Structure of a Language

Grammars: Notation to succinctly represent the structure of a language. Example:

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Introduction Syntax Analysis

Grammars

```
Stmt \longrightarrow if Expr then Stmt else Stmt
```

- Terminal symbols: if, then, else
 - Terminal symbols represent group of characters in input language: *Tokens*.
 - Analogous to words.
- Nonterminal symbols: Stmt, Expr
 - Nonterminal symbols represent a sequence of terminal symbols.
 - Analogous to sentences.

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Phases of Syntax Analysis

1 Identify the words: Lexical Analysis.

Converts a stream of characters (input program) into a stream of tokens.

Also called Scanning or Tokenizing.

2 Identify the sentences: Parsing.

Derive the structure of sentences: construct *parse trees* from a stream of tokens.

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Introduction Syntax Analysis

Applications of Languages

- Command Interpreters: csh, perl, ...
- Programming: FORTRAN, SmallTalk, ...
- Document Structuring: troff, LATEX, HTML, ...
- Page Definition: PostScript, PCL, ...
- Databases: SQL, ...
- Hardware Design: VHDL, VeriLog, ...
- ... and many many more

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Language Processing

Flexible control: programmable combination of primitive operations.

- Express input to the system in a well defined *language*.
- Translate the input into the sequence of primitive operations.
 - Direct execution
 - Byte code emulation
 - Object code compilation

Language processing techniques have evolved over the last 30 years. In almost every domain, at least three steps can be identified: *lexical analysis, parsing, and syntax-directed translation*.

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Introduction Lexical Analysis

Lexical Analysis

Convert a stream of characters into a stream of tokens.

- Simplicity: Conventions about "words" are often different from conventions about "sentences".
- Efficiency: Word identification problem has a much more efficient solution than sentence identification problem.
- Portability: Character set, special characters, device features.

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Terminology

• Token: Name given to a family of words.

e.g., integer_constant

- Lexeme: Actual sequence of characters representing a word. e.g., 32894
- Pattern: Notation used to identify the set of lexemes represented by a token.

e.g.,
$$[0-9]+$$

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Introduction Lexical Analysis

Terminology

A few more examples:

Token	Sample Lexemes	Pattern
while	while	while
integer_constant	32894, -1093, 0	[0-9]+
identifier	buffer_size	[a-zA-Z]+

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Patterns

How do we *compactly* represent the set of all lexemes corresponding to a token?

For instance:

The token $integer_constant$ represents the set of all integers: that is, all sequences of digits (0-9), preceded by an optional sign (+ or -).

Obviously, we cannot simply enumerate all lexemes.

Use Regular Expressions.

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Regular Expressions

Notation to represent (potentially) infinite sets of strings over alphabet Σ .

- a: stands for the set {a} that contains a single string a.
- $a \mid b$: stands for the set $\{a, b\}$ that contains two strings a and b.
 - Analogous to *Union*.
- ab: stands for the set {ab} that contains a single string ab.
 - Analogous to *Product*.
 - (a|b)(a|b): stands for the set $\{aa, ab, ba, bb\}$.
- a^* : stands for the set $\{\epsilon, a, aa, aaa, ...\}$ that contains all strings of zero or more a's.
 - Analogous to *closure* of the product operation.

 ϵ stands for the *empty string*.

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Regular Expressions

Examples of Regular Expressions over {a,b}:

- $(a|b)^*$: Set of strings with zero or more a's and zero or more b's: $\{\epsilon, a, b, aa, ab, ba, bb, aaa, aab, ...\}$
- (a^*b^*) : Set of strings with zero or more a's and zero or more b's such that all a's occur before any b: $\{\epsilon, a, b, aa, ab, bb, aaa, aab, abb, \ldots\}$
- $(a^*b^*)^*$: Set of strings with zero or more a's and zero or more b's: $\{\epsilon, a, b, aa, ab, ba, bb, aaa, aab, ...\}$

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Language of Regular Expressions

Let R be the set of all regular expressions over Σ . Then,

- Empty String: $\epsilon \in R$
- Unit Strings: $\alpha \in \Sigma \Rightarrow \alpha \in R$
- Concatenation: $r_1, r_2 \in R \Rightarrow r_1r_2 \in R$
- Alternative: $r_1, r_2 \in R \Rightarrow (r_1 \mid r_2) \in R$
- Kleene Closure: $r \in R \Rightarrow r^* \in R$

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Regular Expressions

Example: $(a \mid b)^*$

$$L = \bigcup_{i=0}^{\infty} L_i$$
 = $\{\epsilon, a, b, aa, ab, ba, bb, \ldots\}$

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Lexical Analysis

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Regular Expressions Expressions & Meaning

Semantics of Regular Expressions

Semantic Function \mathcal{L} : Maps regular expressions to sets of strings.

$$\mathcal{L}(\epsilon) = \{\epsilon\}$$
 $\mathcal{L}(\alpha) = \{\alpha\} \quad (\alpha \in \Sigma)$
 $\mathcal{L}(r_1 \mid r_2) = \mathcal{L}(r_1) \cup \mathcal{L}(r_2)$
 $\mathcal{L}(r_1 \mid r_2) = \mathcal{L}(r_1) \cdot \mathcal{L}(r_2)$
 $\mathcal{L}(r^*) = \{\epsilon\} \cup (\mathcal{L}(r) \cdot \mathcal{L}(r^*))$

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Computing the Semantics

$$\mathcal{L}(a) = \{a\}$$
 $\mathcal{L}(a \mid b) = \mathcal{L}(a) \cup \mathcal{L}(b)$
 $= \{a\} \cup \{b\}$
 $= \{a, b\}$
 $\mathcal{L}(ab) = \mathcal{L}(a) \cdot \mathcal{L}(b)$
 $= \{a\} \cdot \{b\}$
 $= \{ab\}$
 $\mathcal{L}((a \mid b)(a \mid b)) = \mathcal{L}(a \mid b) \cdot \mathcal{L}(a \mid b)$
 $= \{a, b\} \cdot \{a, b\}$
 $= \{aa, ab, ba, bb\}$

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Regular Expressions Expressions & Meaning

Computing the Semantics of Closure

Example:
$$\mathcal{L}((a \mid b)^*)$$

 $= \{\epsilon\} \cup (\mathcal{L}(a \mid b) \cdot \mathcal{L}((a \mid b)^*))$
 $L_0 = \{\epsilon\} \quad Base \ case$
 $L_1 = \{\epsilon\} \cup (\{a,b\} \cdot L_0)$
 $= \{\epsilon\} \cup (\{a,b\} \cdot \{\epsilon\})$
 $= \{\epsilon,a,b\}$
 $L_2 = \{\epsilon\} \cup (\{a,b\} \cdot L_1)$
 $= \{\epsilon\} \cup (\{a,b\} \cdot \{\epsilon,a,b\})$
 $= \{\epsilon,a,b,aa,ab,ba,bb\}$
:

$$\mathcal{L}((a \mid b)^*) = \mathcal{L}_{\infty} = \{\epsilon, \mathtt{a}, \mathtt{b}, \mathtt{aa}, \mathtt{ab}, \mathtt{ba}, \mathtt{bb}, \ldots\}$$

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Another Example

$\mathcal{L}((a^*b^*)^*)$:

```
 \mathcal{L}(a^*) = \{\epsilon, \mathbf{a}, \mathbf{aa}, \ldots\} 
 \mathcal{L}(b^*) = \{\epsilon, \mathbf{b}, \mathbf{bb}, \ldots\} 
 \mathcal{L}(a^*b^*) = \{\epsilon, \mathbf{a}, \mathbf{b}, \mathbf{aa}, \mathbf{ab}, \mathbf{bb}, \ldots\} 
 \mathcal{L}((a^*b^*)^*) = \{\epsilon\} 
 \cup \{\epsilon, \mathbf{a}, \mathbf{b}, \mathbf{aa}, \mathbf{ab}, \mathbf{bb}, \ldots\} 
 \cup \{\epsilon, \mathbf{a}, \mathbf{b}, \mathbf{aa}, \mathbf{ab}, \mathbf{bb}, \ldots\} 
 \cup \{\epsilon, \mathbf{a}, \mathbf{b}, \mathbf{aa}, \mathbf{ab}, \mathbf{ba}, \mathbf{bb}, \ldots\} 
 \vdots 
 = \{\epsilon, \mathbf{a}, \mathbf{b}, \mathbf{aa}, \mathbf{ab}, \mathbf{ba}, \mathbf{bb}, \ldots\}
```

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Regular Expressions Regular Definitions

Regular Definitions

Assign "names" to regular expressions. For example,

```
\begin{array}{ccc} \text{digit} & \longrightarrow & 0 \mid 1 \mid \cdots \mid 9 \\ \text{natural} & \longrightarrow & \text{digit digit}^* \end{array}
```

SHORTHANDS:

- a+: Set of strings with one or more occurrences of a.
- a?: Set of strings with zero or one occurrences of a.

Example:

$$\texttt{integer} \quad \longrightarrow \quad (+|-)^? \texttt{digit}^+$$

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Regular Definitions: Examples

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Regular Expressions Regular Definitions

Regular Definitions and Lexical Analysis

Regular Expressions and Definitions *specify* sets of strings over an input alphabet.

- They can hence be used to specify the set of *lexemes* associated with a *token*.
- That is, regular expressions and definitions can be used as the *pattern* language

How do we decide whether an input string belongs to the set of strings specified by a regular expression?

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Using Regular Definitions for Lexical Analysis

```
Q: Is <u>ababbaabbb</u> in \mathcal{L}(((a^*b^*)^*)?
A: Hm. Well. Let's see.
\mathcal{L}((a^*b^*)^*) = \{\epsilon\}
\cup \{\epsilon, a, b, aa, ab, bb,
aaa, aab, abb, bbb, \ldots\}
\cup \{\epsilon, a, b, aa, ab, ba, bb,
aaa, aab, aba, abb, baa, bab, bba, bbb, \ldots\}
\vdots
= ???
```

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Regular Expressions Regular Definitions

Recognizers

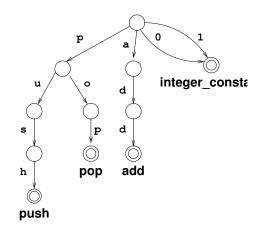
Construct automata that recognize strings belonging to a language.

- Finite State Automata ⇒ Regular Languages
 - ullet Finite State o cannot maintain arbitrary counts.
- Push Down Automata ⇒ Context-free Languages
 - Stack is used to maintain counter, but only one counter can go arbitrarily high.

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Recognizing Finite Sets of Strings

- Identifying words from a small, finite, fixed vocabulary is straightforward.
- For instance, consider a stack machine with push, pop, and add operations with two constants: 0 and 1.
- We can use the *automaton*:



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Finite State Automata Recognizers

Finite State Automata

Represented by a labeled directed graph.

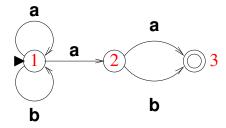
- A finite set of *states* (vertices).
- *Transitions* between states (edges).
- *Labels* on transitions are drawn from $\Sigma \cup \{\epsilon\}$.
- One distinguished *start* state.
- One or more distinguished *final* states.

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Finite State Automata: An Example

Consider the Regular Expression $(a \mid b)*a(a \mid b)$. $\mathcal{L}((a \mid b)*a(a \mid b)) = \{aa, ab, aaa, aab, baa, bab, aaaa, aaab, abaa, abab, baaa, ...\}.$

The following automaton determines whether an input string belongs to $\mathcal{L}((a \mid b)^* a(a \mid b))$:



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Finite State Automata Recognizers

Acceptance Criterion

A finite state automaton (NFA or DFA) accepts an input string x

... if beginning from the start state

... we can trace some path through the automaton

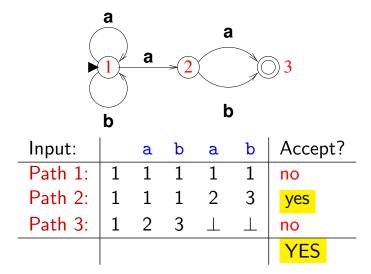
 \dots such that the sequence of edge labels spells x

... and end in a final state.

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Recognition with an NFA

Is $\underline{abab} \in \mathcal{L}((a \mid b)^*a(a \mid b))$?

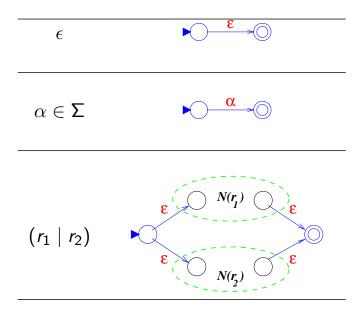


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Finite State Automata Recognizers

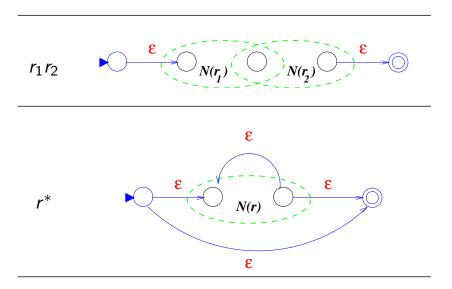
Regular Expressions to NFA

Thompson's Construction: For every regular expression r, derive an NFA N(r) with unique start and final states.



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Regular Expressions to NFA (contd.)

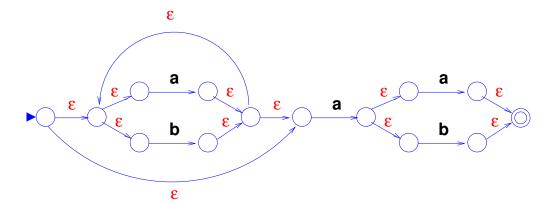


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Finite State Automata Recognizers

Example

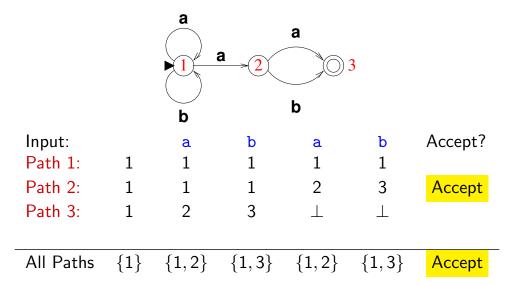
(a | b)*a(a | b):



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Recognition with an NFA

Is $\underline{abab} \in \mathcal{L}((a \mid b)^*a(a \mid b))$?

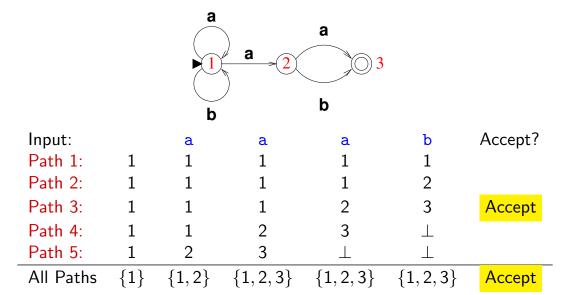


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Finite State Automata Recognizers

Recognition with an NFA (contd.)

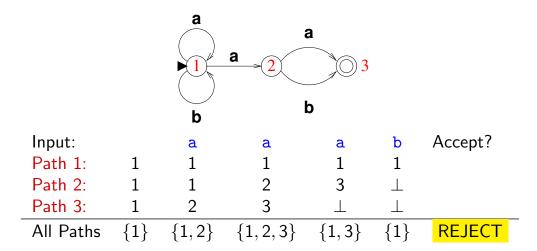
Is $\underline{aaab} \in \mathcal{L}((a \mid b)^*a(a \mid b))$?



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Recognition with an NFA (contd.)

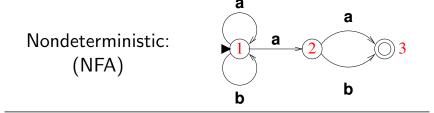
Is
$$\underline{aabb} \in \mathcal{L}((a \mid b)^*a(a \mid b))$$
?



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Finite State Automata DFA & NFA

Determinism $(a \mid b)*a(a \mid b)$:



a

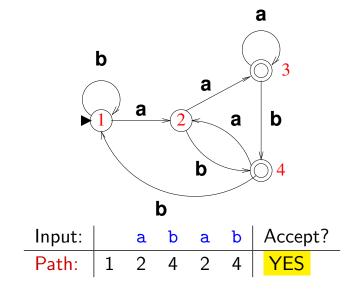
Deterministic:
(DFA)

b
a
b
b
4

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Recognition with a DFA

Is $\underline{abab} \in \mathcal{L}((a \mid b)^*a(a \mid b))$?



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Finite State Automata DFA & NFA

NFA vs. DFA

For every NFA, there is a DFA that accepts the same set of strings.

- NFA may have transitions labeled by ϵ . (Spontaneous transitions)
- All transition labels in a DFA belong to Σ .
- For some string x, there may be many accepting paths in an NFA.
- For all strings x, there is *one unique* accepting path in a DFA.
- Usually, an input string can be recognized *faster* with a DFA.
- NFAs are typically *smaller* than the corresponding DFAs.

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NFA vs. DFA (contd.)

R =Size of Regular Expression

N = Length of Input String

	NFA	DFA
Size of	$O(R)$ $O(2^R)$	
Automaton	O(N)	0(2)
Recognition time	$O(N \times R)$	O(N)
per input string	$ O(N \times N) O(N)$	

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Finite State Automata DFA & NFA

Converting NFA to DFA

Subset construction

Given a set S of NFA states,

- compute $S_{\epsilon} = \epsilon$ -closure(S): S_{ϵ} is the set of all NFA states reachable by zero or more ϵ -transitions from S.
- compute $S_{\alpha} = \text{goto}(S, \alpha)$:
 - S' is the set of all NFA states reachable from S by taking a transition labeled α .
 - $S_{\alpha} = \epsilon$ -closure(S').

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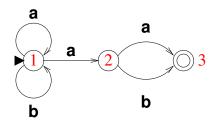
Converting NFA to DFA (contd).

- Each state in DFA corresponds to a set of states in NFA.
- Start state of DFA = ϵ -closure(start state of NFA).
- From a state s in DFA that corresponds to a set of states S in NFA:
 - let $S' = goto(S, \alpha)$ such that S' is non-empty.
 - add an α -transition to state s' that corresponds S' in NFA,
- S contains a final NFA state, and s is the corresponding DFA state $\Rightarrow s$ is a final state of DFA

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Finite State Automata DFA & NFA

$NFA \rightarrow DFA$: An Example

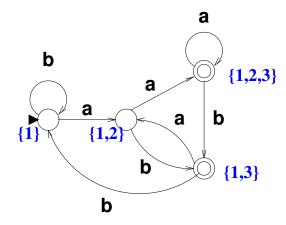


```
\epsilon-closure(\{1\})
                      {1}
goto(\{1\}, a)
                                                               {1, 2, 3}
                      {1, 2}
                                      goto({1,2,3},a)
goto(\{1\},b)
                                      goto(\{1,2,3\},b)
                      {1}
                                                               \{1,3\}
goto({1,2},a)
                                      goto({1,3},a)
                 = \{1, 2, 3\}
                                                               \{1, 2\}
                                      goto({1,3},b)
goto(\{1,2\},b)
                                                               \{1\}
```

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$NFA \rightarrow DFA$: An Example (contd.)

```
\epsilon-closure(\{1\})
                         {1}
goto(\{1\}, a)
                                         goto({1, 2, 3}, a)
                         \{1, 2\}
                                                                     \{1, 2, 3\}
                         {1}
                                                                     \{1, 3\}
goto(\{1\},b)
                                         goto({1,2,3},b)
goto({1,2},a)
                         \{1, 2, 3\}
                                         goto({1,3},a)
                                                                     \{1, 2\}
goto(\{1,2\},b)
                         \{1,3\}
                                         goto({1,3},b)
                                                                     {1}
```



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Generating Lexical Analyzers

Construction of a Lexical Analyzer

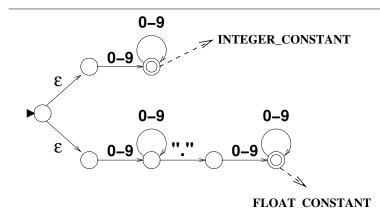
- Regular Expressions and Definitions are used to specify the set of strings (lexemes) corresponding to a *token*.
- An automaton (DFA/NFA) is built from the above specifications.
- Each final state is associated with an *action*: emit the corresponding token.

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Specifying Lexical Analysis

Consider a recognizer for integers (sequence of digits) and floats (sequence of digits separated by a decimal point).

```
[0-9]+ { emit(INTEGER_CONSTANT); }
[0-9]+"."[0-9]+ { emit(FLOAT_CONSTANT); }
```



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Generating Lexical Analyzers

Lex

- Tool for building lexical analyzers.
- Input: lexical specifications (.1 file)
- Output: C function (yylex) that returns a token on each invocation.
- Example:

```
%%
[0-9]+ { return(INTEGER_CONSTANT); }
[0-9]+"."[0-9]+ { return(FLOAT_CONSTANT); }
```

• Tokens are simply integers (#define's).

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Lex Specifications

```
%{
    C header statements for inclusion
%}
    Regular Definitions e.g.:
    digit [0-9]
%%

Token Specifications e.g.:
    {digit}+ { return(INTEGER_CONSTANT); }
%%
    Support functions in C
```

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Generating Lexical Analyzers

Regular Expressions in Lex

Adds "syntactic sugar" to regular expressions:

- Range: [0-7]: Integers from 0 through 7 (inclusive)
 [a-nx-zA-Q]: Letters a thru n, x thru z and A thru Q.
- Exception: [^/]: Any character other than /.
- Definition: {digit}: Use the previously specified regular definition digit.
- Special characters: Connectives of regular expression, convenience features.

```
e.g.: | * ^
```

Special Characters in Lex

For literal matching, enclose special characters in double quotes (") *e.g.:* "*"

Or use "\" to escape. e.g.: *

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Generating Lexical Analyzers

Examples

for	Sequence of f, o, r	
" "	C-style OR operator (two vert. bars)	
.*	Sequence of non-newline characters	
[^*/]+	Sequence of characters except * and /	
\"[^"]*\"	Sequence of non-quote characters	
	beginning and ending with a quote	
({letter} "_")({letter} {digit} "_")*		
C-style identifiers		

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A Complete Example

```
#include <stdio.h>
#include "tokens.h"
%}
digit
       [0-9]
hexdigit [0-9a-f]
%%
"+"
                        { return(PLUS); }
'' _ ''
                        { return(MINUS); }
{digit}+
                        { return(INTEGER_CONSTANT); }
{digit}+"."{digit}+
                        { return(FLOAT_CONSTANT); }
                        { return(SYNTAX_ERROR); }
%%
```

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Generating Lexical Analyzers

Actions

Actions are attached to final states.

- Distinguish the different final states.
- Used to return *tokens*.
- Can be used to set attribute values.
- Fragment of C code (blocks enclosed by '{' and '}').

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Attributes

Additional information about a token's lexeme.

- Stored in variable yylval
- Type of attributes (usually a union) specified by YYSTYPE
- Additional variables:
 - yytext: Lexeme (Actual text string)
 - yyleng: length of string in yytext
 - yylineno: Current line number (number of '\n' seen thus far)
 - enabled by %option yylineno

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Generating Lexical Analyzers

Priority of matching

What if an input string matches more than one pattern?

```
"if" { return(TOKEN_IF); }
{letter}+ { return(TOKEN_ID); }
"while" { return(TOKEN_WHILE); }
```

- A pattern that matches the longest string is chosen.
 Example: if1 is matched with an identifier, not the keyword if.
- Of patterns that match strings of same length, the first (from the top of file) is chosen.

Example: while is matched as an identifier, not the keyword while.

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Constructing Scanners using (f)lex

• Generated scanner in lex.yy.c

```
\begin{array}{ccc} & \text{(g)cc} \\ \text{lex.yy.c} & \longrightarrow & \textit{executable} \end{array}
```

- yywrap(): hook for signalling end of file.
- Use -lfl (flex) or -ll (lex) flags at link time to include default function yywrap() that always returns 1.

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Generating Lexical Analyzers

Implementing a Scanner

```
 \begin{array}{l} \textit{transition}: \textit{state} \times \Sigma \rightarrow \textit{state} \\ \\ \textit{algorithm } \textit{scanner}() \; \{ \\ \textit{current\_state} = \textit{start } \textit{state}; \\ \textit{while } (1) \; \{ \\ \textit{c} = \textit{getc}(); \; / ^* \; \textit{on } \textit{end } \textit{of } \textit{file, } \ldots \; ^* / \\ \\ \textit{if } \; \textit{defined}(\textit{transition}(\textit{current\_state, } \textit{c})) \\ \textit{current\_state} = \textit{transition}(\textit{current\_state, } \textit{c}); \\ \\ \textit{else} \\ \textit{return } \textit{s}; \\ \\ \} \\ \} \\ \end{aligned}
```

Implementing a Scanner (contd.)

Implementing the transition function:

- Simplest: 2-D array.
 Space inefficient.
- Traditionally compressed using row/colum equivalence. (default on (f)lex)

Good space-time tradeoff.

- Further table compression using various techniques:
 - Example: RDM (Row Displacement Method): Store rows in overlapping manner using 2 1-D arrays.

Smaller tables, but longer access times.

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Generating Lexical Analyzers

Lexical Analysis: Summary

Convert a stream of characters into a stream of tokens.

- Make rest of compiler independent of character set
- Strip off comments
- Recognize line numbers
- Ignore white space characters
- Process macros (definitions and uses)
- Interface with **symbol table** (also called "name table").

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