Plane Equation

- **Given three points, they determine a plane**

$$\mathbf{p}_a = \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}$$

$$\mathbf{p}_b = \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix}$$

$$\mathbf{p}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

where $\mathbf{p}_a$**,** $\mathbf{p}_b$**, and** $\mathbf{p}_c$ **are not co-linear!**

- **Normal of the plane**

$$\mathbf{n} = \frac{(\mathbf{p}_c - \mathbf{p}_a) \times (\mathbf{p}_b - \mathbf{p}_a)}{|(\mathbf{p}_c - \mathbf{p}_a) \times (\mathbf{p}_b - \mathbf{p}_a)|}$$

$$\mathbf{n} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

- **Arbitrary point on the plane**

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- **Equation (implicit function)**

$$f(x, y, z) = 0$$

- **Plane equation derivation**

$$(\mathbf{p} - \mathbf{p}_a) \star \mathbf{n} = 0$$

$$(x - x_a)A + (y - y_a)B + (z - z_a)C = 0$$

$$Ax + By + Cz - (Ax_a + By_a + Cz_a) = 0$$

$$Ax + By + Cz + D = 0$$

where

$$D = -(Ax_a + By_a + Cz_a)$$

- **Explicit expression (if C is non-zero)**

$$z = -\frac{1}{C}(Ax + By + D)$$

This can be generalized to both x and y

- **Parametric representation**

$$\mathbf{p}(u,v) = \mathbf{p}_a + (\mathbf{p}_b - \mathbf{p}_a)u + (\mathbf{p}_c - \mathbf{p}_a)v$$

- **Line-Plane intersection**

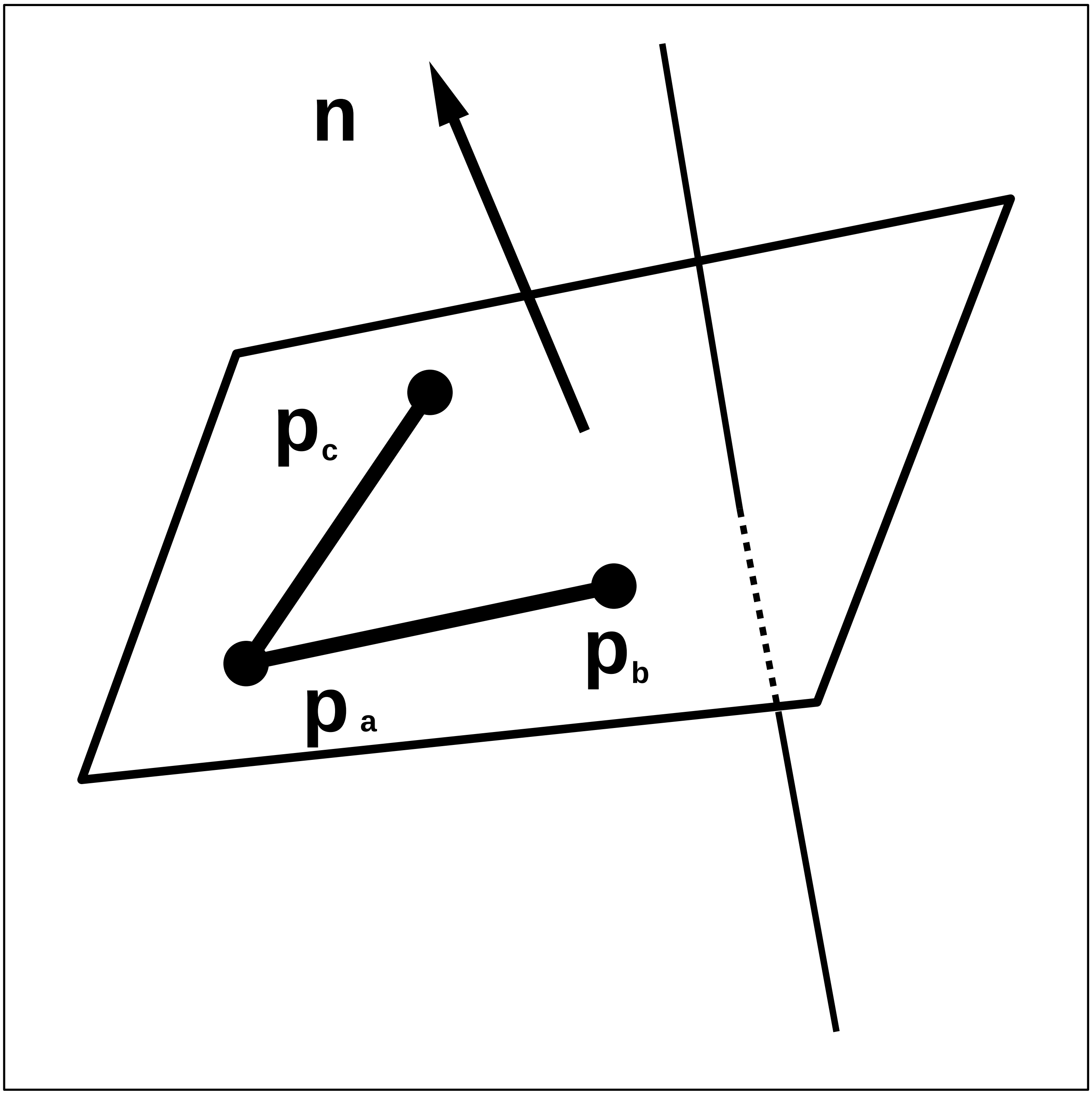
$$\mathbf{l}(u) = \mathbf{p}_0 + (\mathbf{p}_1 - \mathbf{p}_0)u$$

$$(\mathbf{n}) \star (\mathbf{p}_0 + (\mathbf{p}_1 - \mathbf{p}_0)u) + d = 0$$

$$u = -\frac{\mathbf{n} \star \mathbf{p}_0}{\mathbf{n} \star \mathbf{p}_1 - \mathbf{n} \star \mathbf{p}_0} = -\frac{plane(\mathbf{p}_0)}{plane(\mathbf{p}_1) - plane(\mathbf{p}_0)}$$

- **Parametric representation!**

Plane and Intersection



Orthographic View Volume

- **View-volume plane equations**
  - **left plane**
  - **right plane**
  - **bottom plane**
  - **top plane**
  - **front plane**
  - **back plane**
- **Assume all the normals point into the view volume**
- x - left = 0
- - x + right = 0
- y - bottom = 0
- - y + top = 0
- - z - near = 0

- $\mathbf{z + far = 0}$

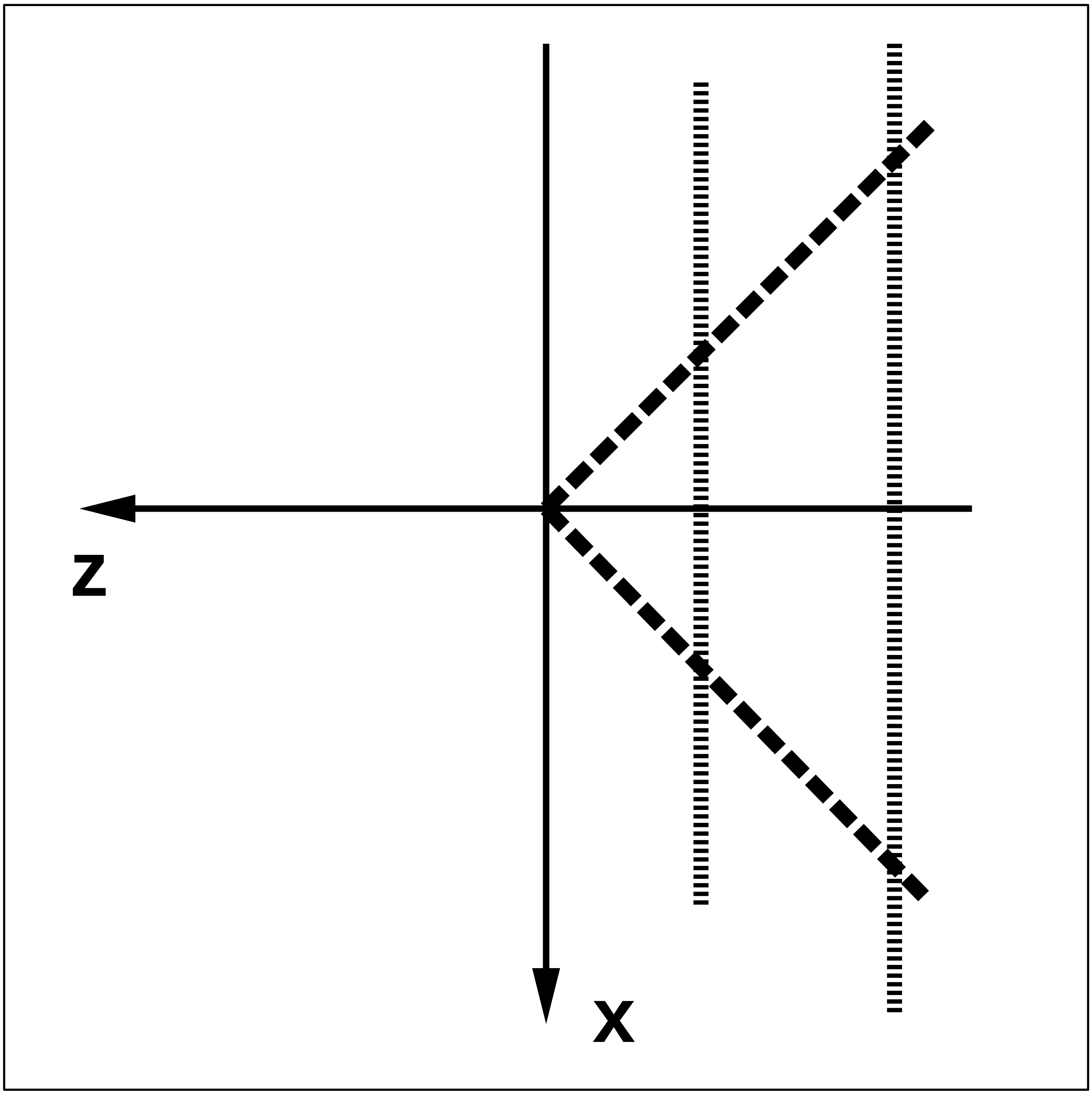
Perspective View Volume

- Again, six planes !
- x + (left * z)/(near) = 0
- - x - (right * z)/(near) = 0
- y + (bottom * z)/(near) = 0
- -y - (top * z)/(near) = 0
- - z - near = 0
- z + far = 0

3D Clipping

- Make use of plane equations
- Determine the sign of the plane equation
- If $plane(\mathrm{p}) > 0$,
  then p is INSIDE!
- Clipping operations
  - point
  - line
  - polygon
  - complicated objects
- Clipping algorithms
- View volume clipping
- 2D algorithms can be generalized to 3D
  - Cohen-Sutherland line-clipping
  - Sutherland-Hodgeman algorithm

View Volume Projection



View Volume Projection