PART 1 (16pts) YES/NO QUESTIONS - (2pts) each

Circle the correct answer. Write SHORT justification. Answers without justification will not receive credit.

1. For any language $L$ there is a deterministic automata $M$, such that $L = L(M)$.

   Justify: Language must be regular.

2. Given $L_1, L_2$ are regular languages over $\Sigma$, then $(L_1 \cap (\Sigma^* - L_1))L_2$ is not regular.

   Justify: Regular languages are closed under intersection and complement

3. There is an algorithm that for any finite automata $M$ computes a regular expression $r$, such that $L(M) = r$.

   Justify: defined in the proof of Main Theorem

4. $L = \{a^{2n} : n \geq 0\}$ is not regular.

   Justify: $L = (aa)^*$

5. $L = \{b^n a^n : n \geq 0\}$ is regular.

   Justify: proved using Pumping Lemma that is not regular

6. The alphabet $\Sigma_G$ in a Generalized Automaton includes some regular expressions.

   Justify: by definition

7. Pumping Lemma proves that a language is not regular.

   Justify: it gives certain characterization of infinite regular languages and can be used for proving that a language is not regular.

8. The class of regular languages is closed with respect to subset relation.

   Justify: $L_2 = \{b^n a^n : n \geq 0\} \subseteq L = b^* a^*$ and $L$ is regular, and $L_1$ is not regular
PART 2: PROBLEMS (15pts)

QUESTION 1 (7pts)

Use the constructions defined in the proof of the theorem:

A language is regular if and only if it is accepted by a finite automata
to construct a finite automata $M$ such that

$$L(M) = (ab \cup c)^*$$

Draw PATTERN diagrams.

Use the constructions described in the proof of the Closure Theorem.

S1. (3pt) Draw diagram of automata $M_a M_b \cup M_c$.

S2. (4pt) Draw diagram of $M = (M_a M_b \cup M_c)^*$. 
QUESTION 2 (8pts)

Evaluate regular expression \( r \), such that \( L(M) = r \) using the **Generalized Automata Construction** for \( M \) given by

\[
M = ((q_1, q_2), \{a, b\}, s = q_1, \\
\Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\}, F = \{q_2\})
\]

Remember using to use the proper names for states.

1. (2pt) Draw a diagram of \( GM \)

The components of \( GM \) are as follows

\[
GM = ((q_1, q_2, q_3, q_4), \{a, b\}, s = q_3, F = \{q_4\} \\
\Delta = \{(q_1, a, q_1), (q_1, a, q_2), (q_2, b, q_2), (q_2, a, q_1)\}, (q_3, e, q_1), (q_2, e, q_4))
\]

The diagram is:

![Diagram of GM](image)

2. (3pt) Draw a diagram of \( GM1 \simeq GM \simeq M \) obtained by elimination of \( q_1 \).

The components of \( GM1 \) are:

\[
GM1 = ((q_2, q_3, q_4), \{a, b\}, s = q_3, F = \{q_4\} \\
\Delta = \{(q_3, a^*b, q_2), (q_2, a, q_2), (q_2, ba^*b, q_2)\}, (q_2, e, q_4))
\]

The diagram is:

![Diagram of GM1](image)

3. (2pt) Draw a diagram of \( GM2 \simeq GM1 \simeq GM \simeq M \) obtained by elimination of \( q_2 \).

The components of \( GM2 \) are:

\[
GM2 = ((q_3, q_4), \{a, b\}, s = q_3, F = \{q_4\} \\
\Delta = \{(q_3, a^*b(ba^*b \cup a)^*, q_4)\}
\]

The diagram is:

![Diagram of GM2](image)

4. (1pt) Write the regular expression \( r \), such that \( L(M) = r \)

\[
L(M) = a^*b(ba^*b \cup a)^*
\]