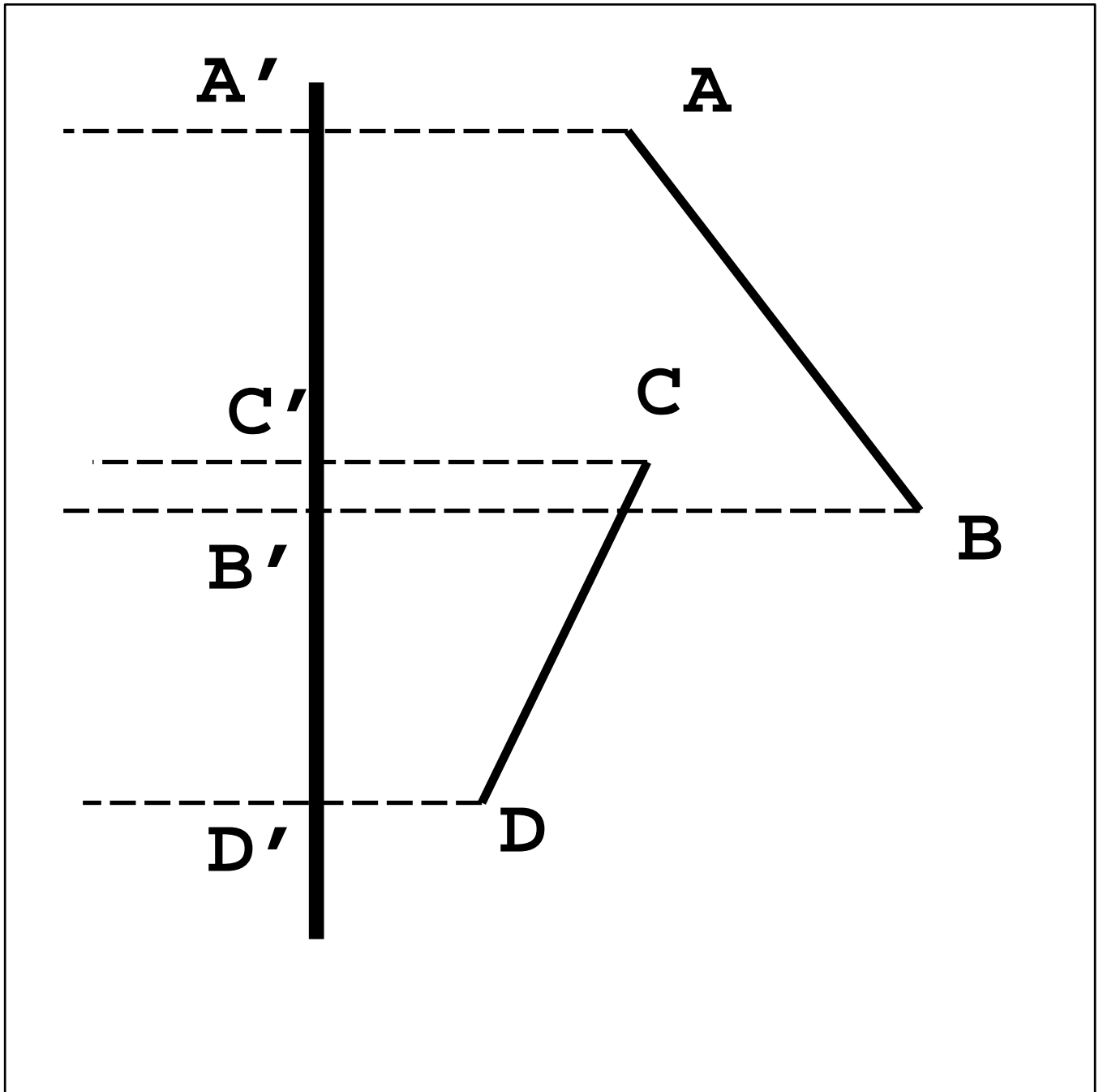# General Transformation

- **Transformation in general is function operation:**
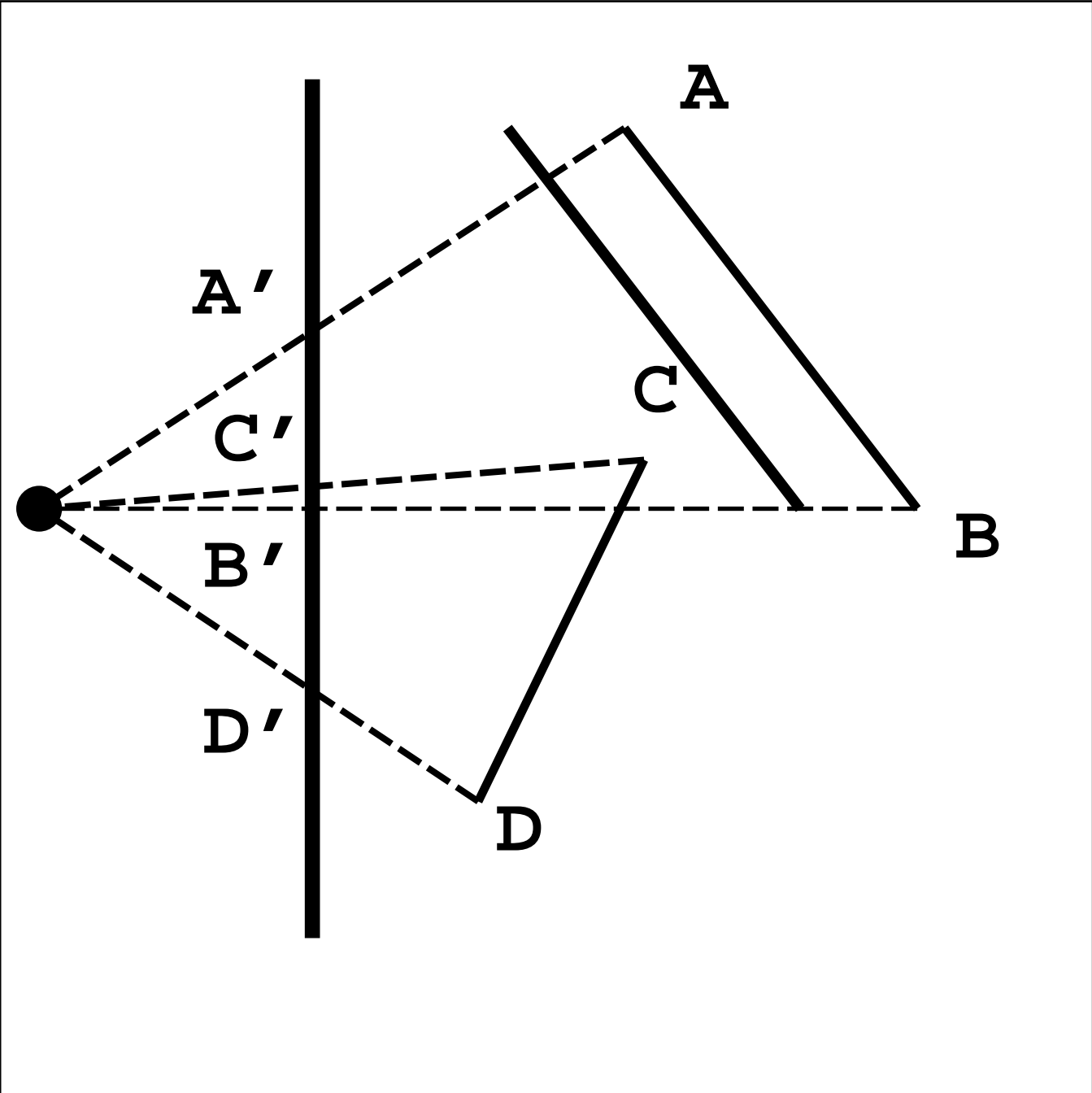
$$f :$$

$$R^n \Rightarrow R^m$$

- **2D transformation:** $n = m = 2$

- **3D transformation:** $n = m = 3$

- **Projection transformation:** $m < n$

- **Example: map 3D object on a plane**

- **Two typical examples:**

  - **parallel projection**
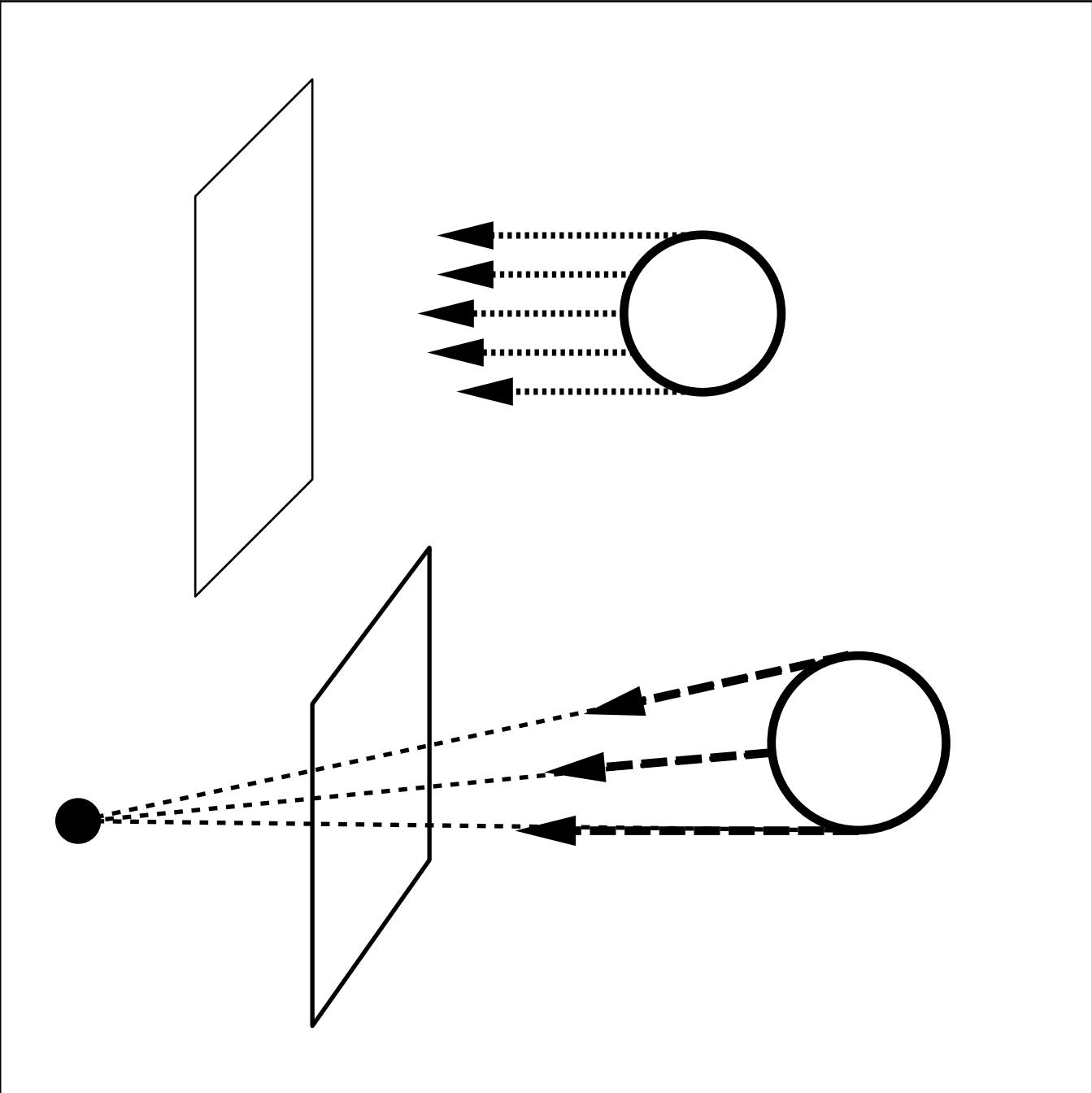 - **perspective projection**

Parallel Projection

Perspective Projection

Projection

- Parallel projection
  preserve the relative length

- Perspective projection
  produce fore-shortening

- Parallel vs. Perspective

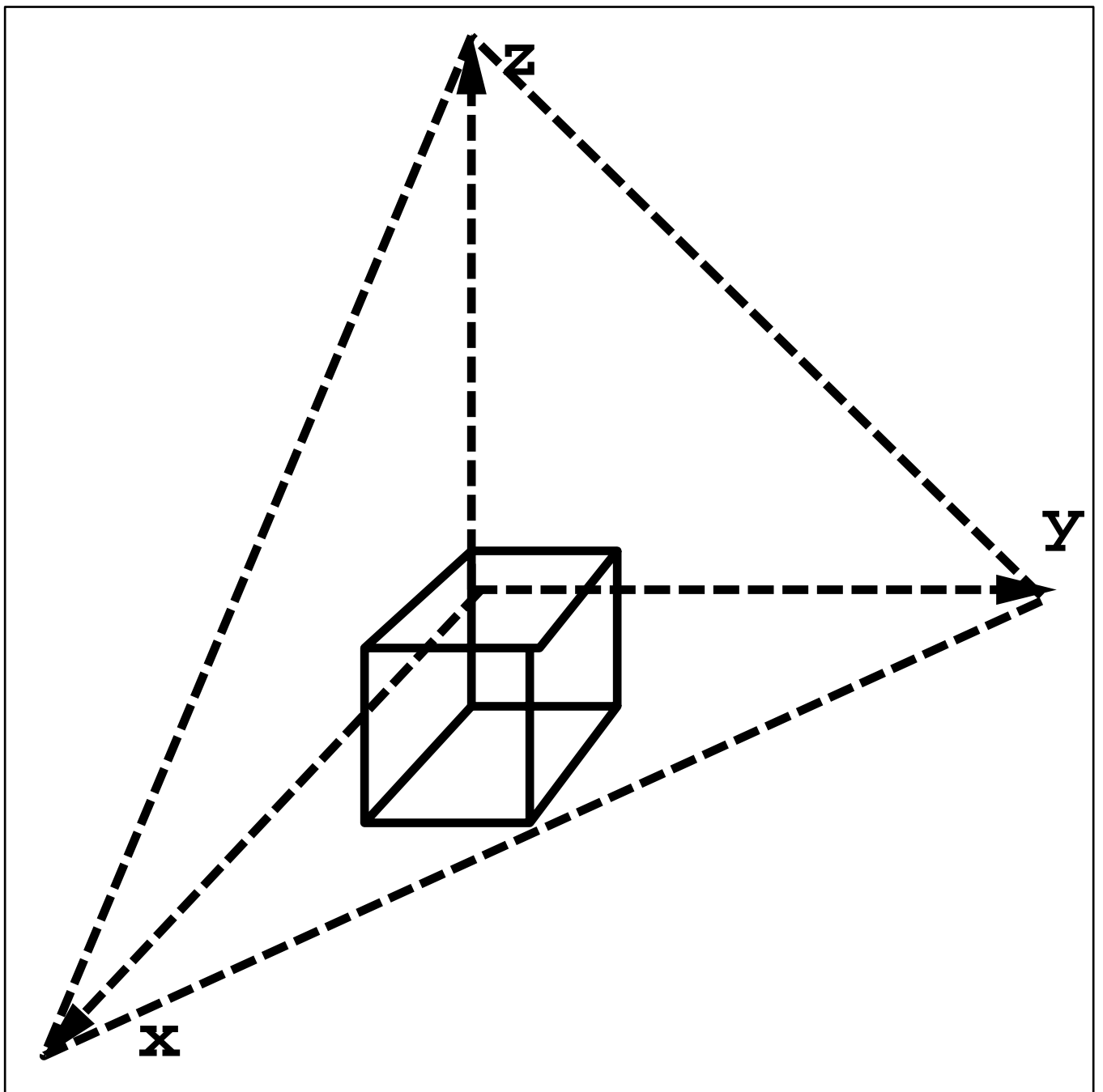 - projection lines
  - center of projection

Projection

Parallel Projection

- Orthographic projection
  (Perpendicular projection)
  direction of projection perpendicular to the
  projection plane

- Top, front, and side projection
  direction of projection parallel to one principal axis

- Axonometric projection
  Not parallel to a principal axis

- Isometric projection
  The normal of the projection plane is $(1, 1, 1)$
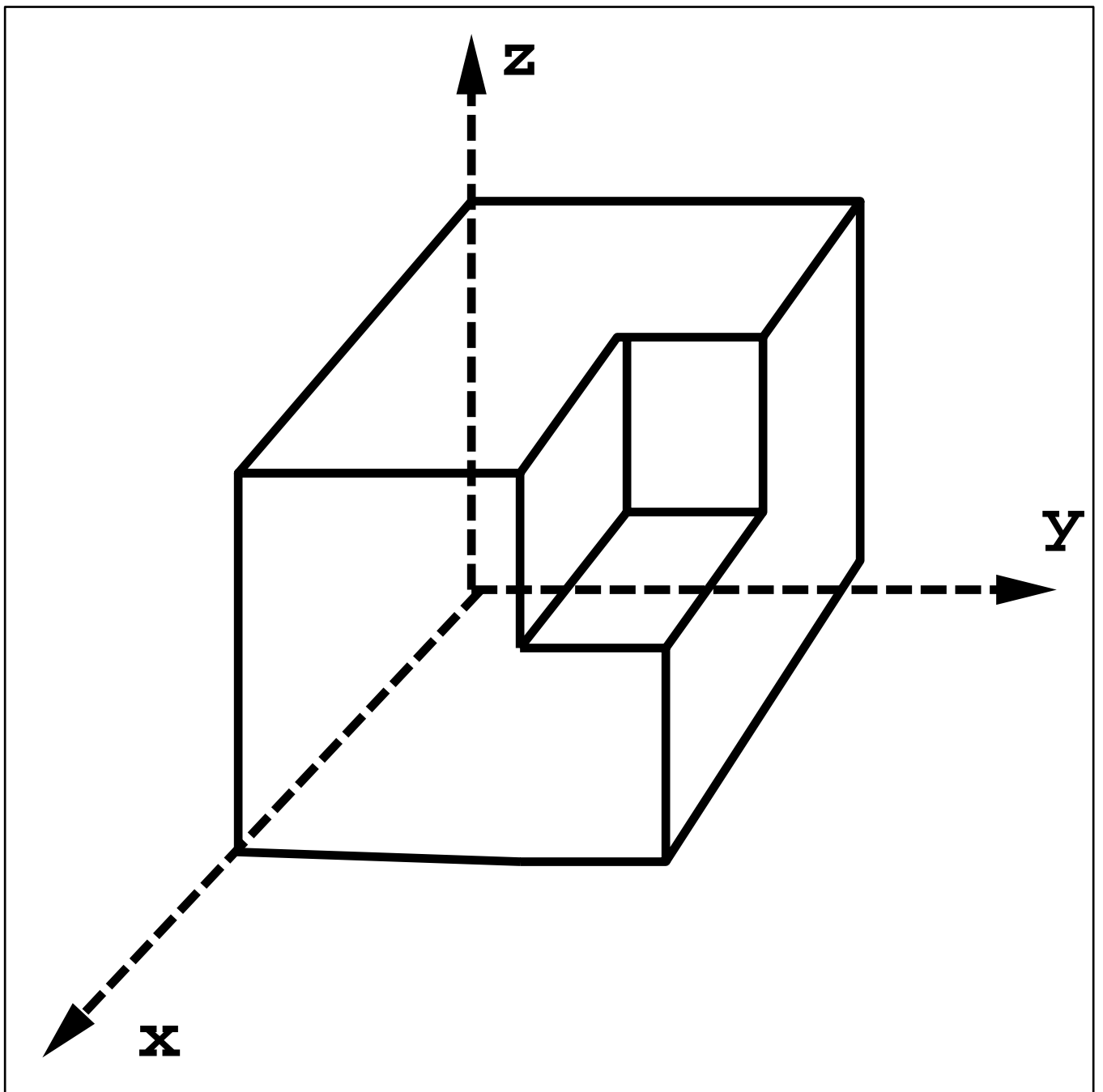 equal angles with each principal axis

Orthographic Projection
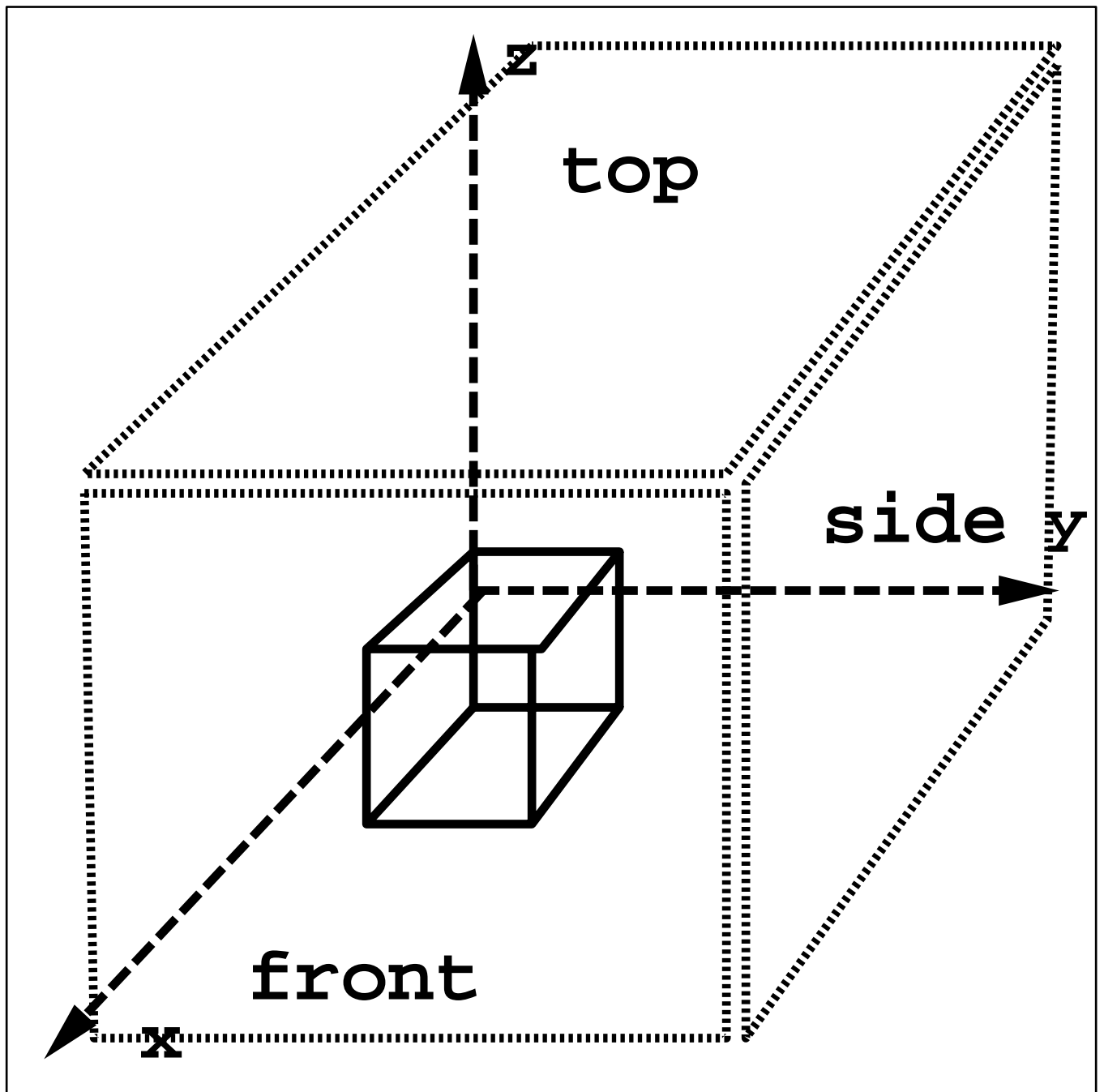
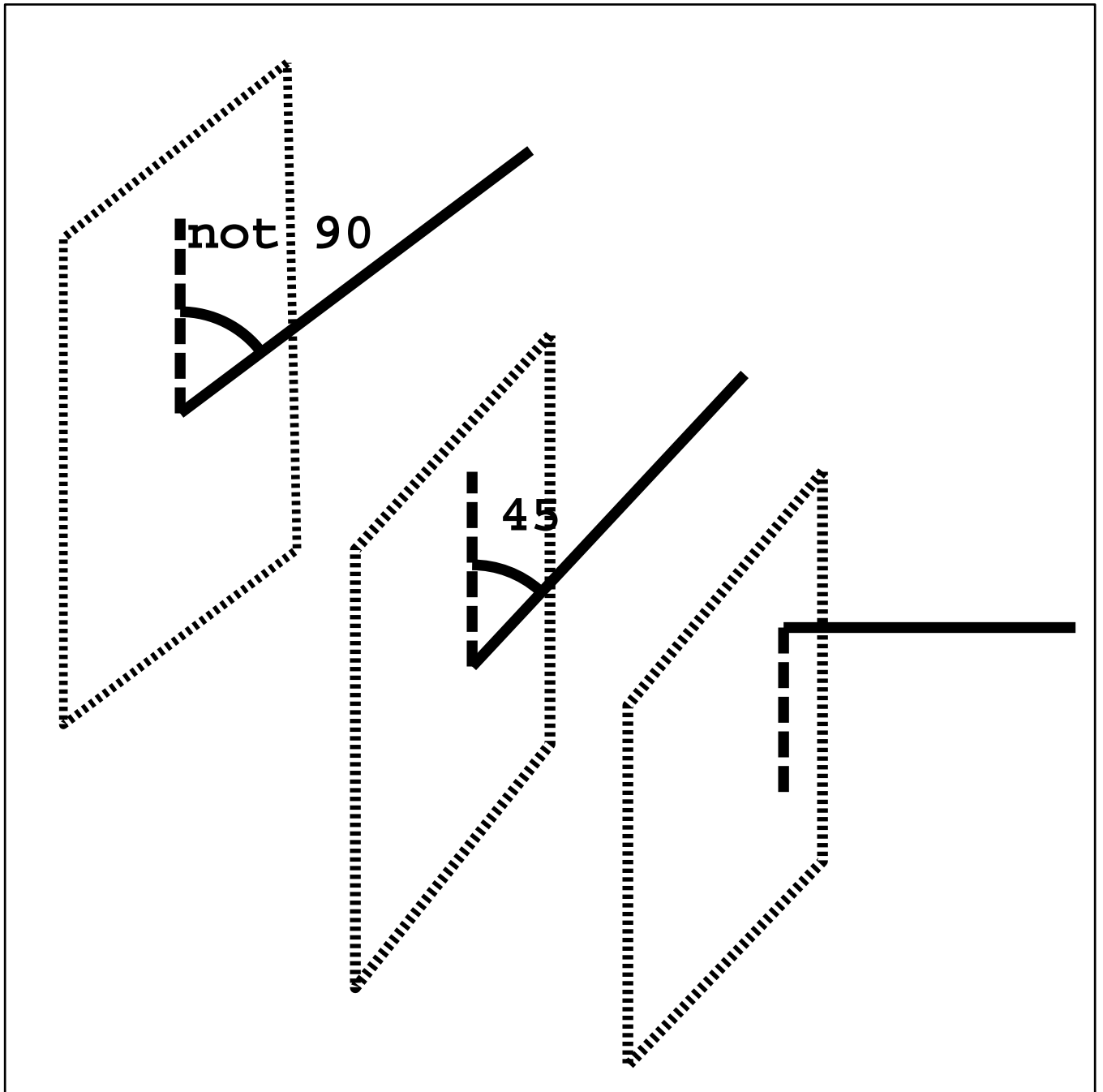# Isometric Projection

Another Example

Oblique Projection

- Oblique projection
  (angle is not 90^0)

- Cavalier projection
  (angle is 45^0)

- Cabinet projection
  (one half of the actual length)

- Others

Oblique Projection

not 90

45

Perspective Projection

- Vanishing point

- Parallel lines that are parallel to projection plane still parallel

- Parallel lines not parallel to projection planes converge to a point

- Infinite number of vanishing points

- Axis (principal) vanishing points
  At most 3 principal vanishing points

- One principal vanishing point

  - example: x-y

- Two principal vanishing points

- Three principal vanishing points

Taxonomy of Projections

- Parallel

  - oblique
    * cabinet
    * cavalier
    * others
  - orthographic
    * top, front, side views
    * axonometric, isometric, others

- Perspective

  - one, two, three

Projection Summary

- Parallel projection

  - specify the direction
  - may not be perpendicular to the projection plane
  - preserve the relative length
  - do not preserve angles
  - preserve parallel lines
  - preserve straight lines

- Perspective projection

  - specify the point
  - do not preserve length
  - do not preserve angles
  - do not preserve parallel lines
  - preserve straight lines