1. TAKE test as a practice - to see how many problems you can solve.

Solutions to all problems and questions are somewhere on our webpage! so you CAN correct yourself-
but do it ONLY AFTER you complete it all by yourself.

This is the goal of the PRACTICE TEST!

2. The real FINAL will have less problems; I will make sure you will be able to complete it within the time allowed.

3. SUBMIT YOUR TEST via Blackboard any day before or on MAY 9

4. I WILL POST THE SOLUTIONS on May 10 for you to study for Final

FINAL is scheduled for MAY 21, 2:15 - 5:00 pm, in FREY HALL 104

1 PART 1: Yes/No Questions

Circle the correct answer (each question is worth 1pt) Write SHORT justification.

1. A is uncountable iff $|A| = c$ (continuum).
   
   Justify: \[ y \ n \]

2. $(ab \cup a^*b)^*$ is a regular language
   
   Justify: \[ y \ n \]

3. There are uncountably many languages over $\Sigma = \{a\}$.
   
   Justify: \[ y \ n \]

4. $L^* = \{w \in \Sigma^* : \exists q \in F(s,w) \vdash^* M(q,e)\}$.
   
   Justify: \[ y \ n \]

5. $L^* = L^+ - \{e\}$.
   
   Justify: \[ y \ n \]
6. $L^* = \{w_1 \ldots w_n, w_i \in L, \ i = 1, \ldots, n\}$.
   Justify: $\ y \ n$

7. $((\phi^* \cap a) \cup (\phi \cup b^*)) \cap \phi^*$ represents a language $L = \{e\}$.
   Justify: $\ y \ n$

8. If $M$ is a FA, then $L(M) \neq \phi$.
   Justify: $\ y \ n$

9. $L(M_1) = L(M_2)$ iff $M_1$ and $M_2$ are finite automata.
   Justify: $\ y \ n$

10. A language is regular if and only if $L = L(M)$ and $M$ is a finite automaton
    Justify: $\ y$

11. If $L$ is regular, there is a PDA $M$ such that $L = L(M)$.
    Justify: $\ y \ n$

12. $((p, e, \beta), (q, \gamma)) \in \Delta$ means: read nothing, move from $p$ to $q$
    Justify: $\ y \ n$

13. Every subset of a regular language is a language.
    Justify: $\ y \ n$

14. Any finite language is CF.
    Justify: $\ y \ n$

15. Intersection of any two regular languages is CF language.
    Justify: $\ y \ n$

16. Union of a regular and a CF language is a CF language.
    Justify: $\ y \ n$

17. If $L$ is regular, there is a CF grammar $G$, such that $L = L(G)$.
    Justify: $\ y \ n$

18. $L = \{a^n b^n c^n : n \geq 0\}$ is CF.
    Justify: $\ y \ n$

19. $L = \{a^n b^n : n \geq 0\}$ is CF.
    Justify: $\ y \ n$
20. Let $\Sigma = \{a\}$, then for any $w \in \Sigma^*, w^R = w$

   Justify: y n

21. Let $G = \{(S, \langle \rangle), \{(\rangle\}, R, S\}$ for $R = \{S \rightarrow SS | (S)\}$. $L(G)$ is regular.

   Justify: y n

22. $L = \{a^n b^m c^n : n, m \in \mathbb{N}\}$ is CF.

   Justify: y n

23. If $L$ is regular, then there is a CF grammar $G$, such that $L = L(G)$.

   Justify: y n

24. Class of context-free languages is closed under intersection.

   Justify: y n

25. A CF language is a regular language.

   Justify: y n

2 PART 2: PROBLEMS

QUESTION 1

Let $L_1, L_2$ be the following languages over $\Sigma = \{a, b\}$:

$L_1 = \{w \in \Sigma^*: \exists u \in \Sigma \{w = uu^R\}\}$,

$L_2 = \{w \in \Sigma^*: w = www\}$.

1. List elements of $\Sigma \Sigma$

2. Show that $L_1$ is a finite set

3. Give examples of 2 words $w$ over $\Sigma$ such that $w \notin L_1$. 
4. Show that $L_2 \neq \emptyset$.

**QUESTION 2**

Let $\Sigma$ be any alphabet, $L_1, L_2$ two languages over $\Sigma$ such that $e \in L_1$ and $e \in L_2$. Show that

$$(L_1 \Sigma^* L_2)^* = \Sigma^*$$

**QUESTION 3**

Use book or lecture definition (specify which are you using) to construct a non-deterministic finite automaton $M$, such that

$L(M) = (ab)^*(ba)^*$.

1. Draw the state diagram.

2 Justify your construction by listing some strings accepted by the state diagram.

**QUESTION 4** Construct a PDA $M$, such that

$L(M) = \{b^n a^{2n} : n \geq 0\}$. 
1. List components: \( M = (K, \Sigma, \Gamma, \Delta, s, F) \)

2. Draw diagram

3. Explain the construction. Write motivation.

4. Trace a transition of \( M \) that leads to the acceptance of the string \( bbaaa \).

QUESTION 5  Given a Regular grammar \( G = (V, \Sigma, R, S) \), where
\[
V = \{a, b, S, A\}, \quad \Sigma = \{a, b\},
\]
\[ R = \{ S \rightarrow aS \mid A \mid e, \quad A \rightarrow aBA \mid a \mid b \}. \]

1. Use the construction in the proof of L-GTheorem:

Language L is regular if and only if there exists a regular grammar G such that \( L = L(G) \)
to construct a finite automaton \( M \), such that \( L(G) = L(M) \).

JUST DRAW a diagram of M

2. Trace a transition of \( M \) that leads to the acceptance of the string \( aaaababa \), and compare with a derivation of the same string in \( G \).
QUESTION 6

Prove that the Class of context-free languages is NOT closed under intersection

EXTRA CREDIT

Use closure under union for CF languages to show that

\[ L = \{ a^n b^n : n \neq m \} \]

is a CF language