CSE303  Q2 PRACTICE SOLUTIONS

PART 1: YES/NO QUESTIONS  Circle the correct answer. Write SHORT justification.

1. The set $K$ of states of any deterministic finite automaton is always non-empty  
   Justify: $s \in K$  

2. Alphabet $\Sigma$ of any deterministic finite automaton is always non-empty  
   Justify: An alphabet $\Sigma$ is any FINITE set, hence it can be empty.  

3. A configuration of a deterministic finite automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^*$.  
   Justify: this is definition  

4. Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a one step computation iff the following condition holds  
   $(q, aw) \vdash_M (q', w)$ iff $\delta(q', a) = q$.  
   Justify: Proper condition is:  
   $(q, aw) \vdash_M (q', w)$ iff $\delta(q, a) = q'$.  

5. Given $M = (K, \Sigma, \delta, s, F)$ we define  
   $L(M) = \{ w \in \Sigma^* : \exists q \in K ((s, w) \vdash^*_M (q, e)) \}$.  
   Justify: Must be: $\exists q \in F ((s, w) \vdash^*_M (q, e))$.  

6. If $M = (K, \Sigma, \delta, s, F)$ is a deterministic, then $M$ is also non-deterministic.  
   Justify: The function $\delta$ is a (special) relation on $K \times \Sigma \times K$, i.e.  
   $\delta = \Delta \subseteq K \times \Sigma \times K \subseteq K \times \Sigma \cup \{e\} \times K \subseteq K \times \Sigma^* \times K$.  

7. A configuration of a non-deterministic finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^*$.  
   Justify: by definition  

TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA
**BOOK DEFINITION:** $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when
\[ \Delta \subseteq K \times (\Sigma \cup \{e\}) \times K \]

OBSERVE that $\Delta$ is always finite because $K, \Sigma$ are finite sets.

**LECTURE DEFINITION:** $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta$ is finite and
\[ \Delta \subseteq K \times \Sigma^* \times K. \]

OBSERVE that we have to say in this case that $\Delta$ is finite because $\Sigma^*$ is infinite.

**SOLVING PROBLEMS** you can use any of these definitions.

**A VERY SHORT QUESTION** Given the automaton $M$ with the following components:
- $\Sigma = \{a, b, c\}$, $K = \{q_0, q_1, q_2\}$, $s = q_0$, $F = \{q_2\}$.
- We define $\Delta$ as follows.
- $\Delta = \{(q_0, abc, q_1), (q_1, e, q_2), (q_0, a, q_2)\}$

1. State and explain whether $M$ represents a deterministic or a non-deterministic automaton.

**Solution** $M$ is non-deterministic. $\Delta$ is not a function on $K \times \Sigma^*$, also $\Delta \subseteq K \times \Sigma^* \times K$ (Lecture definition).

2. Write down a regular expression representing $L(M)$.

**Solution**
\[ L(M) = abc \cup a \]

**PART 2: PROBLEMS**

**QUESTION 1** Construct a deterministic finite automaton $M$ such that
\[ L(M) = \{w \in \{a, b\}^* : \text{neither } bb \text{ nor } aa \text{ is a substring of } w\}. \]

Draw a state diagram and specify all components $K, \Sigma, s, F$ of $M$. Justify your construction.

**Solution**
Components of $M = (K, \Sigma, \delta, s, F)$ are:

$\Sigma = \{a, b\}$, $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $q_3$ is a trap state, $F = \{q_0, q_1, q_2\}$.

We define $\delta$ on non-trap states as follows.

$\delta(q_0, a) = q_1$, $\delta(q_0, b) = q_2$,
$\delta(q_1, b) = q_2$,
$\delta(q_2, a) = q_1$.

$M$ accepts strings $a, aba, ababa...$ or $b, bab, baba...$ etc and never $aa, bb,$ etc...

**QUESTION 2** For the automata $M$ defined below describe the property defining $L(M)$.

Components of $M$ are:

$\Sigma = \{a, b\}$, $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $F = \{q_1\}$.

We define $\delta$ as follows.

$\delta(q_0, a) = q_1$, $\delta(q_0, b) = q_2$,
$\delta(q_1, a) = q_0$, $\delta(q_1, b) = q_3$,
$\delta(q_2, a) = q_3$, $\delta(q_2, b) = q_0$,
$\delta(q_3, a) = q_2$, $\delta(q_3, b) = q_1$.

**Solution**

**Language** of $M$ is:

$L(M) = \{w \in \Sigma^* : w$ has an odd number of $a$'s and an even number of $b$'s$\}$.

**QUESTION 3** Use book or lecture definition (specify which are you using) to construct a non-deterministic finite automaton $M$, such that

$L(M) = (ab)^*(ba)^*$.

Draw a state diagram and specify all components $K, \Sigma, \Delta, s, F$ of $M$. Justify your construction by listing some strings accepted by the state diagram.

**Solution 1** We use the lecture definition.

Components of $M$ are: $\Sigma = \{a, b\}$, $K = \{q_0, q_1\}$, $s = q_0$, $F = \{q_0, q_1\}$.

We define $\Delta$ as follows.

$\Delta = \{(q_0, ab, q_0), (q_0, c, q_1), (q_1, ba, q_1)\}$.

Strings accepted : $ab, abab, abba, ababba, ababbaba,....$

**Solution 2** We use the book definition.
Components of $M$ are: $\Sigma = \{a, b\}$, $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $F = \{q_2\}$.
We define $\Delta$ as follows.
$\Delta = \{(q_0, a, q_1), (q_1, b, q_0), (q_0, e, q_2), (q_2, b, q_3), (q_3, a, q_2)\}$.

Strings accepted: $ab, abab, abba, ababba, ababbaba, \ldots$

QUESTION 4 Let $M$ be defined as follows

$M = (K, \Sigma, s, \Delta, F)$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$
$\Sigma = \{a, b, c\}$, $F = \{q_3\}$ and
$\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\}$.

1. Find the regular expression describing the $L(M)$. Simplify it as much as you can. Explain your steps.

Solution $L(M) = (abc)^*abbb \cup abba \cup ba = (abc)^*abbb \cup (abc)^*baa \cup baa$.

We used the property:

$LL_1 \cup LL_2 = L(L_1 \cup L_2)$.

2. Write down (you can draw the diagram) an automata $M'$ such that $M' \equiv M$ and $M'$ is defined by the BOOK definition.

Solution We apply the "stretching" technique to $M$ and the new $M'$ is as follows.

$M' = (K \cup \{p_1, p_2, \ldots, p_5\}, \Sigma, s = q_0, \Delta', F' = F)$

for $K = \{q_0, q_1, q_2\}$, $s = q_0$
$\Sigma = \{a, b\}$, $F = \{q_3\}$ and
$\Delta' = \{(q_0, b, q_2), (q_0, a, p_1), (p_1, b, p_2), (p_2, c, q_0), (q_0, a, p_3), (p_3, b, q_1), (q_1, b, p_4), (p_4, b, q_3), (q_0, b, q_2), (q_2, a, p_5), (p_5, a, q_3)\}$.

QUESTION 5 Let $M$ be defined as follows

$M = (K, \Sigma, s, \Delta, F)$

for $K = \{q_0, q_1, q_2\}$, $s = q_0$
$\Sigma = \{a, b\}$, $F = \{q_0, q_2\}$ and
$\Delta = \{(q_0, a, q_1), (q_1, b, q_2), (q_1, b, q_0), (q_2, a, q_0)\}$.
Write 4 steps of the general method of transformation a NDFA \( M \), into an equivalent \( M' \), which is a DFA. Reminder: \( E(q) = \{ p \in K : (q,e)^* M(p,e) \} \) and

\[
\delta(Q,\sigma) = \bigcup \{ E(p) : \exists q \in Q(q,\sigma,p) \in \Delta , p \in K \}
\]

Step 1: Evaluate \( \delta(E(q_0), a) \) and \( \delta(E(q_0), b) \).

Step i+1: Evaluate \( \delta \) on all states that result from step i.

Solution

Step 1:

\[
E(q_0) = \{ q_0 \}, E(q_1) = \{ q_1 \}, E(q_2) = \{ q_2 \}
\]

\[
\delta(\{ q_0 \}, a) = E(q_1) = \{ q_1 \} \quad \delta(\{ q_0 \}, b) = \emptyset
\]

Step 2:

\[
\delta(\emptyset, a) = \emptyset, \delta(\emptyset, a) = \emptyset, \delta(\{ q_1 \}, a) = \emptyset, \delta(\{ q_1 \}, b) = E(q_0) \cup E(q_2) = \{ q_0, q_2 \} \in F'
\]

Step 3:

\[
\delta(\{ q_0, q_2 \}, a) = E(q_1) \cup E(q_0) = \{ q_0, q_1 \}, \delta(\{ q_0, q_2 \}, b) = \emptyset
\]

Step 4:

\[
\delta(\{ q_0, q_1 \}, a) = \emptyset \cup E(q_1) = \{ q_1 \}, \delta(\{ q_0, q_1 \}, b) = \emptyset \cup E(q_0) \cup E(q_2) = \{ q_0, q_2 \} \in F'
\]