CSE303 Q1 SOLUTIONS Spring 2023

25 points + 5 extra pts

NAME SOLUTIONS

PART 1: YES/NO QUESTIONS (15pts)

Circle your answer and **write** a short justification **(1pt)** . **No justification- no points**

1. $\{2, \{\emptyset\}, \emptyset\} \cap 2^{\emptyset} = \emptyset$

   Justify: $2^{\emptyset} = \{\emptyset\}$ and $\{2, \{\emptyset\}, \emptyset\} \cap \{\emptyset\} \neq \emptyset$ as $\emptyset \in \{2, \{\emptyset\}, \emptyset\}$

   n

2. For any non-empty sets A, B, any function that maps A into B is a binary relation $R \subseteq A \times B$

   Justify: Functions are, by definition, special binary relations

   y

3. A set $A = \{ (n, m) \in N \times N : \; n^2 < 5, \; m^2 < 6\}$ is countable

   Justify: $A = \{ (n, m) \in N \times N : \; n = 0, 1, 2 \; \textit{and} \; m = 0, 1, 2\}$ is a FINITE set, hence countable

   y

4. A set $A = \{(\{n\}, m) \in 2^N \times N : \; n^2 + 1 < 5, \; m > 6\}$ is countable

   Justify: $A = \{(\{n\}, m) \in 2^N \times N : \; n = 0, 1, \; m > 6\} = B \times C$
   for $B = \{\{0\}, \{1\}\}$ and $C = \{m \in N : m > 6\}$. Both sets are countable as B is finite and C infinitely countable and so is their Cartesian product

   y

5. Set A is countable if and only if $|A| = |N|$, where N is the set of Natural numbers.

   Justify: This is definition of infinitely countable set.
   Definition of a countable set is: a set A is countable iff is finite or infinitely countable.

   n

6. If the set A is uncountable, then $|A| = |R|$ (R is the set of real numbers)

   Justify: Definition of a un countable set is: A is countable iff A is not countable.
   For example $A = 2^R$ is uncountable, but $|R| < 2^R$ by Cantor Theorem.

   n

7. The set $\{\emptyset, \{\emptyset\}\}$ is an alphabet

   Justify: A has two elements so it is a finite set.

   y

8. Let $\Sigma = \emptyset$. There is only one language $L = \emptyset$ over Σ

Justify: $\Sigma^* = \emptyset^* = \{e\}$. We have that and $\emptyset \subseteq \{e\}$ and $\{e\} \subseteq \{e\}$.
Hence there are two languages $L_1 = \emptyset$ and $L_1 = \{e\}$ over Σ

n

9. For any languages $L_1,\ L_2,\ L$ over $\Sigma \neq \emptyset$

$(L_1 \cup L_2) \cap L = (L_1 \cap L) \cup (L_2 \cap L)$

Justify: Languages are sets, so the LAWS of distributivity hold

y

10. Let $\Sigma = \{a, b\}$. There are more than 20 words of length 4 over Σ.

Justify: By Counting Functions Theorem there are exactly $2^4 = 16$ words of length 4 over Σ and $16 < 20$.

n

11. For any languages $L_1,\ L_2,\ L$ over $\Sigma \neq \emptyset$

$\Sigma^* - (L_1 \cup L_2) = (\Sigma^* - L_1) \cap (\Sigma^* - L_2)$

Justify: Languages are sets so de Morgan Laws hold for them.

y

12. $L^* = \{w_1 ... w_n : w_i \in L,\ i = 1, 2, ..n,\ n \geq 0\}$

Justify:Definition of Kleene Star operation on languages

y

13. $L^+ = LL^*$.

Justify: the problem is only with cases $e \in L$ or $e \notin L$.
When $e \in L$, then $e \in L^+$, and always $e \in L^*$, and $e \in LL^*$.
When $e \notin L$, then $e \notin L^+$, and always $e \in L^*$, but then $e \notin LL^*$ as $xe = x$ for $x \neq e$, hence $L^+ = LL^*$ **y**

14. A regular language is a regular expression.

Justify:A language $L \subseteq \Sigma^*$ is regular if and only if L is represented by a regular expression, i.e. when there is $\alpha \in \mathcal{R}$ such that $L = \mathcal{L}(\alpha)$

n

15. $(\emptyset^* \cap a) \cup b^*) \cap \emptyset^*$ describes a language with only one element.

Justify: we have that $\emptyset^* = \{e\}$, $(\emptyset^* \cap a) = \{e\} \cap \{a\} = \emptyset$, and so
$(\emptyset^* \cap a) \cup b^*) \cap \emptyset^* = (\emptyset \cup \{b\}^*) \cap \{e\} = \{b\}^* \cap \{e\} = \{e\}$ **y**

PART 2: PROBLEMS (15pts)

QUESTION 1 (8pts)

Given an alphabet $\Sigma = \{a, b, c\}$ and a regular expression $\alpha = ((c^*a) \cup b)$

1. (1pt) Write α in the shorthand notation

$$c^*a \cup b$$

2. (3pts) Given the representation function $\mathcal{L} : \mathcal{R} \longrightarrow 2^{\Sigma^*}$.

Use the definition of the function $\mathcal{L}$ to evaluate, step by step $L = \mathcal{L}(\alpha)$.

$L = \mathcal{L}(\alpha) = \mathcal{L}(c^*a \cup b) = \mathcal{L}(c^*a) \cup \mathcal{L}(b) = \mathcal{L}(c^*)\mathcal{L}(a) \cup \{b\} = \{c\}^*\{a\} \cup \{b\}$

3. (2pts) List elements of the infinite set $L = \alpha$ (short notation)

$$\{e, c, cc, ccc, ...\}\{a\} \cup \{b\} = \{a, ca, cca, ccca, ...\} \cup \{b\} = \{b, a, ca, cca, ccca, ...\}$$

4. (2pts) Write a property $P(w)$ describing $L = \alpha$, i.e. write the language L as $L = \{w \in \Sigma^* : P(w)\}$

$L = \{w \in \Sigma^* :$ $w = a$ or $w = b$ or w is any nonempty finite sequence of c' s followed by a }

PROBLEM 2 (7pts)

Let $\Sigma = \{0, 1\}$. Let $L \subseteq \Sigma^\star$ be defined as follows.

$L = \{w \in \{0, 1\}^* :$ w has **three** occurrences of 1 the **first** and the **second** of which **are not** consecutive }

1. (4pts) Write a regular expression α, such that $L = \alpha$ (short notation)

$$L = 0^*10^*010^*10^*$$

2. (3pts) Explain carefully our answer.

L obviously has has **three** occurrences of 1.

The **first** and the **second** occurrences of 1 **are not** consecutive because in the expression $10^*\mathbf{0}1$ the **0** separates them.