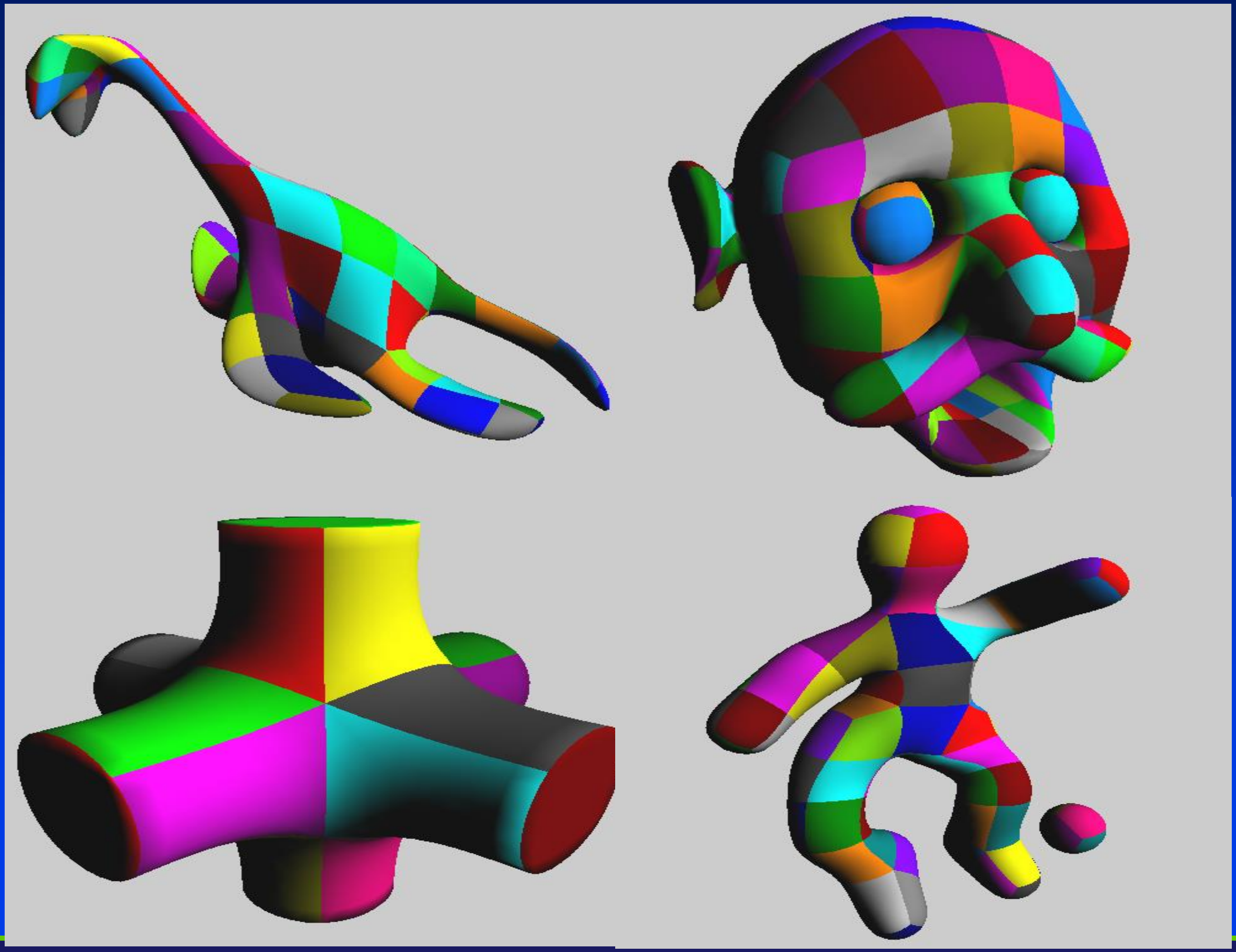


Catmull-Clark Patches

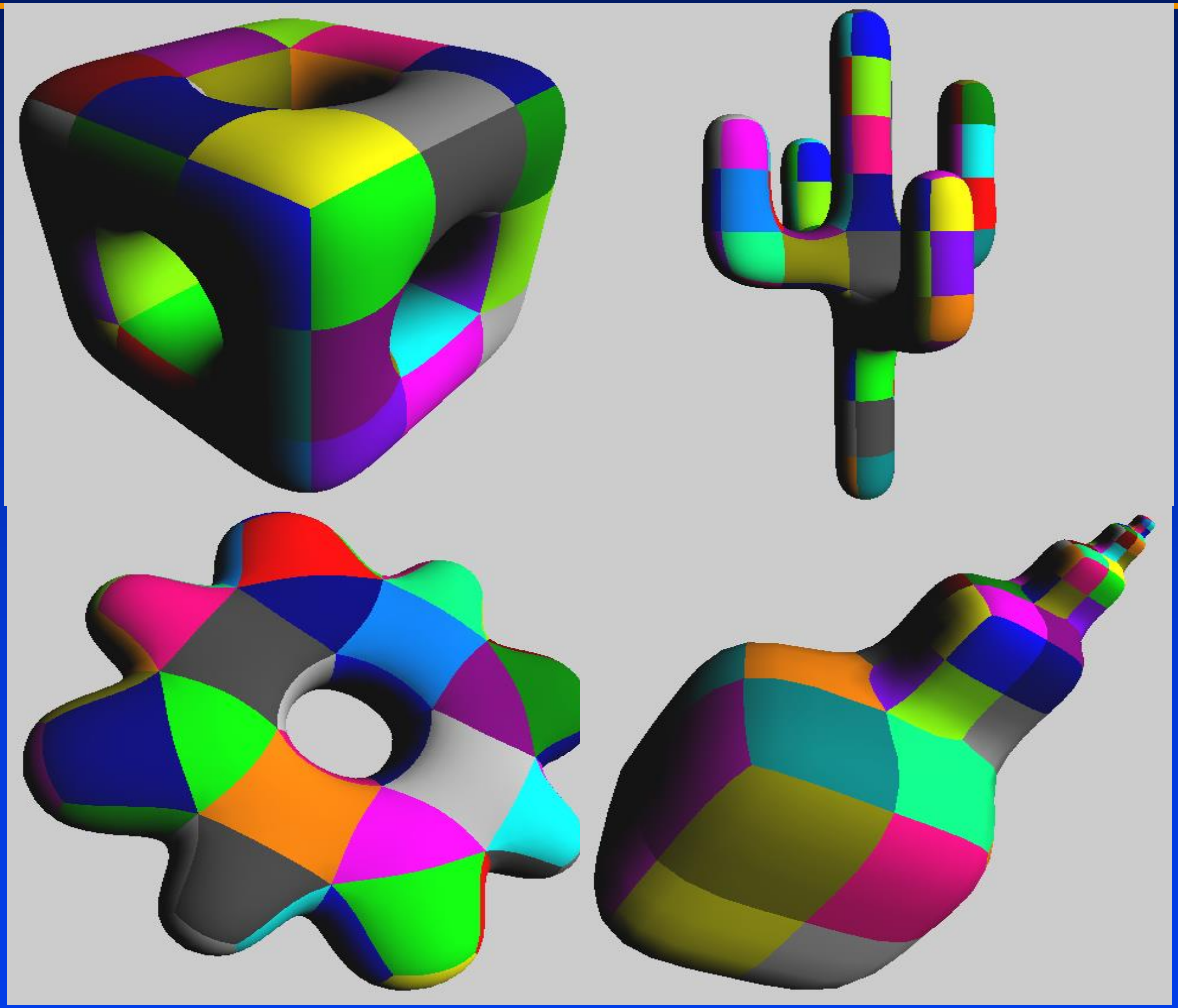




Interactive Sculpting



More Examples



Subdivision Solids

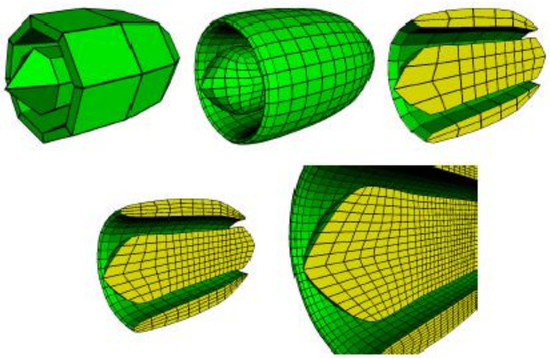
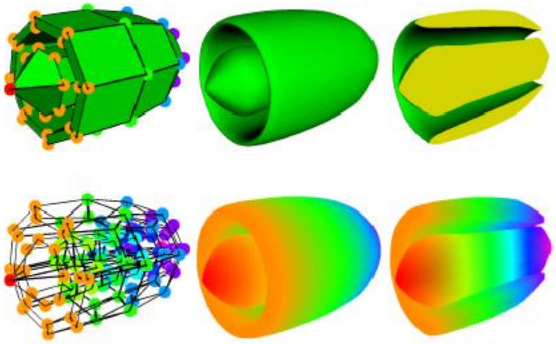


Fig. 22. Jet engine model comprised of two disconnected parts.



(a) (b) (c)

Fig. 23. Material properties can be interpolated smoothly throughout the entire volumetric domain. (a) Control mesh with color. (b-c) Model after three levels of subdivision.

Scenes and Sculptures