CSE328 Fundamentals of Computer Graphics (Theory, Algorithms, and Applications)

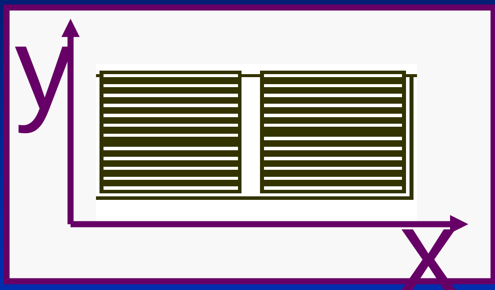
Hong Qin
Department of Computer Science
Stony Brook University (SUNY at Stony Brook)
Stony Brook, New York 11794-2424
Tel: (631)632-8450; Fax: (631)632-8334
qin@cs.stonybrook.edu
http://www.cs.stonybrook.edu/~qin

2D Transformations

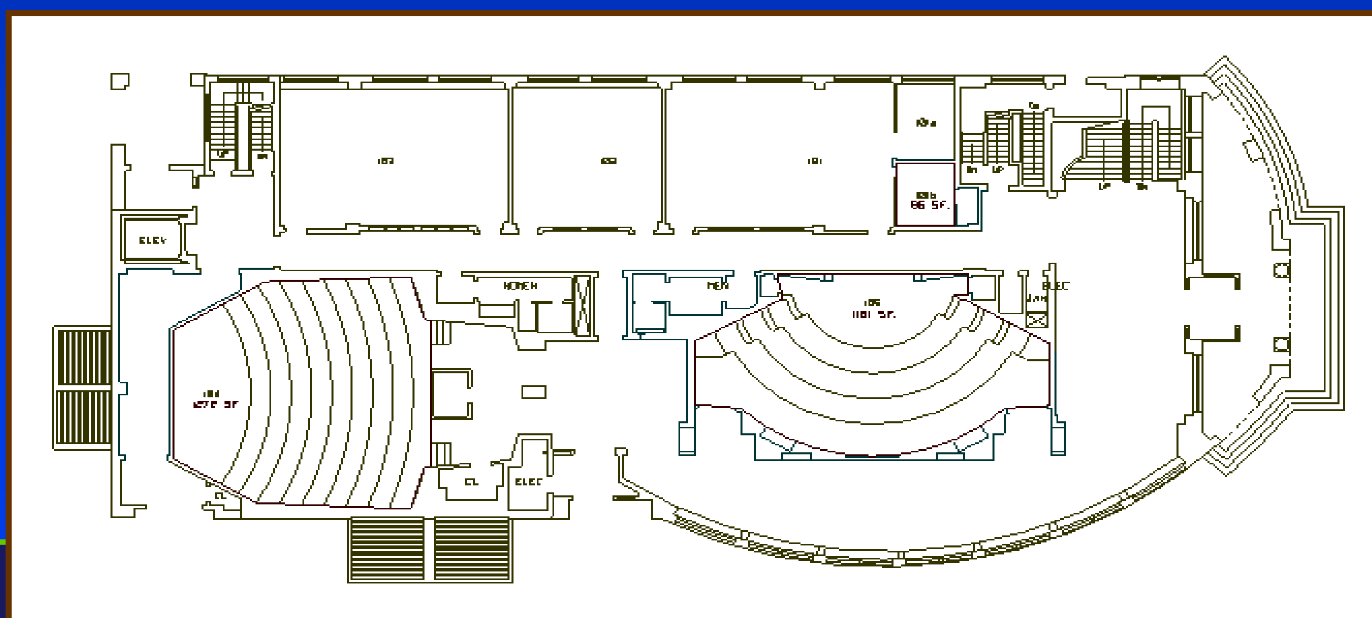
- From local, model coordinates to global, world coordinates

From Model Coordinates to World Coordinates (Local to Global)

Model coordinates (local)



World coordinates (global)



Modeling Transformations

- 2D transformations
- Specify transformations for objects
  - Allows definitions of objects in their own coordinate systems
  - Allows use of object definition multiple times in a scene
  - Please pay attention to how OpenGL provides a transformation stack because they are so frequently reused

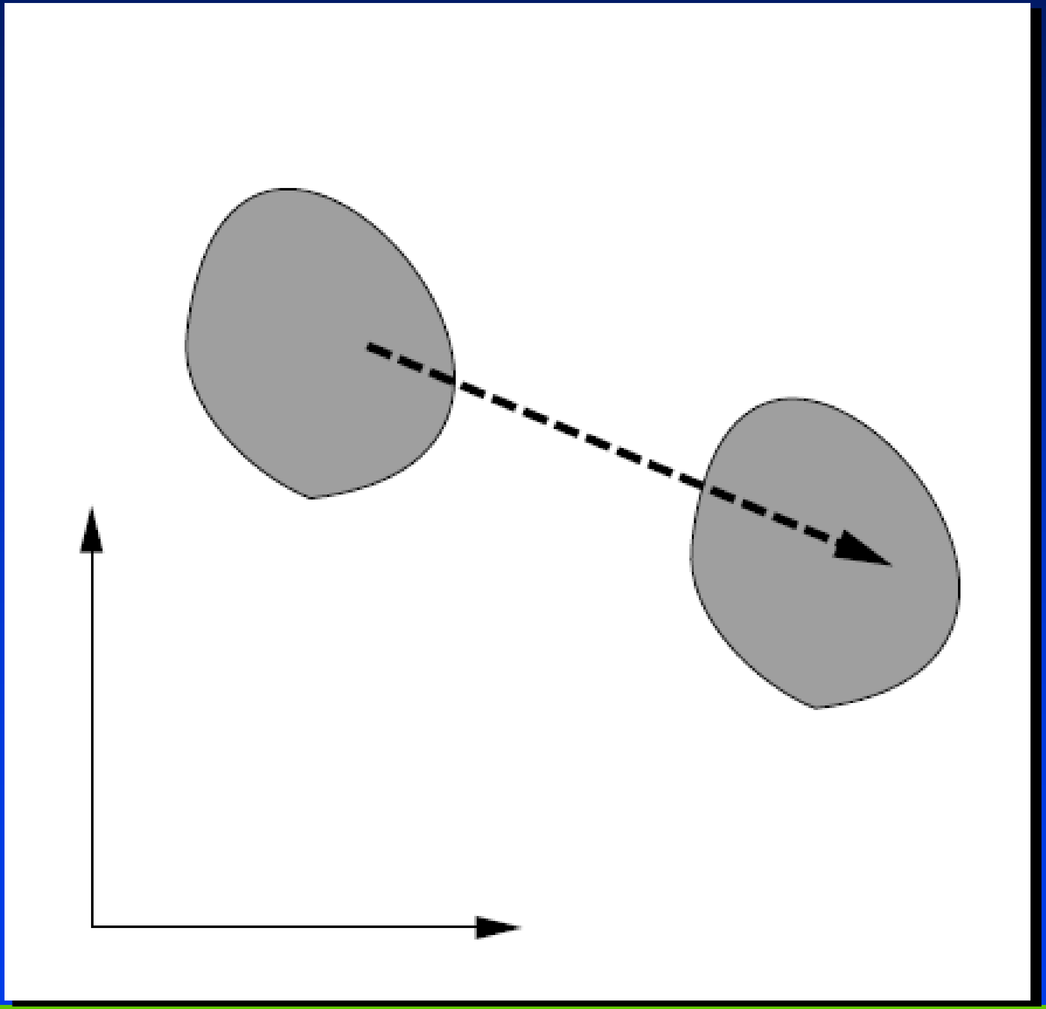
Overview

- 2D Transformations
  - Basic 2D transformations
  - Matrix representation
  - Matrix composition
- Generalizations to 3D Transformations
  - Basic 3D transformations
  - Same as 2D

Basic 2D Transformations

- Translation:
  - $x' = x + t_x$
  - $y' = y + t_y$
- Scale:
  - $x' = x * s_x$
  - $y' = y * s_y$
- Shear:
  - $x' = x + h_x * y$
  - $y' = y + h_y * x$
- Rotation:
  - $x' = x * \cos\Theta - y * \sin\Theta$
  - $y' = x * \sin\Theta + y * \cos\Theta$

2D Translation



2D Translation

- Current position

$$\mathbf{p} = \begin{bmatrix} x \\ y \end{bmatrix}$$

- Translation operation

$$T(\delta x, \delta y) = \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}$$

$$\mathbf{p} + T(\delta x, \delta y) = \mathbf{p}'$$

$$x' = x + \delta x$$

$$y' = y + \delta y$$

Scaling

- Scaling a coordinate means multiplying each of its components by a scalar
- Uniform scaling means this scalar is the same for all components:

- Non-uniform scaling: different scalars per component:

- How can we represent scaling in matrix form?

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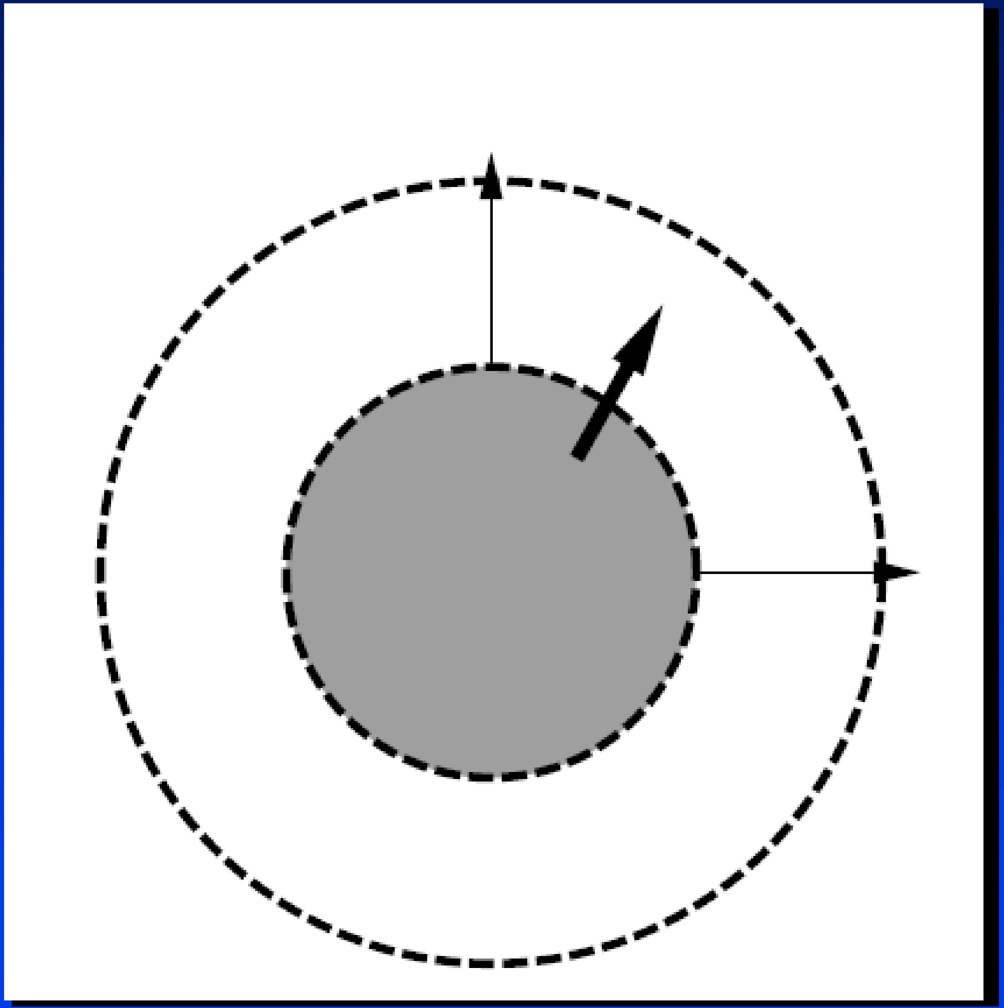
2D Scaling



Scaling Operation in Matrix Form

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} ax \\ by \end{bmatrix}$$

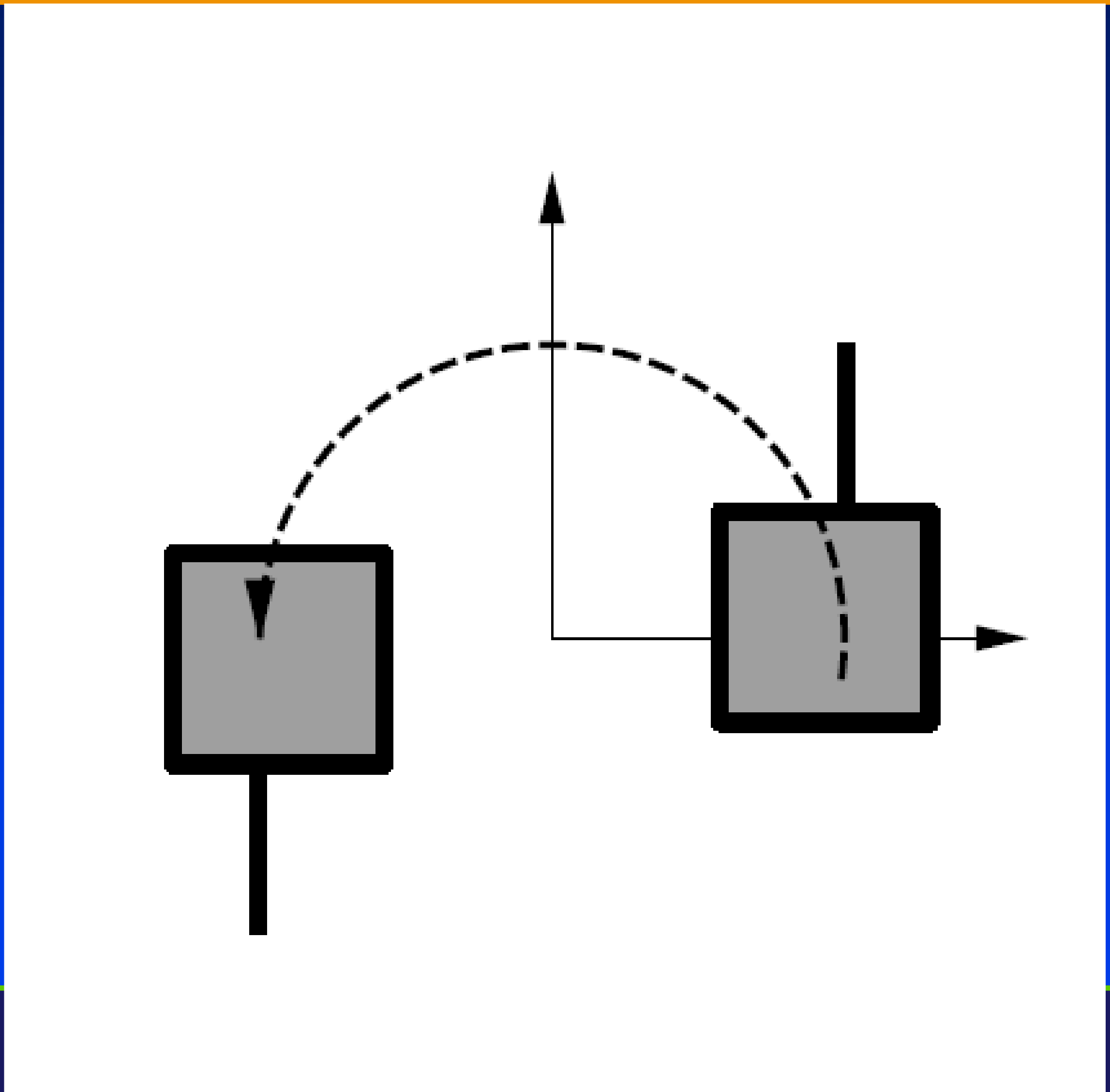
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling

- Matrix multiplication
- Scaling operation: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} ax \\ by \end{bmatrix}$
- Or, in matrix form: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\text{scaling matrix}} \begin{bmatrix} x \\ y \end{bmatrix}$
- Note any difference from T

scaling matrix

2D Rotation



2-D Rotation

$$x' = x \cos(\theta) - y \sin(\theta)$$

$$y' = x \sin(\theta) + y \cos(\theta)$$

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$R(\theta)\mathbf{p} = \mathbf{p}'$$

Positive angles are “counter-clockwise”!

Derivation of 2D Rotation

- $x = r \cos(\phi)$
- $y = r \sin(\phi)$
- $x' = r \cos(\phi + \theta)$
- $y' = r \sin(\phi + \theta)$

- $x' = r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta)$
- $y' = r \sin(\phi) \sin(\theta) + r \cos(\phi) \cos(\theta)$

- $x' = x\, \mathbf{cos}(\theta) - y\, \mathbf{sin}(\theta)$
- $y' = x\, \mathbf{sin}(\theta) + y\, \mathbf{cos}(\theta)$

2-D Rotation

- It is straightforward to see this procedure in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

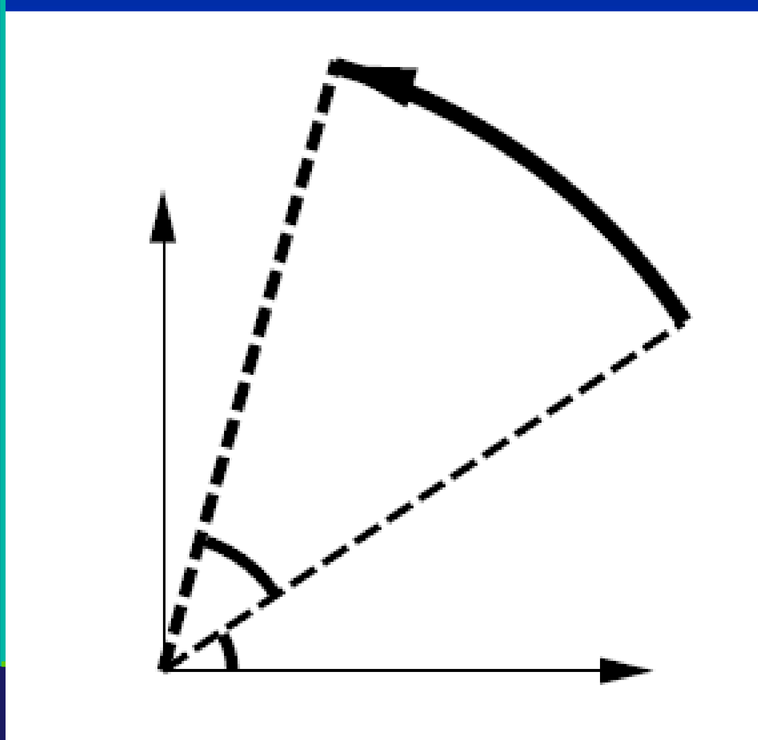
- Important results from trigonometry!

Observation - Even though sin(θ) and cos(θ) are nonlinear functions of θ,

- ***x' is a linear combination of x and y***
- ***y' is a linear combination of x and y***

2D Rotation's Geometric Understanding

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



$$x' = r\cos(\theta_1)\cos(\theta_2) - r\sin(\theta_1)\sin(\theta_2)$$

$$y' = r\cos(\theta_1)\sin(\theta_2) + r\sin(\theta_1)\cos(\theta_2)$$

Basic 2D Transformations

- Translation:
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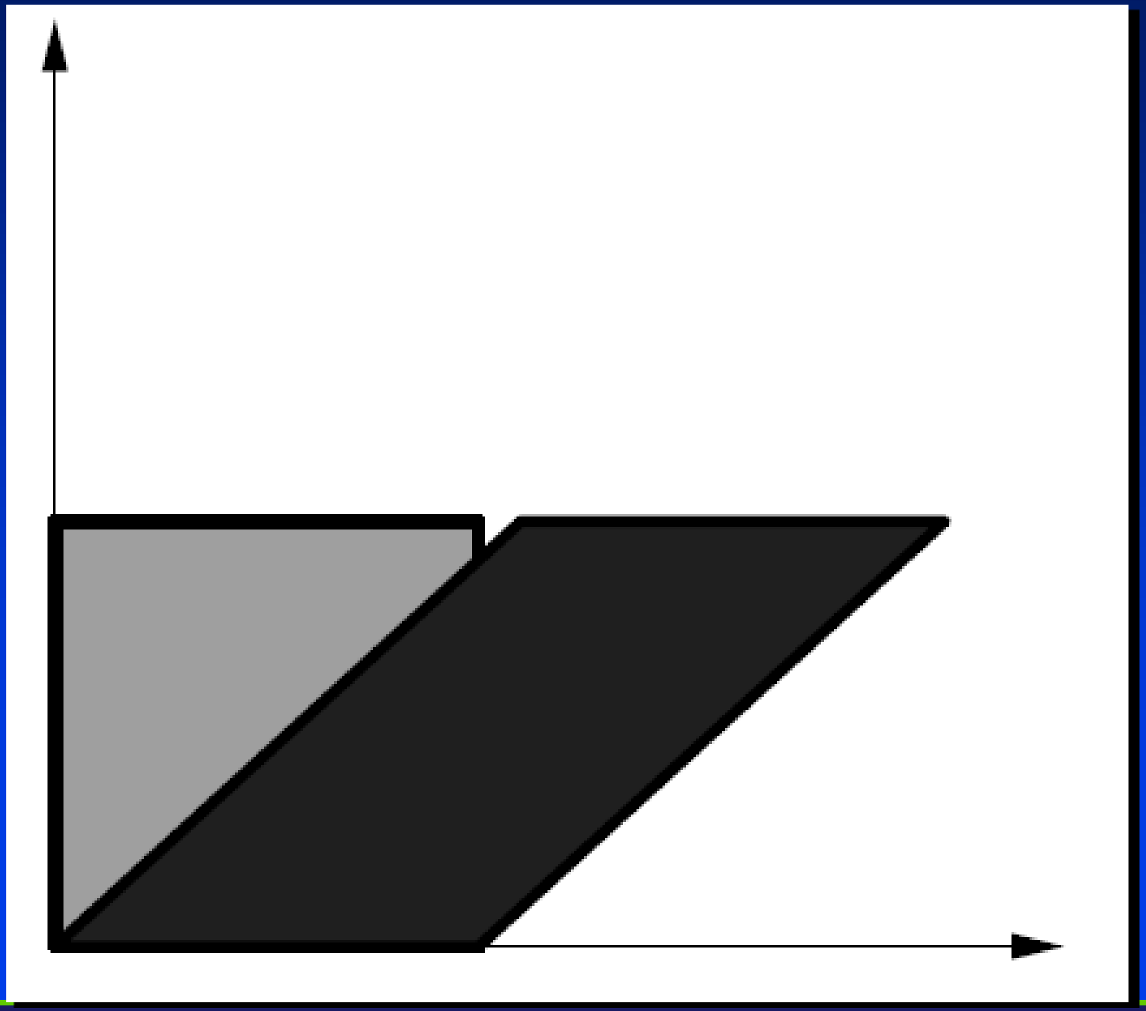
  - $y' = x*\sin\Theta + y*\cos\Theta$

2D Shear



2D Shear and Geometric Meaning

- Shear operation along the x-axis

$$\mathbf{p} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{p}' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x + ay \\ y \end{bmatrix}$$

$$Sh_x(a) = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{p}' = Sh_x(a)\mathbf{p}$$

- Shear operation along the y-axis

$$Sh_y(b) = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$$

$$\mathbf{p}' = Sh_y(b)\mathbf{p} = \begin{bmatrix} x \\ bx + y \end{bmatrix}$$

2D Shear

- Consider more complicated cases!
- Various examples are shown in the class!

Basic 2D Transformations

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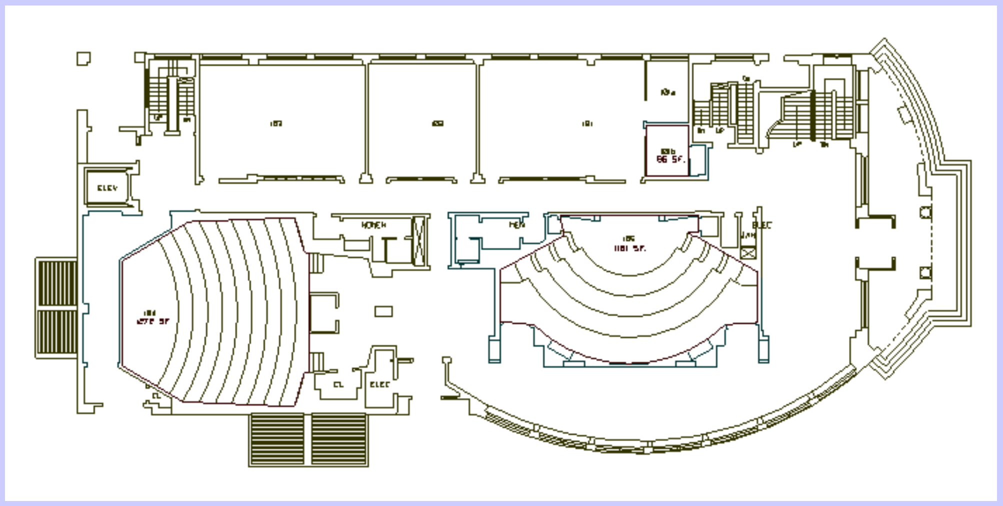

Transformations can be combined (with simple algebra)

Combining Transformations

- Transformations can be combined (with simple algebra)