3D Graphics Concepts

- 3D coordinate system
  - x, y, and z
  - depth information
- Geometric modeling of various objects
  - point, line, polygon
  - curve, surface, solid
- Geometric transformation
- 3D Viewing
  - parallel projection
  - perspective projection
- Display methods of 3D objects
  - wireframe
  - shaded objects
  - visible object identification

– **realistic rendering techniques**
– **3D stereoscopic viewing**

3D Transformation

- 3D translation

$$\mathbf{T}(\delta x, \delta y, \delta z) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

- 3D scaling

$$\mathbf{S}(a, b, c) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax \\ by \\ cz \end{bmatrix}$$

- 3D rotation

$$\mathbf{R}(z, \theta) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x\cos(\theta) - y\sin(\theta) \\ x\sin(\theta) + y\cos(\theta) \\ z \end{bmatrix}$$

- Note that, a positive rotation about z-axis
  is defined as one rotation
  from positive x-axis to positive y-axis

- We can similarly define rotations about

the other two axes
About x-axis:
from positive y-axis to positive z-axis
About y-axis:
from positive z-axis to positive x-axis

- One example:
One scaling operation $\mathrm{S}(a, b, c)$ followed
by a translation $\mathrm{T}(d, e, f)$
Let's write the matrix formulation

$$\begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$

- Our goal is to combine them into an integrated representation

- We use homogeneous representation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Longleftrightarrow \begin{bmatrix} xh \\ yh \\ zh \\ h \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- **Scaling operation in matrix form**

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Translation operation in matrix form**

$$\begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **The previous example (in matrix form)**

$$\begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x_{new} \\ y_{new} \\ z_{new} \\ 1 \end{bmatrix}$$

Rotation

- **W.r.t. z-axis**

$$\mathbf{R}(z,\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **W.r.t. x-axis**

$$\mathbf{R}(x,\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **W.r.t. y-axis**

$$\mathbf{R}(y,\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Complex Operations

- **Shearing operation**
- **Composition of transformations**
- **Inversion of transformations**
- **A series of transformations can be accumulated into a single transformation matrix**
- **One example**
  Rotate along an axis (defined by $x = 2$ and $y = 3$) by -90^0
- **The operation consists of three steps**
  (1) $\mathrm{A} = \mathrm{T}(-2, -3, 0)$
  (2) $\mathrm{B} = \mathrm{R}(z, -90^0)$
  (3) $\mathrm{C} = \mathrm{T}(2, 3, 0)$
- **The new object after this transformation is**

$$Obj_{new} = \mathbf{C} \star \mathbf{B} \star \mathbf{A} \star Obj_{old}$$

- **So, general form**

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} X & X & X & t_x \\ X & X & X & t_y \\ X & X & X & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- **Composite transformations are non-commutative**

- **Transformations can be inverted**