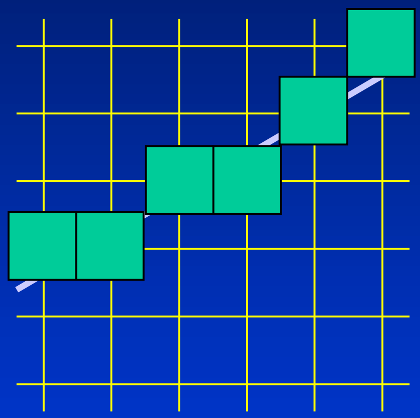
CSE328 Fundamentals of Computer Graphics: Theory, Algorithms, and Applications

Hong Qin
Department of Computer Science
Stony Brook University (SUNY at Stony Brook)
Stony Brook, New York 11794-2424
Tel: (631)632-8450; Fax: (631)632-8334
qin@cs.stonybrook.edu
http://www.cs.stonybrook.edu/~qin

Rasterization

Per-pixel operations: ray-casting/ray-tracing

Screen = matrix



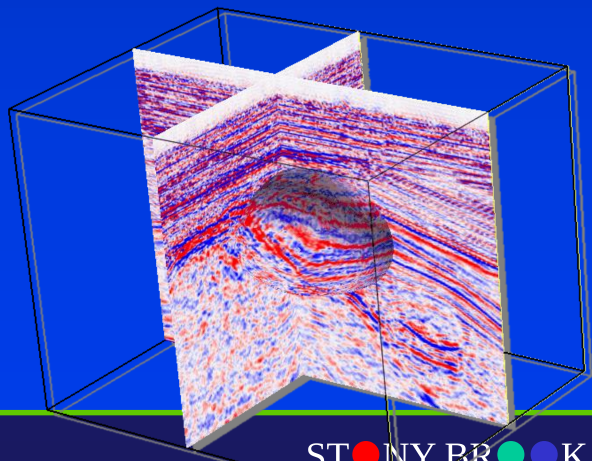
Scan conversion of lines:

- naive version
- Bresenham algorithm
- (mid-point algorithm)

Scan conversion of polygons

Aliasing / antialiasing

Texturing

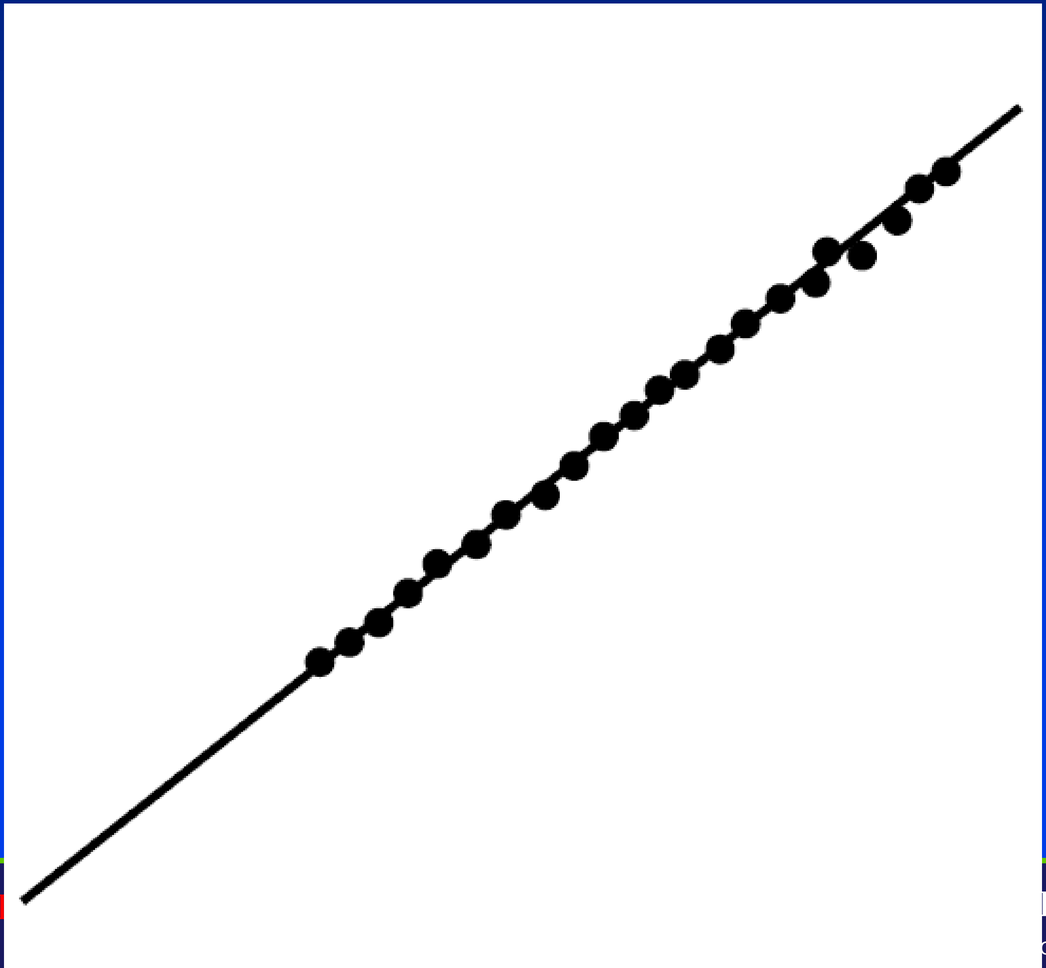


Drawing of Line Geometry

- Why line drawing – the line is the most fundamental drawing primitive with many uses
  - Charts, engineering drawings, illustrations, 2D pencil-based animation, curve approximation
- Some desirable properties for any line drawing algorithm
  - A line should be straight; endpoint interpolation; uniform density for all lines; efficient
- Our current goal – efficient and correct line drawing algorithm
- Draw-line(x_0, y_0, x_1, y_1)

Line Drawing

- Convert a continuous line to a set of discretized points
- Rasterization



Algorithm Assumption

- Point samples on 2D integer lattice
- Bi-level display: on or off
- Line endpoints are all integer coordinates
- All line slopes are: $|k| <= 1$
- Lines are ONE pixel thick
- Are the above assumptions reasonable?

Line Geometry

- Explicit representation
- $y = mx + b$
- The geometric meanings of these parameters: m – slope of the line; b – where it intercept y-axis (where $x = 0$)
- More derivations
  - $dy = y1 – y0$
  - $dx = x1 – x0$
  - $m = (dy) / (dx)$

Simple Algorithm

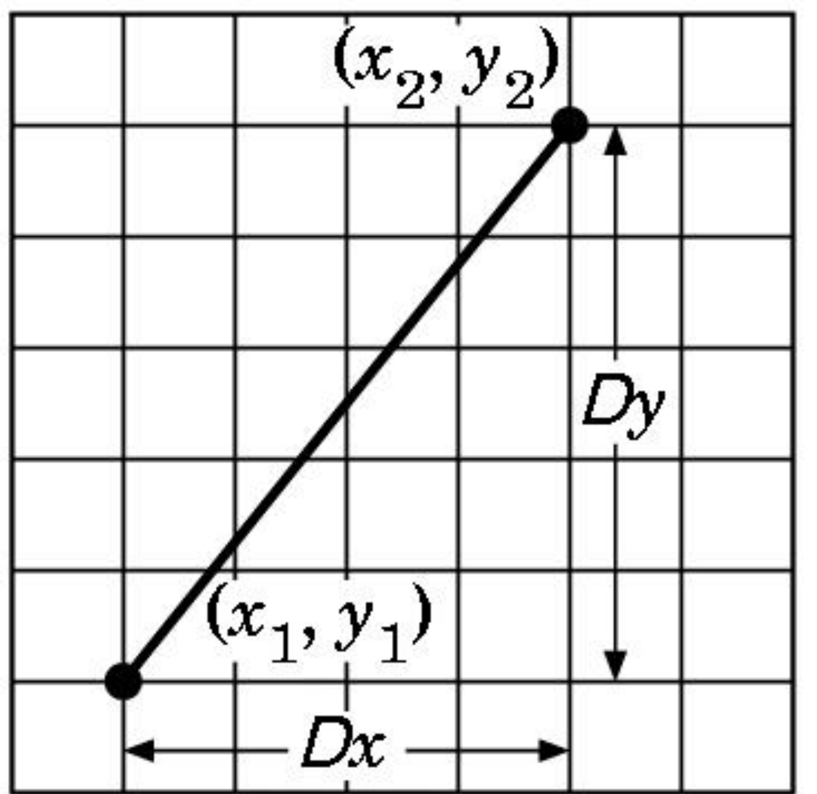
- Draw-line(x0, y0, x1, y1)
  1. Let dy = y1 – y0; dx = x1 – x0
  2. For x = x0 to x1
  3. y = rounding-operation(y0 + (x – x0) (dy / dx)
  4. draw-point(x,y)
  5. End for
- Why does the above procedure work?
- Explicit definition of the line geometry
  – $y = (dy / dx)(x - x0) + y0 = mx + b$

Rendering Line Segments (Rasterization)

- One of the fundamental tasks in 2D computer graphics is 2D line drawing: How to render a line segment from (x_1, y_1) to (x_2, y_2)?

- Use the equation
  $y = mx + h$ (explicit)

- What about horizontal vs. vertical lines?



Further Improvement

- A more efficient algorithm
  1. x = x0; y = y0
  2. draw-point(x,y)
  3. For x from x0 + 1 to x1
  4. y = y + (dy / dx)
  5. End for
- Note that, m = (dy / dx), and m is a float or double

DDA Algorithm

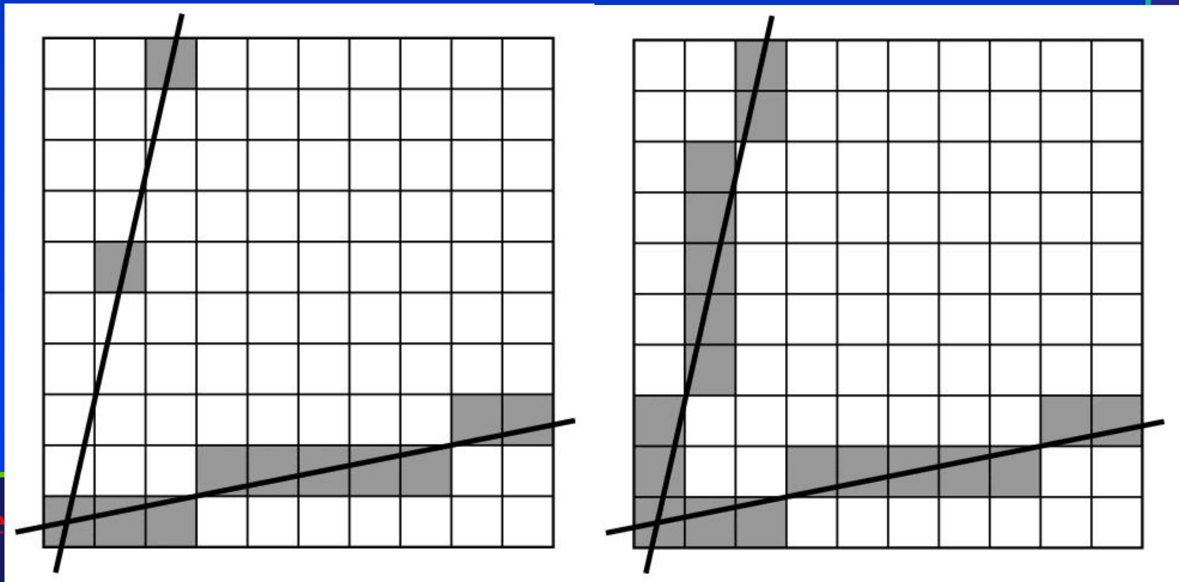
- Digital Differential Analyzer (DDA)

  for (x=x_1; x<=x_2; x++)

  y += m;

  draw_pixel(x, y, color)

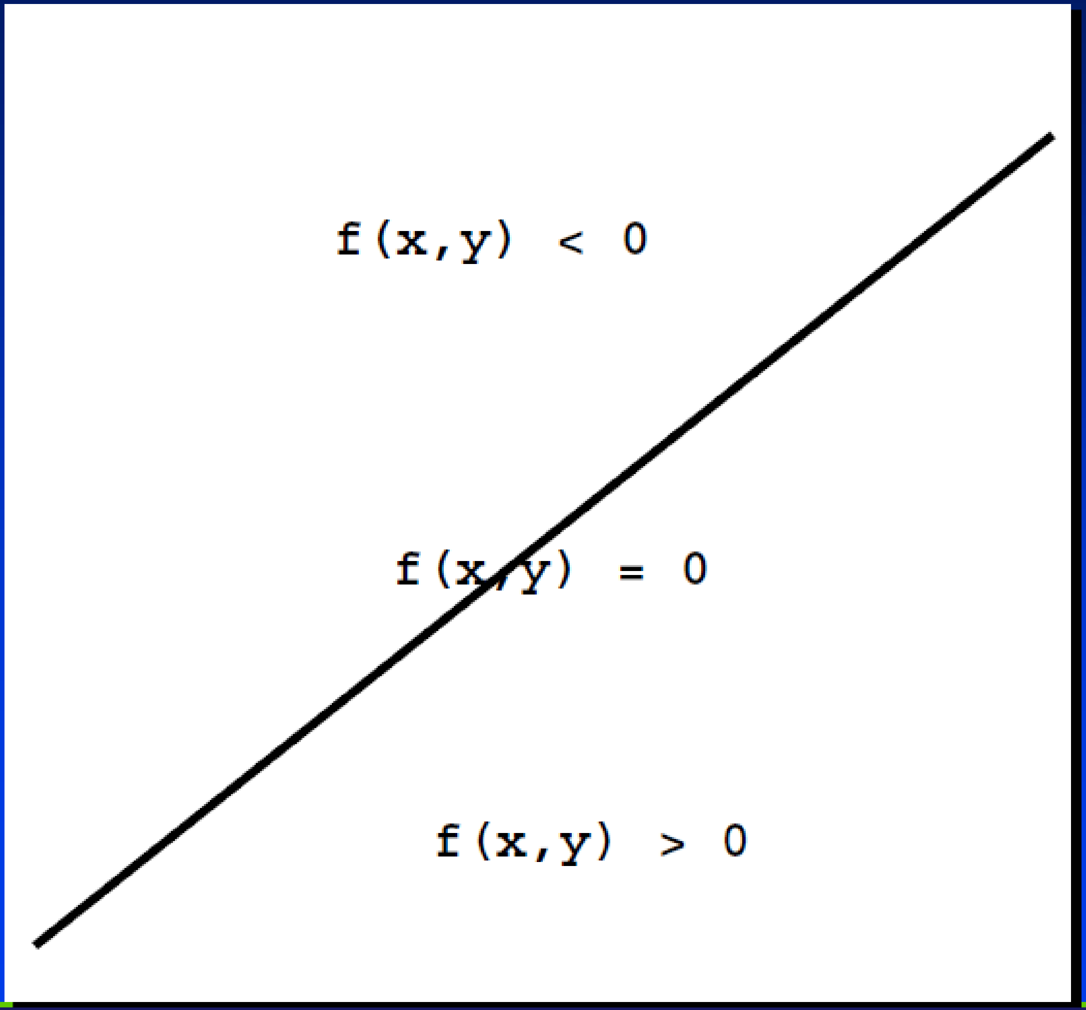
- Handle slopes 0 <= m <= 1; handle others symmetrically
- Does this need floating point operations?



Further Improvement

- We are now seeking an integer-ONLY algorithm to handle all line geometry
- The above procedures will fail
- We must explore new schemes (beyond the line geometry we have already know till now)

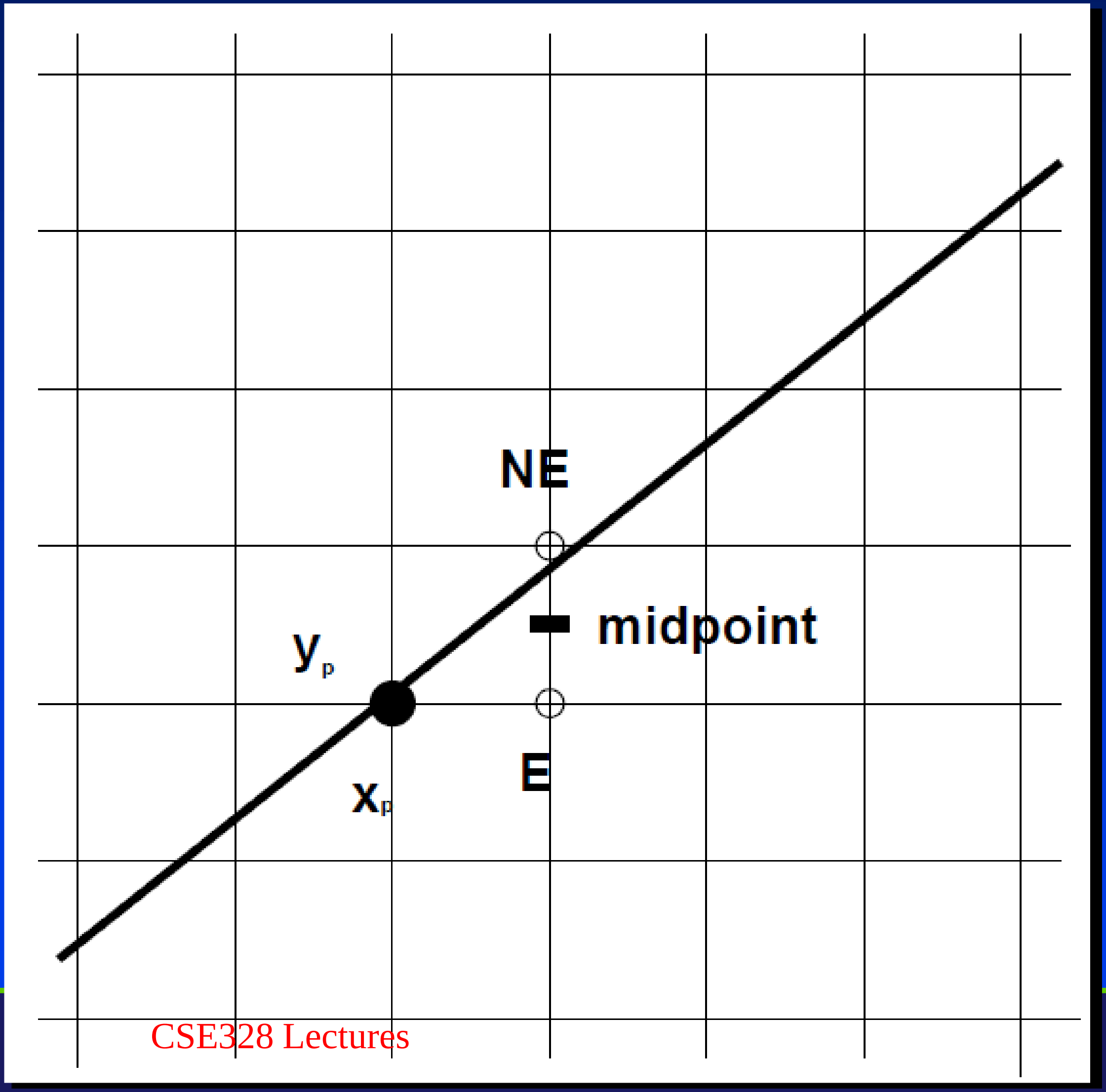
Implicit Equation



Midpoint Algorithm

- Implicit expression for the line geometry
  - $f(x,y) = (x - x0)*(dy) - (y - y0)*(dx)$
- What does this formulation provide us (compared with the previous derivations)?
- Fundamental ideas – spatial partitioning based on the signs!
  - If $f(x,y) = 0$, then (x, y) is on the line
  - If $f(x,y) > 0$, then (x,y) is below the line
  - If $f(x,y) < 0$, then (x, y) is above the line

Midpoint Motivation



Midpoint Motivation

- We are actually considering d = f(xp + 1, yp +0.5)
- There are three different cases
  - If d < 0, line is below the (current) midpoint, then choose E
  - If d >0, lie is above the midpoint, choose NE
  - If d =0, line is passing through the midpoint, either E or NE

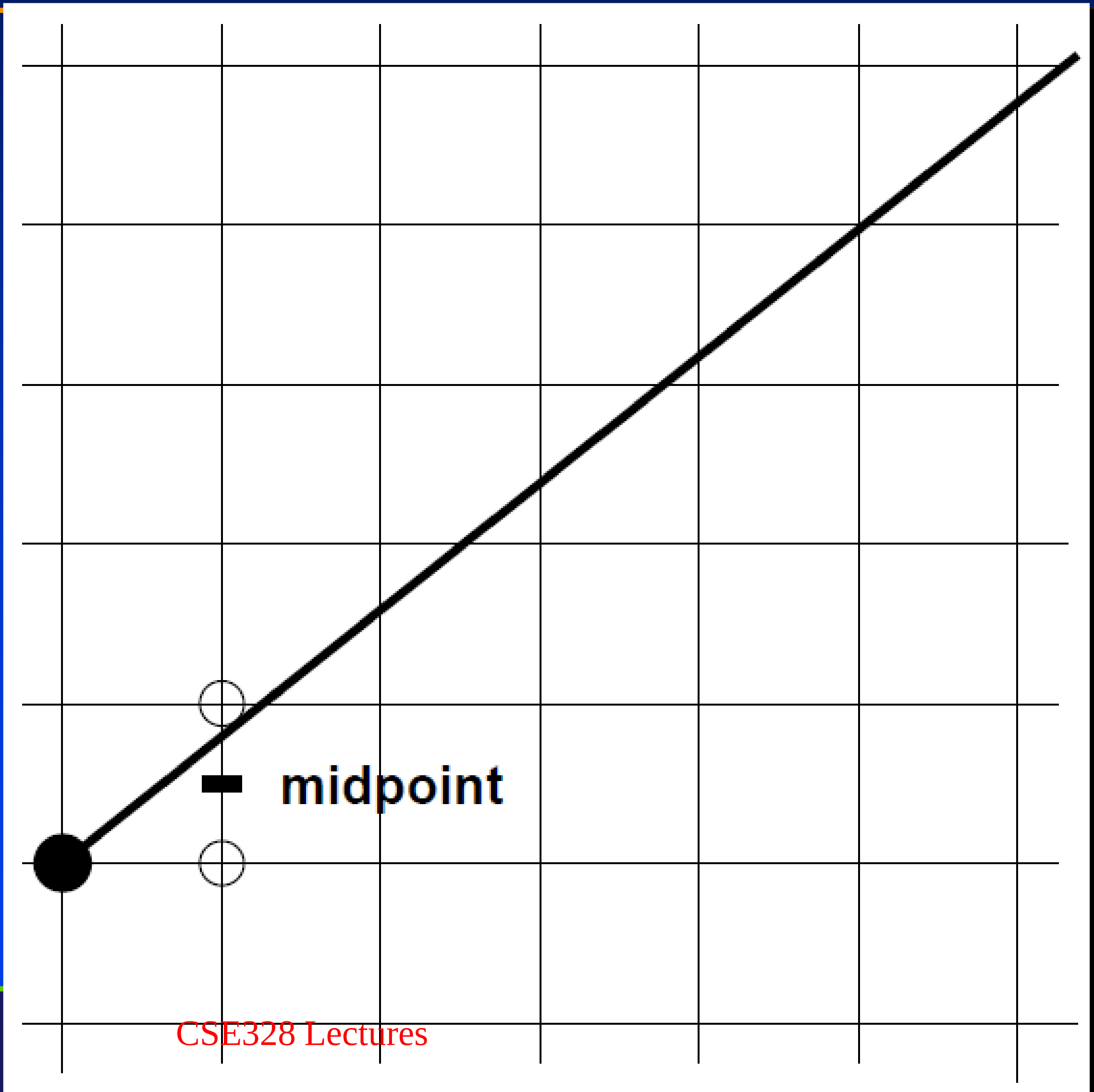
Recursive Algorithm

- Midpoint algorithm is a recursive algorithm!
- For any recursive algorithm, we MUST consider the subsequent steps (by traversing all cases respectively)!
- If E is chosen, then the NEW E is (xp + 2, yp), the NEW NE is (xp + 2, yp +1), the NEW midpoint is (xp + 2, yp + 0.5)
  - d_new = f (xp + 2, yp + 0.5)
  - d_old = f (xp + 1, yp +0.5)
  - d_new = d_old + (dy)

Recursive Algorithm

- If NE is chosen, the NEW E is (xp +2, yp +1), the NEW NE is (xp + 2, yp + 2), the NEW midpoint is (xp + 2, y + 1.5)
  - d_new = f(xp + 2, yp + 1.5)
  - d_old = f(xp +1, yp + 0.5)
  - d_new = d_old + (dy – dx)
- This process MUST repeat recursively, stepping along x from x0 to x1

Midpoint Initialization



Initialization

- How about the initialization process
- At the beginning,
  - $xp = x0$
  - $yp = y0$
  - $d\_old = f(x0 + 1, y0 + 0.5) = (dy) - (dx) * (1/2)$

Midpoint Algorithm

- draw-line(x0, y0, x1, y1)
  - Int x0, y0, x1, y1
  - { int dx, dy, inc_E, inc_NE, x, y,
  - real d
  - dx = x1 – x0
  - dy = y1 – y0
  - d = (dy) – (dx) * (1/2)
  - inc_E = dy
  - inc_NE = dy – dx
  - y = y0
  - for x from x0 to x1
  - if d>0, then d = d + inc_NE, y + 1, else d = d + inc_E
  - end for
  - }

Midpoint Algorithm

- d is NOT an integer, however, ONLY the sign MATTERS!
- We prefer an integer-ONLY algorithm!!!
  - $g(x,y) = 2 f(x,y)$
  - d becomes 2d
  - then $d = 2(dy) - (dx)$

Modifying the Previous Algorithm

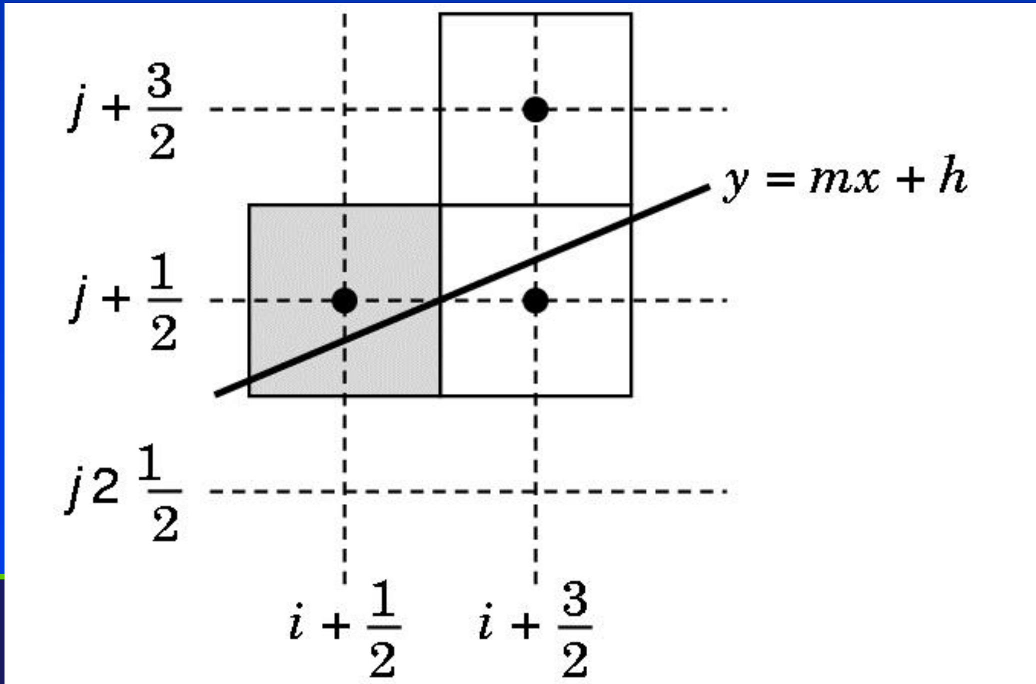
- Make it an integer-ONLY algorithm
- Our earlier assumptions
  – slopes: 0 <= (dy) / (dx) <=1
  – line endpoints are all integer coordinates
- How about other cases

Handling All Other Cases

- Generalizations
  - negative slope
  - slope larger than 1
- If the slope is larger than 1, we use symmetry to switch x and y (you are NOT displaying (x,y), you should display (y,x))!
- In negative slope, we should use x and (-y)

Bresenham’s Algorithm

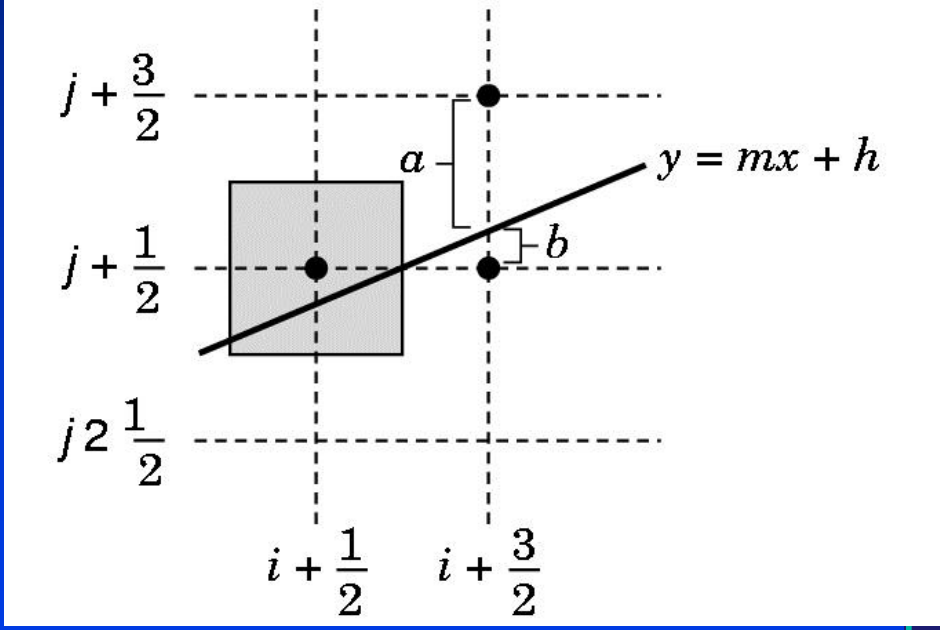
- The DDA algorithm requires a floating point *add* and *round* for each pixel: can we eliminate?
- Note that at each step we will go E or NE. How to decide which?



Department of Computer Science
Center for Visual Computing

Bresenham Decision Variable

- Bresenham algorithm uses decision variable d=a-b, where a and b are distances to NE and E pixels
- If d>=0, go NE;
  if d<0, go E
- Let $d=(x_2-x_1)(a-b) = d_x(a-b)$
  [only sign matters]
- Substitute for a and b using line equation to get integer math (but lots of it)



- $d=(a-b)\ d_x = (2j+3)\ d_x - (2i+3)\ d_y - 2(y_1 d_x - x_1 d_y)$
- But note that $d_{k+1}=d_k + 2d_y$ (E) or $2(d_y-d_x)$ (NE)

Bresenham's Algorithm

- Set up loop computing *d* at x_1, y_1

```
for (x=x1; x<=x2; )
    x++;
    d += 2dy;
    if (d >= 0) {
        y++;
        d -= 2dx; }
    drawpoint(x,y);
```

- Pure integer math, and not much of it
- So easy that it is built into one graphics instruction (for several points in parallel)

Extensions to Handle Curves

- Generalizations to handle all cases for line drawing
- Algorithms for circle-drawing
- Algorithms for ellipses, conic section drawing
- Algorithms for cubic curve drawing
- Algorithms to handle any type of curves?

Circles

- Implicit expression of a circle f(x,y)=0

$$f(x,y) = (x - x_0)^2 + (y - y_0)^2 - r^2$$

- Remember the key idea is that, ONLY the sign matters!
  - If f(x,y)=0, then (x,y) is on the circle
  - If f(x,y)>0, then (x,y) is outside the circle
  - If f(x,y)<0, then (x,y) is inside the circle
- Equations for ellipses?
- The key message: the slope is controllable!!!