CSE303 PRACTICE MIDTERM Spring 2019
(15 extra pts)

NAME **ID:**

1. TAKE test as a practice - to see how much points you would get.

Solutions to all problems and questions are somewhere on our webpage! so you CAN correct yourself- but do it **ONLY AFTER you complete it all by yourself**.

This is the **goal** of the PRACTICE TEST!

2. The **real midterm will have less problems**; I will make sure you will be able to complete it within 1 hour and 15 minutes.

3. SUBMIT YOUR TEST via Blackboard any day before or on March 25

4. I WILL POST THE SOLUTIONS on MONDAY for you to STUDY for MIDTERM

MIDTERM is THURSDAY, March 28

1 YES/NO questions

Circle the correct answer (each question is worth 1pt) Write SHORT justification.

1. For any function f from $A \neq \emptyset$ onto A, f has property $f(a) \neq a$ for certain $a \in A$.
   Justify:

   y n

2. For any binary relation $R \subseteq A \times A$, R^{-1} exists.
   Justify:

   y n

3. All infinite sets have the same cardinality.
   Justify:

   y n

4. Set A is uncountable iff $R \subseteq A$ (R is the set of *real* numbers).
   Justify:

   y n

5. Let $A \neq \emptyset$ such that there are exactly 25 partitions of A. It is possible to define 20 equivalence relations on A.
   Justify:

   y n

6. There is a relation that is equivalence and *order* at the same time.
Justify:

y **n**

7. Let $A = \{n \in N : n^2 + 1 \leq 15\}$. It is possible to define 8 *alphabets* $\Sigma \subseteq A$.
Justify:

y **n**

8. There is exactly as many languages over alphabet $\Sigma = \{a\}$ as real numbers.
Justify:

y **n**

9. Let $\Sigma = \{a, b\}$. There are more than 20 words of length 4 over Σ.
Justify:

y **n**

10. $L^* = \{w_1...w_n : w_i \in L, i = 1, 2, ..n, n \geq 1\}$.
Justify:

y **n**

11. $L^+ = LL^*$.
Justify:

y **n**

12. $L^+ = L^* - \{e\}$.
Justify:

y **n**

13. If $L = \{w \in \{0,1\}^* :$ w has an unequal number of 0's and 1's $\}$, then $L^* = \{0,1\}^*$.
Justify:

y **n**

14. For any languages L_1, L_2,

$$L_1^* = L_2^* \ \ iff \ \ L_1 = L_2$$

Justify:

y **n**

15. For any languages L_1, L_2, $(L_1 \cup L_2) \cap L_1 = L_1$.
Justify:

y **n**

16. For any languages L_1, L_2, $(L_1 \cup L_2)^* = L_1^*$.
Justify:

y **n**

17. $((\emptyset^* \cap a) \cup b^*) \cap \emptyset^*$ describes a language with only one element.
Justify:

y **n**

18. $((\emptyset^* \cap a) \cup b^*) \cap a*$ is a finite regular language.
Justify:

y **n**

19. $(\{a\} \cup \{e\}) \cap \{ab\}^*$ is a finite regular language.
Justify:

y **n**

20. Any regular language has a finite description.
Justify:

y **n**

21. Any finite language is regular.
Justify:

y **n**

22. Every deterministic automaton is also non-deterministic.
Justify:

y **n**

23. The set of all configurations of a given finite state automaton is always non-empty.
Justify:

y **n**

24. Let M be a finite state automaton, $L(M) = \{\omega \in \Sigma^* : (q, \omega) \overset{*,M}{\longmapsto} (s, e)\}$.
Justify:

y **n**

25. For any automaton M, $L(M) \neq \emptyset$.
Justify:

y **n**

26. $L(M_1) = L(M_2)$ iff M_1, M_2 are deterministic.
Justify:

y **n**

27. DFA and NDFA compute the same class of languages.
Justify:

y **n**

28. Let M_1 be a deterministic, M_2 be a nondeterministic FA, $L_1 = L(M_1)$ and $L_2 = L(M_2)$ then there is a deterministic automaton M such that $L(M) = (L^* \cup (L_1 - L_2)^*)L_1$
Justify:

y **n**

TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA

BOOK DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$.
OBSERVE that Δ is always finite because K, Σ are finite sets.

LECTURE DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when Δ is finite and $\Delta \subseteq K \times \Sigma^* \times K$.
OBSERVE that we have to say in this case that Δ is finite because Σ^* is infinite.

SOLVING PROBLEMS you can use any of these definitions.

2 Problems

PROBLEM 1 (10 pts)

Let L be a language defines as follows

$$L = \{w \in \{a,b\}^* : every\ a\ is\ either\ immediately\ proceeded\ or\ followed\ by\ b\}.$$

1. Describe a regular expression r such that $\mathcal{L}(r) = L$. Explain shortly your solution.

2. Construct a finite state automata M, such that $L(M) = L$.

State Diagram of M is:

Some elements of $L(M)$ as defined by the state diagram are:

Components of M are:

PROBLEM 2 (6 pts)

1. Let M be a deterministic finite automaton. Under exactly what conditions $e \in L(M)$?

2. Let M be a non-deterministic finite automaton. Under exactly what conditions $e \in L(M)$?

PROBLEM 3 (16 pts) Let

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $\Sigma = \{a\, b\}$, $F = \{q_1, q_2, q_3\}$, and

$$\Delta = \{(q_0, a, q_1), (q_0, b, q_3), (q_1, a, q_2), (q_1, b, q_1), (q_3, a, q_3), (q_3, b, q_2)\}$$

1 Draw the State Diagram of M.

2. List few elements of L(M).

3. Write a regular expression for the language accepted by M. Explain and simplify the solution.

4. Define a deterministic M' such that $M \approx M'$, i.e. $L(M) = L(M')$.

State Diagram of M' is:

Justify why $M \approx M'$.

PROBLEM 4 (15 pts)

Let M be defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2, q_3, \},\ s = q_0$

$\Sigma = \{a, b, c\}, \quad F = \{q_0, q_2, q_3\}$ and

$\Delta = \{(q_0, abc, q_0), (q_0, a, q_1), (q_0, e, q_3), (q_1, bc, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_3), (q_3, a, q_3)\}$.

1. Draw the State Diagram of M.

2. Find the regular expression describing the $L(M)$. Simplify it as much as you can. Explain your steps. Does $e \in L(M)$?

3. Write down (you can draw the diagram) an automata M' such that $M' \equiv M$ and M' is defined by the **BOOK definition**.

PROBLEM 5 (15 pts.) For M defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2, q_3\},\ s = q_0$
$\Sigma = \{a. b\},\ F = \{q_2\}$ and

$$\Delta = \{(q_0, a, q_3), (q_0, e, q_3), (q_0, b, q_1), q_0, e, q_1), (q_1, a, q_2), (q_2, b, q_3), (q_2, e, q_3)\}$$

1. Draw the State Diagram of M.

2. Write 2 steps of the general method of transformation a NDFA M, into an equivalent M', which is a DFA, where M is given by a following state diagram.

Step 1: Evaluate $\delta(E(q_0), a)$ and $\delta(E(q_0), b)$.

Step 2: Evaluate δ on all states that result from step 1.

Reminder: $E(q) = \{p \in K : (q, e) \overset{*,M}{\longmapsto} (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup\{E(p) : \exists q \in Q,\ (q, \sigma, p) \in \Delta\}.$$

EXTRA SPACE