CSE303 Q3 SOLUTIONS

YES/NO questions Circle the correct answer. Write SHORT justification.

1. For any language $L \subseteq \Sigma^*, \Sigma \neq \emptyset$ there is a deterministic automata M, such that $L = L(M)$.
   Justify: only when L is regular **n**
2. Any regular language has a finite representation.
   Justify: definition; regular expression is a finite string **y**
3. Any finite language is regular.
   Justify: any finite language is a finite union of one element regular languages **y**
4. Given L_1, L_2 languages over Σ, then $((L_1 \cap (\Sigma^* - L_2)) \cup L_2)L_1$ is regular.
   Justify: only when both are regular languages **n**
5. For any deterministic automata M, $L(M) = \bigcup\{R(1, j, n) : q_j \in F\}$, where $R(1, j, n)$ is the set of all strings in Σ^* that may drive M from state initial state to state q_j without passing through any intermediate state numbered $n+1$ or greater, where n is the number of states of M.
   Justify: basic fact and definition **y**
6. Σ in any Generalized Finite Automaton includes some regular expressions.
   Justify: GFA recognizes regular expressions over Σ **n**
7. For any finite automata M, there is a regular expression r, such that $L(M) = r$ (short hand notation).
   Justify: main theorem **y**
8. Pumping Lemma says that we can always prove that a language is not regular.
   Justify: PL gives a certain characterization of infinite regular languages **n**
9. Pumping Lemma serves as a tool for proving that a language is not regular.
   Justify: when the language is infinite and we can get contradiction **y**
10. $L = \{w \in \{a, b\}^* : w = w^R\}$ is regular.
    Justify: not regular, proof by PL **n**
11. $L = \{a^n a^n : n \geq 0\}$ is not regular.
    Justify: $L = (aa)^*$ and hence regular **n**

12. $L = \{a^n c^n : n \geq 0\}$ is regular.
Justify: not regular, proof by PL **n**
13. Let L be a regular language, and $L_1 \subseteq L$, then L_1 is regular.
Justify: $L_1 = \{a^n b^n : n \geq 0\}$ is a non-regular subset of regular $L = a^* b^*$ **n**
14. Let L be a regular language. The language $L^R = \{w^R : w \in L\}$ is regular.
Justify: L^R is accepted by finite automata M^R constructed from M such that $L(M) = L$ **y**

PROBLEMS

QUESTION 1 Using the construction in the proof of theorem

A language is regular iff it is accepted by a finite automata

construct a a finite automata M accepting

$$L_1 = \mathcal{L} = ((ab)^* \cup (bc)^*)ba$$

You can just draw a diagrams.

1. Diagrams for $M1, M2, M3$ such that $L(M1) = ab, L(M2) = bc, L(M3) = ba$

Solution

M1 components:

$$K = \{q_1, q_2\}, \Sigma = \{a, b, c\}, s = q_1, F = \{q_2\},$$
$$\Delta_{M1} = \{(q_1, ab, q_2)\}$$

M2 components:

$$K = \{q_3, q_4\}, \Sigma = \{a, b, c\}, s = q_3, F = \{q_4\},$$
$$\Delta_{M2} = \{(q_2, bc, q_4)\}$$

M3 components:

$$K = \{q_5, q_6\}, \Sigma = \{a, b, c\}, s = q_5, F = \{q_6\},$$
$$\Delta_{M3} = \{(q_5, ba, q_6)\}$$

2. Diagrams for $M4, M5$ such that $L(M4) = L(M1)^*, L(M5) = L(M2)^*$

Solution

M4 components:

$$K = \{q_1, q_2, q_7\}, \Sigma = \{a, b, c\}, s = q_7, F = \{q_2, q_7\},$$

$$\Delta_{M4} = \{(q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1)\}$$

M5 components:

$$K = \{q_3, q_4, q_8\}, \Sigma = \{a, b, c\}, s = q_8, F = \{q_4, q_8\},$$

$$\Delta_{M4} = \{(q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3)\}$$

3. Diagram for $M6$ such that $L(M5) = L(M4) \cup L(M5)$

Solution

M5 components:

$$K = \{q_1, q_2, q_3, q_4, q_7, q_8, q_9\}, \Sigma = \{a, b, c\}, s = q_9, F = \{q_2, q_4, q_7, q_8\},$$

$$\Delta_{M5} = \{(q_9, e, q_7), (q_9, e, q_8), (q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3)\}$$

4. Diagram for $M = M5M3$, i.e M is such that $L(M) = L(M5)L(M3)$.

M components:

$$K = \{q_1, q_2, q_3, q_4, q_7, q_8, q_9\}, \Sigma = \{a, b, c\}, s = q_9, F = \{q_6\},$$

$$\begin{aligned}\Delta_{M5} &= \Delta_{M4} \cup \{(q_7, e, q_5), (q_8, e, q_5), (q_2, e, q_5), (q_4, e, q_5), (q_5, ba, q_6)\} \\ &= \{(q_9, e, q_7), (q_9, e, q_8), (q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3), \\ &\quad (q_7, e, q_5), (q_8, e, q_5), (q_2, e, q_5), (q_4, e, q_5), (q_5, ba, q_6)\}\end{aligned}$$

QUESTION 2 For the automaton M

$$M = (\{q_1, q_2\},\ \{a, b\},\ s = q_1,$$

$$\Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\},\ F = \{q_2\})$$

Q2(a) (2pts) Evaluate 4 steps, in which you must include at least one $R(i, j, 0)$, in the construction of regular expression that defines $L(M)$ that uses the formulas:

$$L(M) = \bigcup \{R(1, j, n) :\ q_j \in F\}$$

$$R(i, j, k) = R(i, j, k-1) \cup R(i, k, k-1) R(k, k, k-1)^* R(k, j, k-1)$$

where n is the number of states of M, $k = 1, ...n$ and

$R(i, j, 0)$ is either $\{a \in \Sigma \cup \{e\} : \ (q_i, a, q_j) \in \Delta\}$ if $i \neq j$, or is

$\{e\} \cup \{a \in \Sigma \cup \{e\} : \ (q_i, a, q_j) \in \Delta\}$ if $i = j$.

Solution

Step 1 $L(M) = R(1, 2, 2)$

Step 2 $R(1, 2, 2) = R(1, 2, 1) \cup R(1, 2, 1)R(2, 2, 1)^* R(2, 2, 1)$

Step 3 $R(1, 2, 1) = R(1, 2, 0) \cup R(1, 1, 0)R(1, 1, 0)^* R(1, 2, 0)$

Step 4 $R(1, 2, 0) = \{b\}, \ R(1, 1, 0) = \{e\} \cup \{a\}$

$R(1, 2, 1) = \{e\} \cup \{a\} \cup (\{e\} \cup \{a\})(\{e\} \cup \{a\})^* \{b\}$

Question 3 Evaluate r, such that

$$\mathcal{L}(r) = L(M)$$

using the Generalized Automata Construction, as described in example 2.3.2 page 80.

$$M = (\{q_1, q_2\}, \ \{a, b\}, \ s = q_1,$$

$$\Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\}, \ F = \{q_2\})$$

Step 1: Construct a generalized GM that extends M, i.e. such that $L(M) = L(GM)$

Solution

$$GM = (\{q_1, q_2, q_3, q_4\}, \ \{a, b\}, \ s = q_3, \ F = \{q_4\}$$

$$\Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\}, (q_3, e, q_1), (q_2, e, q_4))$$

Step 2: Construct $GM1 \simeq GM \simeq M$ by elimination of q_1.

Solution

$$GM1 = (\{q_2, q_3, q_4\}, \ \{a, b\}, \ s = q_3, \ F = \{q_4\}$$

$$\Delta = \{(q_3, a^* b, q_2), (q_2, a, q_2), (q_2, ba^* b, q_2)\}, (q_2, e, q_4))$$

Step 3: Construct $GM2 \simeq GM1 \simeq GM \simeq M$ by elimination of q_2.

Solution

$$GM2 = (\{q_3, q_4\},\ \{a, b\},\ s = q_3,\ F = \{q_4\}$$

$$\Delta = \{(q_3, a^*b(ba^*b \cup a)^*, q_4))$$

Answer : the language is

$$L(M) = a^*b(ba^*b \cup a)^*$$

QUESTION 4 Show that the class of regular languages is not closed with respect to subset relation.

Solution Consider

$$L_1 = \{a^n b^n : n \in N\}, \quad L_2 = a^*b^*$$

$L_1 \subseteq L_2$ and L_1 is a non-regular subset of a regular L_2.

QUESTION 5

1. If L_1, L_2 are regular languages, is $L_1 \cap L_2$ also regular? Explain.

Solution YES, class of regular languages is closed under $\cap$.

2. If $L_1 \cap L_2$ is a regular language, are L_1 and L_2 also regular? Explain.

Solution NO. Take

$$L_1 = \{a^n b^n : n \in N\}, \quad L_2 = \{a^n : n \in Prime\}$$

$L_1 \cap L_2 = \emptyset$ is a regular language and L_1, L_2 are not regular

QUESTION 6 Show that the language

$$L = \{xyx^R :\ x, y \in \Sigma^*\}$$

is regular for any Σ.

Solution Take $x = e \in \Sigma^*$. The language

$$L_1 = \{eye^R :\ e, y \in \Sigma^*\} \subseteq L$$

and $L_1 = \Sigma^*$. We get $\Sigma^* \subseteq L \subseteq \Sigma^*$ and hence $L = \Sigma^*$ is regular.

QUESTION 7 Show that if L is regular, so is the language

$$L_1 = \{xy :\ x \in L, y \notin L\}.$$

Solution Observe that $L_1 = L(\Sigma^* - L)$ and L regular, hence $\Sigma^* - L)$ is regular (closure under complement), so is L_1 by closure under concatenation.