PART 1  YES/NO QUESTIONS - (2pts) each

1. For any language $L$ there is a deterministic automata $M$, such that $L = L(M)$.
   Justify: language must be regular
   $\text{n}$

2. Given $L_1, L_2$ regular languages over $\Sigma$, then $(L_1 \cap (\Sigma^* - L_1))L_2$ is not regular.
   Justify: Regular languages are closed under intersection, complement, and concatenation
   $\text{n}$

3. There is an algorithm that for any finite automata $M$ computes a regular expression $r$, such that $L(M) = r$.
   Justify: as defined in the proof of Main Theorem
   $\text{y}$

4. $L = \{a^{2n} : n \geq 0\}$ is not regular.
   Justify: $L = (aa)^*$
   $\text{n}$

5. $L = \{b^n a^n : n \geq 0\}$ is regular.
   Justify: We proved using Pumping Lemma that I is not regular
   $\text{n}$

PART 2: PROBLEMS

QUESTION 1  (7pts) Use the constructions defined in the proof of the theorem: A language is regular iff it is accepted by a finite automata to construct a finite automata $M$ such that

$$L(M) = (ab \cup c)^*$$

Draw PATTERN diagrams. Use the constructions described in the proof of the Closure Theorem.

S1. (3pt) Draw diagram of automata $M_a M_b \cup M_c$.

The question is about the direct application of the definitions of automaton $M_\sigma$ for any $\sigma \in \Sigma$, and of the concatenation and union of two automata as defined in the proof Closure Theorem

S2. (4pt) Draw diagram of $M = (M_a M_b \cup M_c)^*$.

The question is about the direct application of the definition of construction of automata that is a Kleene star of a given one.
QUESTION 2 (8pts)

Evaluate regular expression $r$, such that $L(M) = r$ using the Generalized Automata Construction for $M$ given by

$$M = ([q_1, q_2], \{a, b\}, s = q_1,$$

$$\Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\}, F = \{q_2\})$$

Remember using to use the proper names for states.

1. (2pt) Draw a diagram of $GM$ defined below

$$GM = ([q_1, q_2, q_3, q_4], \{a, b\}, s = q_3, F = \{q_4\})$$

$$\Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\}, (q_3, e, q_1), (q_2, e, q_4))$$

2. (3pt) Draw a diagram of $GM1 \simeq GM \simeq M$ obtained by elimination of $q_1$.

$$GM1 = ([q_2, q_3, q_4], \{a, b\}, s = q_3, F = \{q_4\})$$

$$\Delta = \{(q_2, a^*b, q_2), (q_2, a, q_2), (q_2, b, q_4)\}, (q_2, e, q_4))$$

3. (2pt) Draw a diagram of $GM2 \simeq GM1 \simeq GM \simeq M$ obtained by elimination of $q_2$.

$$GM2 = ([q_3, q_4], \{a, b\}, s = q_3, F = \{q_4\})$$

$$\Delta = \{(q_3, a^*b(a^*b \cup a)^*, q_4)\})$$

4. (1pt) Write the regular expression $r$, such that $L(M) = r$

The regular expression $r$, such that $L(M) = r$ is $a^*b(ba^*b \cup a)^*$, i.e.

$$L(M) = a^*b(ba^*b \cup a)^*$$