1. TAKE test as a practice - to see how much points you would get.

Solutions to all problems and questions are somewhere on our webpage! so you CAN correct yourself-but do it **ONLY AFTER you complete it all by yourself.**

This is the **goal** of the PRACTICE TEST!

2. The **real midterm will have less problems**; I will make sure you will be able to complete it within 1 hour and 15 minutes.

3. SUBMIT YOUR TEST via Blackboard any day before or on March 25

4. I WILL POST THE SOLUTIONS on MONDAY for you to STUDY for MIDTERM

MIDTERM is **THURSDAY, March 28**

1. **YES/NO questions**

Circle the correct answer (each question is worth 1pt) Write SHORT justification.

1. For any function \( f \) from \( A \neq \emptyset \) onto \( A \), \( f \) has property \( f(a) \neq a \) for certain \( a \in A \).
   
   **Justify:**
   
   \( y \)  \( n \)

2. For any binary relation \( R \subseteq A \times A \), \( R^{-1} \) exists.
   
   **Justify:**
   
   \( y \)  \( n \)

3. All infinite sets have the same cardinality.
   
   **Justify:**
   
   \( y \)  \( n \)

4. Set \( A \) is uncountable iff \( R \subseteq A \) (\( R \) is the set of **real** numbers).
   
   **Justify:**
   
   \( y \)  \( n \)

5. Let \( A \neq \emptyset \) such that there are exactly 25 partitions of \( A \). It is possible to define 20 equivalence relations on \( A \).
   
   **Justify:**
   
   \( y \)  \( n \)
6. There is a relation that is equivalence and order at the same time.
   Justify:
   \( \text{y n} \)

7. Let \( A = \{ n \in N : n^2 + 1 \leq 15 \} \). It is possible to define 8 alphabets \( \Sigma \subseteq A \).
   Justify:
   \( \text{y n} \)

8. There is exactly as many languages over alphabet \( \Sigma = \{ a \} \) as real numbers.
   Justify:
   \( \text{y n} \)

9. Let \( \Sigma = \{ a, b \} \). There are more than 20 words of length 4 over \( \Sigma \).
   Justify:
   \( \text{y n} \)

10. \( L^* = \{ w_1...w_n : w_i \in L, i = 1, 2, n, n \geq 1 \} \).
    Justify:
    \( \text{y n} \)

11. \( L^+ = LL^* \).
    Justify:
    \( \text{y n} \)

12. \( L^+ = L^* - \{ e \} \).
    Justify:
    \( \text{y n} \)

13. If \( L = \{ w \in \{ 0, 1 \}^* : w \text{ has an unequal number of } 0's \text{ and } 1's \} \), then \( L^* = \{ 0, 1 \}^* \).
    Justify:
    \( \text{y n} \)

14. For any languages \( L_1, L_2 \),
    \[ L_1^* = L_2^* \ \text{iff} \ \ L_1 = L_2 \]
    Justify:
    \( \text{y n} \)

15. For any languages \( L_1, L_2 \), \( (L_1 \cup L_2) \cap L_1 = L_1 \).
    Justify:
    \( \text{y n} \)

16. For any languages \( L_1, L_2 \), \( (L_1 \cup L_2)^* = L_1^* \).
    Justify:
    \( \text{y n} \)

17. \((\emptyset \cap a) \cup b^* \) \cap \emptyset^* describes a language with only one element.
    Justify:
    \( \text{y n} \)

18. \((\emptyset \cap a) \cup b^* \) \cap a^* is a finite regular language.
    Justify:
    \( \text{y n} \)

19. \((\{ a \} \cup \{ e \}) \cap \{ ab \}^* \) is a finite regular language.
    Justify:
    \( \text{y n} \)
20. Any regular language has a finite description.
   Justify: y n

21. Any finite language is regular.
   Justify:

22. Every deterministic automaton is also non-deterministic.
   Justify: y n

23. The set of all configurations of a given finite state automaton is always non-empty.
   Justify: y n

24. Let $M$ be a finite state automaton, $L(M) = \{ \omega \in \Sigma^* : (q, \omega) \xrightarrow{s} (s, e) \}$.
   Justify: y n

25. For any automaton $M$, $L(M) \neq \emptyset$.
   Justify: y n

26. $L(M_1) = L(M_2)$ iff $M_1$, $M_2$ are deterministic.
   Justify: y n

27. DFA and NDFA compute the same class of languages.
   Justify: y n

28. Let $M_1$ be a deterministic, $M_2$ be a nondeterministic FA, $L_1 = L(M_1)$ and $L_2 = L(M_2)$ then there is a deterministic automaton $M$ such that $L(M) = (L^* \cup (L_1 - L_2)^*)L_1$
   Justify: y n

TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA

BOOK DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$.

OBSERVE that $\Delta$ is always finite because $K, \Sigma$ are finite sets.

LECTURE DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta$ is finite and $\Delta \subseteq K \times \Sigma^* \times K$.

OBSERVE that we have to say in this case that $\Delta$ is finite because $\Sigma^*$ is infinite.

SOLVING PROBLEMS you can use any of these definitions.
2 Problems

PROBLEM 1 (10 pts)

Let \( L \) be a language defined as follows

\[
L = \{ w \in \{ a, b \}^* : \text{every } a \text{ is either immediately preceded or followed by } b \}.
\]

1. Describe a regular expression \( r \) such that \( L(r) = L \). Explain shortly your solution.

2. Construct a finite state automata \( M \), such that \( L(M) = L \).

State Diagram of \( M \) is:

Some elements of \( L(M) \) as defined by the state diagram are:

Components of \( M \) are:
PROBLEM 2 (6 pts)

1. Let $M$ be a deterministic finite automaton. Under exactly what conditions $e \in L(M)$?

2. Let $M$ be a non-deterministic finite automaton. Under exactly what conditions $e \in L(M)$?

PROBLEM 3 (16 pts) Let

$M = (K, \Sigma, s, \Delta, F)$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $\Sigma = \{a, b\}$, $F = \{q_1, q_2, q_3\}$, and

$\Delta = \{(q_0, a, q_1), (q_0, b, q_3), (q_1, a, q_2), (q_1, b, q_1), (q_3, a, q_3), (q_3, b, q_2)\}$

1. Draw the State Diagram of $M$.

2. List few elements of $L(M)$.

3. Write a regular expression for the language accepted by $M$. Explain and simplify the solution.
4. Define a deterministic $M'$ such that $M \approx M'$, i.e. $L(M) = L(M')$.

**State Diagram** of $M'$ is:

Justify why $M \approx M'$.

**PROBLEM 4** (15 pts)

Let $M$ be defined as follows

$$M = (K, \Sigma, s, \Delta, F)$$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$

$\Sigma = \{a, b, c\}$, $F = \{q_0, q_2, q_3\}$ and

$\Delta = \{(q_0, abc, q_0), (q_0, a, q_1), (q_0, c, q_3), (q_1, bc, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_3), (q_3, a, q_3)\}$.

1. Draw the State Diagram of $M$.

2. Find the regular expression describing the $L(M)$. Simplify it as much as you can. Explain your steps. Does $e \in L(M)$?
3. Write down (you can draw the diagram) an automata $M'$ such that $M' \equiv M$ and $M'$ is defined by the BOOK definition.

PROBLEM 5 (15 pts.) For $M$ defined as follows

$$M = (K, \Sigma, s, \Delta, F)$$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$

$\Sigma = \{a, b\}$, $F = \{q_2\}$ and

$$\Delta = \{(q_0, a, q_3), (q_0, e, q_3), (q_0, b, q_1), (q_0, e, q_1), (q_1, a, q_2), (q_2, b, q_3), (q_2, e, q_3)\}$$

1. Draw the State Diagram of $M$.

2. Write 2 steps of the general method of transformation a NDFA $M$, into an equivalent $M'$, which is a DFA, where $M$ is given by a following state diagram.

Step 1: Evaluate $\delta(E(q_0), a)$ and $\delta(E(q_0), b)$.

Step 2: Evaluate $\delta$ on all states that result from step 1.

Reminder: $E(q) = \{p \in K : (q, e) \xrightarrow{M} (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup\{E(p) : \exists q \in Q, (q, \sigma, p) \in \Delta\}.$$