cse303
ELEMENTS OF THE THEORY OF COMPUTATION

Professor Anita Wasilewska
LECTURE 6a
REVIEW for Q2

Q2 covers Lecture 5 and Lecture 6

Chapter 2 - Deterministic Finite Automata DFA

Chapter 2 - Nondeterministic Finite Automata NDFA

1. Some YES-NO Questions
2. Some Very Short Questions
3. Some Homework Problems
CHAPTER 2 PART 1
Deterministic Finite Automata DFA
Nondeterministic Finite Automata NDFA
Write your answers and only after writing them check the solutions

Q1 Alphabet $\Sigma$ of any deterministic finite automaton $M$ is always non-empty

Q2 The set $K$ of states of any deterministic finite automaton is always non-empty

Q3 A configuration of a DF Automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^* \times K$

Q4 Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a transition relation iff the following condition holds

$(q, aw) \vdash_M (q', w)$ iff $\delta(q', a) = q$
Short YES/NO Questions

Q5  A configuration of a non-deterministic finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^*$

Q6  Given $M = (K, \Sigma, \delta, s, F)$ we define $L(M) = \{ w \in \Sigma^* : ((s, w) \vdash^* M(q, e)) \text{ for some } q \in K \}$

Q7  Given $M = (K, \Sigma, \delta, s, F)$ we define $L(M) = \{ w \in \Sigma^* : \exists q \in K ((s, w) \vdash^* M(q, e)) \}$

Q8  If $M = (K, \Sigma, \delta, s, F)$ is a deterministic, then $M$ is also non-deterministic

Q9  For any automata $M$, we have that $L(M) \neq \emptyset$

Q10 For any DFA $M = (K, \Sigma, \delta, s, F)$, $e \in L(M)$ if and only if $s \in F$
Short YES/NO Questions

Q11  $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when
      $\Delta \subseteq K \times \Sigma^* \times K$

Q12  $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when
      $\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$

Q13  The set of all configurations of any non-deterministic
      state automata is always non-empty

Q14  We say that two automata $M_1, M_2$ (deterministic or
      nondeterministic) are the same, i.e. $M_1 = M_2$ if and only if
      $L(M_1) = L(M_2)$

Q15  For any DFA $M$, there is is a NDFA $M'$, such that
      $M \approx M'$
Q16 For any NDFA $M$, there is a DFA $M'$, such that $M \approx M'$

Q17 If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then $M$ is also non-deterministic, as defined in the lecture

Q18 We define, for any (deterministic or non-deterministic $M = (K, \Sigma, \Delta, s, F)$ a computation of the length $n$ from $(q, w)$ to $(q', w')$ as a sequence

$$(q_1, w_1), (q_2, w_2), \ldots, (q_n, w_n), \quad n \geq 1$$

of configurations, such that $q_1 = q$, $q_n = q'$, $w_1 = w$, $w_n = w'$ and $(q_i, w_i) \vdash^* M(q_{i+1}, w_{i+1})$ for $i = 1, 2, \ldots, n - 1$

Statement: For any $M$ a computation $(q, w)$ exists
Here are solutions to some short YES/NO Questions for material covered in Chapter 2, Part 1

Solving Quizzes and Tests you have to write a short solutions and circle the answer

You will get 0 pts if you only circle your answer without providing a solution, even if it is correct

Here are some questions

**Q1** Alphabet $\Sigma$ of any deterministic finite automaton $M$ is always non-empty

no An alphabet $\Sigma$ is, by definition, any finite set, hence it can be empty

**Q2** The set $K$ of states of any deterministic finite automaton is always non-empty

yes $s \in K$
Short YES/NO Questions

**Q3** A configuration of a DF Automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^* \times K$

**no** Configuration is any element $(q, w) \in K \times \Sigma^*$

**Q4** Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a **transition relation** iff the following condition holds

$$(q, aw) \vdash_M (q', w) \text{ iff } \delta(q', a) = q$$

**no** Proper condition is:

$$(q, aw) \vdash_M (q', w) \text{ iff } \delta(q, a) = q'$$

**Q5** A configuration of a non-deterministic finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^*$

**yes** by definition
Short YES/NO Questions

Q6  Given $M = (K, \Sigma, \delta, s, F)$ we define
$L(M) = \{ w \in \Sigma^* : ((s, w) \vdash^* M(q, e)) \text{ for some } q \in K \}$
no  Must be: for some $q \in F$

Q7  Given $M = (K, \Sigma, \delta, s, F)$ we define
$L(M) = \{ w \in \Sigma^* : \exists q \in K ((s, w) \vdash^* M(q, e)) \}$
no  Must be: $\exists q \in F ((s, w) \vdash^* M(q, e))$

Observe that Q7 is really the Q6 written in symbolic way correctly using the symbol of existential quantifier
Short YES/NO Questions

Q8  If \( M = (K, \Sigma, \delta, s, F) \) is a deterministic, then \( M \) is also non-deterministic

**yes**  The function \( \delta \) is a (special) relation on \( K \times \Sigma \times K \), i.e.
\[ \delta \subseteq K \times \Sigma \times K \subseteq K \times \Sigma \cup \{e\} \times K \subseteq K \times \Sigma^* \times K \]

Q9  For any automata \( M \), we have that \( L(M) \neq \emptyset \)

**no**  Take \( M \) with \( \Sigma = \emptyset \) or \( F = \emptyset \) then we get \( L(M) = \emptyset \)

Q10  For any DFA \( M = (K, \Sigma, \delta, s, F) \), \( e \in L(M) \) if and only if \( s \in F \)

**yes**  this is the **DFA Theorem**
Short YES/NO Questions

Q11  $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta \subseteq K \times \Sigma^* \times K$

no  we must say: $\Delta$ is finite

Q12  $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$

yes  this is book definition; do not need to say $\Delta$ is a finite set, as the set $K \times (\Sigma \cup \{e\}) \times K$ is always finite

Q13  The set of all configurations of any non-deterministic state automata is always non-empty

yes  the set of all configuration of NDFA is by definition $K \times \Sigma^* = \{(q, w) : q \in K, w \in \Sigma^*\}$ and we have that $(s, e) \in K \times \Sigma^*$ even when $\Sigma = \emptyset$ as always $s \in K$, $e \in \Sigma^*$
Short YES/NO Questions

Q14  We say that two automata $M_1, M_2$ (deterministic or nondeterministic) are the same, i.e. $M_1 = M_2$ if and only if $L(M_1) = L(M_2)$

no  we say that $M_1, M_2$ are equivalent, i.e. $M_1 \approx M_2$ if and only if $L(M_1) = L(M_2)$

Q15  For any DFA $M$, there is a NDFA $M'$, such that $M \approx M'$

yes  This is the **Equivalency Theorems 1**

Q16  For any NDFA $M$, there is a DFA $M'$, such that $M \approx M'$

yes  This is the **Equivalency Theorems 2**
Q17 If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then $M$ is also non-deterministic, as defined in the lecture

**yes** $\Sigma \cup \{e\} \subseteq \Sigma^*$

Q18 We define, for any (deterministic or non-deterministic $M = (K, \Sigma, \Delta, s, F)$ a computation of the length $n$ from $(q, w)$ to $(q', w')$ as a sequence

$$(q_1, w_1), (q_2, w_2), ..., (q_n, w_n), \quad n \geq 1$$

of configurations, such that

$q_1 = q, \quad q_n = q', \quad w_1 = w, \quad w_n = w'$ and $(q_i, w_i) \vdash^* M (q_{i+1}, w_{i+1})$ for $i = 1, 2, ..n - 1$

**Statement:** For any $M$ a computation $(q, w)$ exists

**yes** By definition a computation of length one (case $n=1$) always exists
Very Short Questions

For all short questions given on Quizzes and Tests you will have to do the following

1. Decide and explain whether the diagram represents a DFA, NDFA or does not

2. List all components of $M$ when it represents DFA, NDFA

3. Describe $L(M)$ as a regular expression when it represents DFA, NDFA
Very Short Questions

Consider a diagram $M_1$

1. Yes, it represents a DFA; $\delta$ is a function on $\{q_0, q_1\} \times \{a\}$ and initial state $s = q_0$ exists
2. $K = \{q_0, q_1\}$, $\Sigma = \{a\}$, $s = q_0$, $F = \{q_1\}$,
   
   $\delta(q_0, a) = q_1$, $\delta(q_1, a) = q_1$
3. $L(M_1) = aa^*$
Very Short Questions

Consider a diagram \( M2 \)

1. Yes, it represents a DFA; \( \delta \) is a function on \( \{ q_0 \} \times \{ a \} \) and initial state \( s = q_0 \) exists
2. \( K = \{ q_0 \}, \Sigma = \{ a \}, s = q_0, F = \emptyset, \delta(q_0, a) = q_0 \)
3. \( L(M2) = \emptyset \)
Consider a diagram $M_3$

1. Yes, it represents a DFA; initial state $s = q_0$ exists
2. $K = \{q_0\}$, $\Sigma = \emptyset$, $s = q_0$, $F = \emptyset$, $\delta = \emptyset$
3. $L(M_3) = \emptyset$
Consider a diagram M4

1. Yes, it represents a DFA; initial state $s = q_0$ exists
2. $K = \{q_0\}$, $\Sigma = \{a\}$, $s = q_0$, $F = \{q_0\}$, $\delta(q_0, a) = q_0$
3. $L(M4) = a^*$

Remark $e \in L(M4)$ by DFA Theorem, as $s = q_0 \in F = \{q_0\}$
Very Short Questions

Consider a diagram M5

1. NO! it is NOT neither DFA nor NDFA - initial state does not exist
Very Short Questions

Consider a diagram $M_6$

1. It is not a DFA; Initial state does exist, but $\delta$ is not a function; $\delta(q_0, b)$ is not defined and we didn’t say "plus trap states"
2. It is a NDFA
3. $L(M_6) = \emptyset$
Consider a diagram M7

1. Yes! it is a DFA with trap states
   Initial state exists and we can complete definition of $\delta$ by adding a trap state as pictured below
Very Short Questions

Consider again diagram M7

2. If we do not say "plus trap states" it represents a NDFA with 
\[ \Delta = \{(q_0, a, q_1), (q_1, a, q_1), (q_1, b, q_1)\} \]

3. \[ L(M7) = \emptyset \] as \[ F = \emptyset \]
Very Short Questions

There is much more Short Questions examples in the section SHORT PROBLEMS at the end of Lecture 5
Some Homework Problems

Problem 1
Construct deterministic $M$ such that

$$L(M) = \{ w \in \Sigma^* : w \text{ has an odd number of } a \text{'s and an even number of } b \text{'s } \}$$

Solution
Here is the short diagram - we must say: plus trap states
Problem 2

Construct a DFA $M$ such that

$L(M) = \{ w \in \{a, b\}^* : \text{every substring of length 4 in word } w \text{ contains at least one } b \}$

Solution  Here is a short pattern diagram (the trap states are not included)
Some Homework Problems

Problem 3
Construct a DFA $M$ such that

$$L(M) = \{ w \in \{a, b\}^* : \text{every word } w \text{ contains an even number of sub-strings } ba \}$$

Solution

Here is a pattern diagram

Zero is an even number so we must have that $e \in L(M)$, i.e. we have to make the initial state also a final state.
Problem 4

Construct a DFA $M$ such that

$$L(M) = \{ w \in \{a, b\}^* : w \text{ has } abab \text{ as a substring} \}$$

Solution  The essential part of the diagram must produce $abab$ and it can be surrounded by proper elements on both sides and can be repeated.

Here is the essential part of the diagram

![Diagram](image)
We complete the **essential part** following the fact that it can be surrounded by proper elements on both sides and can be repeated.

Here is the **diagram** of M.

Observe that this is a **pattern diagram**; you need to add names of states only if you want to list all components. M does not have trap states.
Problem 5

Use book or lecture definition (specify which are you using) to construct a non-deterministic finite automaton $M$, such that

$$L(M) = (ab)^*(ba)^*$$

Specify all components $K, \Sigma, \Delta, s, F$ of $M$ and draw a state diagram.

Justify your construction by listing some strings accepted by the state diagram.
Solution 1
We use the lecture definition
Components of $M$ are:

$$\Sigma = \{a, b\}, \quad K = \{q_0, q_1\}, \quad s = q_0, \quad F = \{q_0, q_1\}$$

We define $\Delta$ as follows

$$\Delta = \{(q_0, ab, q_0), (q_0, e, q_1), (q_1, ba, q_1)\}$$

Strings accepted: $ab, abab, abba, babba, ababbaba, ...$
Problem 5 Solutions

Solution 2

We use the **book definition**

Components of $M$ are:

$$
\Sigma = \{a, b\}, \quad K = \{q_0, q_1, q_2, q_3\}, \quad s = q_0, \quad F = \{q_2\}
$$

We define $\Delta$ as follows

$$
\Delta = \{(q_0, a, q_1), (q_1, b, q_0), (q_0, e, q_2), (q_2, b, q_3), (q_3, a, q_2)\}
$$

Strings accepted: $ab, abab, abba, babba, ababbaba, ....$
Some Homework Problems

Problem 6
Let $M = (K, \Sigma, s, \Delta, F)$ for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $\Sigma = \{a, b, c\}$, $F = \{q_3\}$ and

$\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\}$

Find the regular expression describing the $L(M)$.
Simplify it as much as you can. Explain your steps

Solution

$L(M) = (abc)^*abbb \cup abbb \cup (abc)^*baa \cup ba = (abc)^*abbb \cup (abc)^*baa(abc)^*(abbb \cup baa)$

We used the property: $LL_1 \cup LL_2 = L(L_1 \cup L_2)$
Some Homework Problems

Problem 7
Let \( M = (K, \Sigma, s, \Delta, F) \) for \( K = \{q_0, q_1, q_2, q_3\}, s = q_0, \Sigma = \{a, b, c\}, F = \{q_3\} \) and \( \Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\} \)

Write down (you can draw the diagram) an automaton \( M' \) such that \( M' \equiv M \) and \( M' \) is defined by the BOOK definition.

Solution
We apply the "stretching" technique to \( M \) and the new \( M' \) is as follows.

\( M' = (K \cup \{p_1, p_2, ..., p_5\}, \Sigma, s = q_0, \Delta', F' = F) \)

\( \Delta' = \{(q_0, b, q_2), (q_0, a, p_1), (p_1, b, p_2), (p_2, c, q_0), (q_0, a, p_3), (p_3, b, q_1), (q_1, b, p_4), (p_4, b, q_3), (q_0, b, q_2), (q_2, a, p_5), (p_5, a, q_3)\} \)
Some Homework Problems

I will NOT include this problem on Q2, but you have to know how to solve similar problems for Midterm

Problem 8
Let \( M \) be defined as follows

\[ M = (K, \Sigma, s, \Delta, F) \]

for \( K = \{q_0, q_1, q_2\} \), \( s = q_0 \), \( \Sigma = \{a, b\} \), \( F = \{q_0, q_2\} \) and

\[ \Delta = \{(q_0, a, q_1), (q_1, b, q_2), (q_1, b, q_0), (q_2, a, q_0)\} \]

Write 4 steps of the general method of transformation a NDFA \( M \), into an equivalent \( M' \), which is a DFA
Problem 8

Reminder

\[ E(q) = \{ p \in K : (q, e) \vdash^* M(p, e) \} \] and

\[ \delta(Q, \sigma) = \bigcup_{p \in K} \{ E(p) : \exists q \in Q (q, \sigma, p) \in \Delta \} \]

Step 1
Evaluate \( \delta(E(q_0), a) \) and \( \delta(E(q_0), b) \)

Step i+1]
Evaluate \( \delta \) on all states that result from Step i
Problem 8

Solution

\[ \delta(Q, \sigma) = \bigcup_{p \in K} \{ E(p) : \exists q \in Q \ (q, \sigma, p) \in \Delta \} \]

Step 1

\[ E(q_0) = \{ q_0 \}, \ E(q_1) = \{ q_1 \}, \ E(q_2) = \{ q_2 \} \]

\[ \delta(\{ q_0 \}, a) = E(q_1) = \{ q_1 \}, \ \delta(\{ q_0 \}, b) = \emptyset \]

Step 2

\[ \delta(\emptyset, a) = \emptyset, \ \delta(\emptyset, a) = \emptyset, \ \delta(\{ q_1 \}, a) = \emptyset, \]
\[ \delta(\{ q_1 \}, b) = E(q_0) \cup E(q_2) = \{ q_0, q_2 \} \in F' \]
Problem 8

Solution

\[ \delta(Q, \sigma) = \bigcup_{p \in K} \{ E(p) : \exists q \in Q (q, \sigma, p) \in \Delta \} \]

Step 3

\[ \delta(\{q_0, q_2\}, a) = E(q_1) \cup E(q_0) = \{q_0, q_1\}, \quad \delta(\{q_0, q_2\}, b) = \emptyset \]

Step 4

\[ \delta(\{q_0, q_1\}, a) = \emptyset \cup E(q_1) = \{q_1\}, \]
\[ \delta(\{q_0, q_1\}, b) = \emptyset \cup E(q_0) \cup E(q_2) = \{q_0, q_2\} \in F' \]