CSE303 MIDTERM SOLUTIONS

1 YES/NO questions (20pts)

1. All infinite sets have different cardinality.
   Justify: The sets N (natural numbers) and Z (integers) have the same cardinality: $|N| = |Z| = \aleph_0$. This is not the only example. **n**

2. Regular language is a regular expression.
   Justify: By definition: "A language L is regular iff there is a regular expression α such that $L = \mathcal{L}(\alpha)$. **y**

3. $L^+ = \{w_1...w_n : w_i \in L, i = 1, 2, ..n,\ n \geq 2\}$.
   Justify: the correct condition is $n \geq 1$. **n**

4. $L^+ = L^* - \{e\}$.
   Justify: It holds only when $e \notin L$. When $e \in L$ we get that $e \in L^+$ and $e \notin L^* - \{e\}$. **n**

5. Let $\alpha = (\emptyset^* \cap b^*) \cup \emptyset^*$. The language defined by α is empty.
   Justify: $L = \mathcal{L}(\alpha) = (\{e\} \cap \{b\}^*) \cup \{e\} = \{e\} \cup \{e\} = \{e\} \neq \emptyset$ **n**

6. A configuration of any finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^* \times K$.
   Justify: it is element of $K \times \Sigma^*$ **n**

7. Let M be a finite state automaton, $L(M) = \{\omega \in \Sigma^* : (s, \omega) \overset{*,M}{\longmapsto} (q, e)\}$.
   Justify: only when $q \in F$ **n**

8. For any M,$L(M) \neq \emptyset$ if and only if the set F of its final states is non-empty.
   Justify: Let M be such that $\Sigma = \emptyset, F \neq \emptyset, s \notin F$, we get $L(M) = \emptyset$. **n**

9. The set F of final states of any non-deterministic finite automaton is always non-empty
   Justify: The definition says that F is a finite set, i.e. can be empty, hence for some M, $L(M) = \emptyset$ **n**

10. DFA and NDFA recognize the same class of languages.
    Justify: theorem proved in class **y**

2 Two definitions of a non-deterministic automaton

BOOK DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when

$$\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$$

OBSERVE that Δ is always finite because K, Σ are finite sets.

LECTURE DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when Δ is finite and

$$\Delta \subseteq K \times \Sigma^* \times K$$

OBSERVE that we have to say in this case that Δ is finite because Σ^* is infinite.

SOLVING PROBLEMS you can use any of these definitions.

3 Very short questions (15pts)

For the QUESTIONS below do the following.

1. Draw the DIAGRAM

2. Determine whether it defines a finite state automaton.

3. Determine whether it is a deterministic / non-deterministic automaton.

4. Describe the language by writing a regular expression $\delta(q_1, a) = q_2$, that defines it.

Q1 Solution: $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $K = \{q_0\} = F,\ s = q_0,\ \Sigma = \emptyset, \Delta = \emptyset$.

M is deterministic and

$$L(M) = \{e\} \neq \emptyset$$

Q2 Solution: $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $\Sigma = \{a, b\}, K = \{q_0, q_1\},\ s = q_0, F = \{q_0\}, \Delta = \{(q_0, a, q_1), (q_1, b, q_0)\}$.

M is non deterministic; Δ is not a function on $K \times \Sigma$.

$$L(M) = (ab)^*$$

Q3 Solution: $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $\Sigma = \{a, b\}, K = \{q_0, q_1, q_2\}, F = \{q_1\}$,
$\Delta = \{(q_0, a, q_1), (q_0, b, q_1), (q_1, a, q_1), (q_1, e, q_2), (q_2, ab, q_2)\}$.

It is NOT an automaton. It has no initial state.

4 Problems

PROBLEM 1 (20pts)

Given and automata

$$M = (K, \Sigma, \delta, s, F)$$

such that $\Sigma = \{a, b\}$, $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $F = \{q_0\}$ and q_3 is a trap state.

We define δ on **non-trap states** as follows.

$\delta(q_0, a) = q_1,\ \delta(q_0, b) = q_0,\quad \delta(q_1, a) = q_2,\quad \delta(q_1, b) = q_1,\quad \delta(q_2, a) = q_0$

Solution

We define δ on the trap state q_3 as follows.
$\delta(q_2, b) = q_3,\quad \delta(q_3, a) = q_3,\quad \delta(q_3, b) = q_3$

$$e,\ bbb,\ abbaa,\ abbaaabbaa \in L(M), \text{ and } a,\ aa,\ aba,\ baba \notin L(M)$$

Language of M is:

$$L(M) = b^* \cup (b^*ab^*aa)^*$$

Observe that $e \in L(M)$ as the initial state is also a final state and $b^* \in L(M)$

$b^*ab^*aa \in L(M)$ and then all repetitions of of this are in $L(M)$ as a loop on q_0

PROBLEM 2 (20pts)

Let

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0\},\ s = q_0,\ \Sigma = \{a, b\},\ F = \{q_0\}$ and

$$\Delta = \{(q_0, aba, q_0), (q_0, ab, q_0)\}$$

Solution

1. List some elements of $L(M)$.

$$e, ab, abab, ababa, ababaaba, ...$$

2. Write a regular expression for the language accepted by M.

$$L = (ab \cup aba)^*$$

PROBLEM 3 (25pts)

Let M be defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2, q_3, \},\ s = q_0$
$\Sigma = \{a, b, c\},\quad F = \{q_0, q_2, q_3\}$ and
$\Delta = \{(q_0, abc, q_0), (q_0, a, q_1), (q_0, e, q_3), (q_1, bc, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_3), (q_3, a, q_3)\}$.

1. Draw the state diagram of M.

2. Find the regular expression describing the $L(M)$. Explain your steps.

$$L = \alpha_1 \cup \alpha_2 \cup \alpha_3 \cup \alpha_4$$

where

$\alpha_1 = (abc)^*$ - loop on q_0,

$\alpha_2 = (abc)^* a (bc)^* ba^*$ - path from q_0 to q_2,

$\alpha_3 = (abc)^* a (bc)^* ba^* ba^*$ - path from q_0 to q_3 via q_2,

$\alpha_3 = (abc)^* a^*$ - path from q_0 directly to q_3

This is not the only solution.

Observe that $e \in L$ as $q_0 \in F$ and also $(q_0, e, q_3) \in \Delta$ and $q_3 \in F$.

This is not the only solution.

3. DRAW then DIAGRAM of an automata M' such that $M' \equiv M$ and M' is defined by the **BOOK definition**.

We apply the "stretching" technique to M and the new M' COMPONENTS are as follows.

$$M' = (K \cup \{p_1, p_2, p_3\}\ \Sigma,\ s = q_0,\ \Delta',\ F' = F\ )$$

for $K = \{q_0, q_1, q_2\},\ s = q_0$
$\Sigma = \{a, b\},\ F = \{q_0, q_2, q_3\}$ and
$\Delta' = \{(q_0, a, q_1), (q_0, e, q_3), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_3), (q_3, a, q_3)\}$
$\cup\{(q_0, a, p_1), (p_1, b, p_2), (p_2, c, q_0), (q_1, b, p_3), (p_3, b, q_1)\}$.

EXTRA CREDIT (10pts)

For M defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2, q_3\},\ s = q_0$
$\Sigma = \{a, b\},\ F = \{q_2, q_3\}$ and

$$\Delta = \{(q_0, a, q_1), (q_0, e, q_1), (q_0, b, q_2), (q_1, b, q_3), (q_1, e, q_3), (q_2, b, q_2), (q_2, e, q_3), (q_3, a, q_3)\}$$

1. Write 4 steps of the general method of transformation the NDFA M, into an equivalent deterministic M'.

2. Draw the State Diagram of M' thus far constructed.

Reminder: $E(q) = \{p \in K : (q, e) \overset{*,M}{\longmapsto} (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup\{E(p) : \exists q \in Q,\ (q, \sigma, p) \in \Delta\}.$$

Step 1:

$$E(q_0) = \{q_0, q_1, q_3\},\ E(q_1) = \{q_1, q_3\},\ E(q_2) = \{q_2, q_3\},\ E(q_3) = \{q_3\}.$$

Step 2:

$$\delta(E(q_0), a) = \delta(\{q_0, q_1, q_3\}, a) = E(q_1) \cup E(q_3) = \{q_1, q_3\} \in F,$$

$$\delta(E(q_0), b) = \delta(\{q_0, q_1, q_3\}, b) = E(q_2) \cup E(q_3) \cup \emptyset = \{q_2, q_3\} \in F,$$

Step 3:

$$\delta(\{q_1, q_3\}, a) = \emptyset \cup E(q_3) = \{q_3\} \in F,$$

$$\delta(\{q_1, q_3\}, b) = E(q_3) \cup \emptyset = \{q_3\} \in F,$$

$$\delta(\{q_2, q_3\}, a) = \emptyset \cup E(q_3) = \{q_3\} \in F,$$

$$\delta(\{q_2, q_3\}, b) = E\{q_2\} \cup \emptyset = \{q_2, q_3\} \in F$$

Step 4:

$$\delta(\{q_3\}, a) = E(q_3) = \{q_3\} \in F, \quad \delta(\{q_3\}, b) = \emptyset,$$

$$\delta(\emptyset, a) = \emptyset, \quad \delta(\emptyset, b) = \emptyset$$

End of the construction.

You must Draw the DIAGRAM