1. TAKE test as a practice - to see how many problems you can solve.

Solutions to all problems and questions are somewhere on our webpage! so you CAN correct yourself-but do it ONLY AFTER you complete it all by yourself.

This is the goal of the PRACTICE TEST!

2. The real FINAL will have less problems; I will make sure you will be able to complete it within the time allowed.

3. SUBMIT YOUR TEST via Blackboard before or on Saturday, December 7

4. I will post SOLUTIONS on Monday, December 9 for you to study for Final

FINAL is scheduled for Wednesday, DECEMBER 18, 8:30pm - 11:00 pm, in FREY HALL 102

1 PART 1: Yes/No Questions

Circle the correct answer. Write SHORT justification.

1. $A$ is uncountable iff $|A| = c$ (continuum).
   Justify: y n

2. $(ab \cup a^*b)^*$ is a regular language
   Justify: y n

3. There are uncountably many languages over $\Sigma = \{a\}$.
   Justify: y n

4. $L^* = \{w \in \Sigma^* : \exists q \in F(s, w) \vdash_M \phi(q, e)\}$.
   Justify: y n

5. $L^* = L^+ - \{e\}$.
   Justify: y n

6. $L^* = \{w_1 \ldots w_n, w_i \in L, i = 1, \ldots, n\}$.
   Justify: y n

7. $((\phi \cap a) \cup (\phi \cup b^*)) \cap \phi^*$ represents a language $L = \{e\}$.
   Justify: y n
8. If $M$ is a FA, then $L(M) \neq \phi$.
   Justify:
   
9. $L(M_1) = L(M_2)$ iff $M_1$ and $M_2$ are finite automata.
   Justify:

10. A language is regular if and only if $L = L(M)$ and $M$ is a finite automaton
   Justify:

11. If $L$ is regular, there is a PDA $M$ such that $L = L(M)$.
   Justify:

12. $((p, e, \beta), (q, \gamma)) \in \Delta$ means: read nothing, move from $p$ to $q$
   Justify:

13. Every subset of a regular language is a language.
   Justify:

14. Any finite language is CF.
   Justify:

15. Intersection of any two regular languages is CF language.
   Justify:

16. Union of a regular and a CF language is a CF language.
   Justify:

17. If $L$ is regular, there is a CF grammar $G$, such that $L = L(G)$.
   Justify:

18. $L = \{a^n b^n c^n : n \geq 0\}$ is CF.
   Justify:

19. $L = \{a^n b^n : n \geq 0\}$ is CF.
   Justify:

20. Let $\Sigma = \{a\}$, then for any $w \in \Sigma^*$, $w^R = w$
   Justify:

21. Let $G = (\{S, (,)\}, \{(,)\}, R, S)$ for $R = \{S \rightarrow SS | (S)\}$. $L(G)$ is regular.
   Justify:
22. \( L = \{a^n b^m c^n : n, m \in N\} \) is CF.
   Justify: \( y \) \( n \)

23. If \( L \) is regular, then there is a CF grammar \( G \), such that \( L = L(G) \).
   Justify: \( y \) \( n \)

24. Class of context-free languages is closed under intersection.
   Justify: \( y \) \( n \)

25. A CF language is a regular language.
   Justify: \( y \) \( n \)

2 \hspace{1cm} \textbf{PART 2}

\textbf{QUESTION 1}

Let \( L_1, L_2 \) be the following languages over \( \Sigma = \{a, b\} \):

\[
L_1 = \{w \in \Sigma^* : \exists u \in \Sigma \Sigma (w = uu^R)\}, \\
L_2 = \{w \in \Sigma^* : \, ww = www\}.
\]

1. List elements of \( \Sigma \Sigma \)

2. Show that \( L_1 \) is a finite set

3. Give examples of 2 words \( w \) over \( \Sigma \) such that \( w \notin L_1 \).

4. Show that \( L_2 \neq \emptyset \).
QUESTION 2

Let $\Sigma$ be any alphabet, $L_1, L_2$ two languages over $\Sigma$ such that $e \in L_1$ and $e \in L_2$. Show that

$$(L_1 \Sigma^* L_2)^* = \Sigma^*$$

QUESTION 3

Use book or lecture definition (specify which are you using) to construct a non-deterministic finite automaton $M$, such that

$L(M) = (ab)^*(ba)^*$.

1. Draw the state diagram.

2. Justify your construction by listing some strings accepted by the state diagram.

QUESTION 4

Construct a PDA $M$, such that

$L(M) = \{b^n a^{2n} : n \geq 0\}$.

1. List componenets: $M = (K, \Sigma, \Gamma, \Delta, s, F)$
2. Draw diagram

3. Explain the construction. Write motivation.

4. Trace a transitions of $M$ that leads to the acceptance of the string $bbaaaa$. 

QUESTION 5  Given a Regular grammar $G = (V, \Sigma, R, S)$, where

- $V = \{a, b, S, A\}$, $\Sigma = \{a, b\}$,
- $R = \{S \rightarrow aS | A | e, \ A \rightarrow abA | a | b\}$.

1. Use the construction in the proof of L-GTheorem:

Language $L$ is regular if and only if there exists a regular grammar $G$ such that $L = L(G)$ to construct a finite automaton $M$, such that $L(G) = L(M)$.

JUST DRAW a diagram of $M$
2. Trace a transition of $M$ that leads to the acceptance of the string $aaaababa$, and compare with a derivation of the same string in $G$.

QUESTION 6

Prove that the Class of context-free languages is NOT closed under intersection
EXTRA CREDIT

Use closure under union for CF languages to show that

\[ L = \{a^n b^n : n \neq m\} \]

is a CF language