PART 1  (10pts)  YES/NO QUESTIONS - (2 pts) each

1. For any finite language $L$ there is a deterministic automata $M$, such that $L = L(M)$.
   \[ \text{Justify}: \text{Any finite language is regular, so it is true by the Main Theorem.} \]

2. The alphabet $\Sigma_G$ in a Generalized Automaton includes some regular expressions.
   \[ \text{Justify}: \text{by definition } \Sigma_G = \Sigma \cup R_0, \text{ where } R_0 \text{ is a FINITE subset of the set } R \text{ go regular expressions.} \]

3. Pumping Lemma proves that a language is not regular.
   \[ \text{Justify}: \text{Pumping Lemma serves as a tool for proving that a language is not regular} \]

4. $L = \{a^n b^n : n \geq 0\}$ is not regular.
   \[ \text{Justify}: L = (aa)^n \]

5. The class of regular languages is closed with respect to subset relation.
   \[ \text{Justify}: \text{Consider } L_1 = \{a^n b^n : n \in N\}, L_2 = a^* b^*. \text{ Observe that } L_1 \subseteq L_2 \text{ and } L_1 \text{ is a non-regular subset of a regular } L_2. \]

PART 2: PROBLEMS

QUESTION 1 (5pts) Use the constructions defined in the proof of the theorem:  \textbf{A language is regular \iff it is accepted by a finite automata} to construct a finite automata $M$ such that

$L(M) = (a \cup b^*)^*$

Draw PATTERN diagrams. Use the constructions described in the proof of the \textbf{Closure Theorem}.

S1. (2pts) Draw diagram of automata $M_a, M_b, M_b^*$.

Here are the components of the automata, draw diagrams.

$M_a = ( \{q_1, q_2\}, \Sigma = \{a\}, s = q_1, \Delta = \{(q_1, a, q_2)\}, F = \{q_2\} )$

$M_b = ( \{q_1, q_2\}, \Sigma = \{b\}, s = q_1, \Delta = \{(q_1, b, q_2)\}, F = \{q_2\} )$

$M_b^* = ( \{q_1, q_2, q_3\}, \Sigma = \{b\}, s = q_1, \Delta = \{(q_1, e, q_2), (q_2, b, q_3), (q_3, e, q_2), (q_3, e, q_3)\}, F = \{q_1, q_3\} )$

S2. (3pts) Draw diagrams of $M_a \cup M_b$ and $(M_a \cup M_b)^*$

$M_a \cup M_b = ( \{q_1, q_2, q_3, q_4, q_5, q_6\}, \Sigma = \{a, b\}, s = q_1, \Delta, F = \{q_3, q_4, q_6\} )$

$\text{for } \Delta = \{(q_1, e, q_2), (q_2, a, q_3), (q_4, e, q_4), (q_5, b, q_6), (q_6, e, q_3)\}$

$(M_a \cup M_b)^* = ( \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \Sigma = \{a, b\}, s = q_0, \Delta, F = \{q_0, q_3, q_4, q_6\} )$

$\text{for } \Delta = \{(q_0, e, q_1), (q_1, e, q_2), (q_2, a, q_3), (q_4, e, q_4), (q_5, e, q_5), (q_5, b, q_6), (q_6, e, q_3)\}$
QUESTION 2 (10pts) Given an automaton

\[ M = ( \{ q_1, q_2 \}, \{ a, b \}, s = q_1, \Delta, F = \{ q_1, q_2 \} ) \]

for \( \Delta = \{ (q_1, b, q_1), (q_1, a, q_2), (q_2, a, q_2), (q_2, b, q_2) \} \)

S1. (1pt) Draw a diagram of \( M \) and write \( L(M) = r \) for \( r \in \mathcal{R} \)

Components are given, draw Diagram and

\[ L(M) = b^* \cup b^*a(a \cup b)^* \]

S2. (3pts) Write \( L(M) \) using the formula \( L(M) = \bigcup \{|R(1, j, n): q_j \in F\} \).

Observe that \( n = |K| \) so in our case \( n = |\{a, b\}| = 2 \) and

\[ L(M) = \bigcup \{|R(1, j, 2): q_j \in F\} = R(1, 1, 2) \cup R(1, 2, 2) \]

Evaluate \( R(1, 1, 0) \) and \( R(1, 2, 0) \).

Observe that \( (q_1, b, q_1) \in \Delta \) and \( (q_1, a, q_2) \in \Delta \), so we get

\[ R(1, 1, 0) = \{e\} \cup \{b\}, \quad R(1, 2, 0) = \{a\} \]

S3. (6pts) Evaluate regular expression \( r \), such that \( L(M) = r \) using the **Generalized Automata Construction**

1. (1pt) Draw a diagram of \( GM \)

Here are the components of \( GM \), draw diagram.

\[ GM = (\{q_1, q_2, q_3, q_4\}, \{a, b\} \cup \mathcal{R}_0, s = q_3, F = \{q_4\} \]

\[ \Delta_G = (\{q_3, e, q_1\}, (q_1, b, q_1), (q_1, a, q_2), (q_2, a, q_2), (q_2, b, q_2), (q_1, e, q_4), (q_2, e, q_4)) \]

2. (2pts) Draw a diagram of \( GM1 \approx GM \approx M \) obtained by elimination of \( q_1 \).

Here are the components of \( GM \), draw diagram.

\[ GM1 = (\{q_2, q_3, q_4\}, \{a, b\} \cup \mathcal{R}_0, s = q_3, F = \{q_4\} \]

\[ \Delta_{G1} = (\{q_3, b^*a, q_2\}, (q_3, b^*, q_4) (q_2, a, q_2), (q_2, b, q_2), (q_2, e, q_4)) \]

3. (2pt) Draw a diagram of \( GM2 \approx GM1 \approx GM \approx M \) obtained by elimination of \( q_2 \).

Here are the components of \( GM \), draw diagram.

\[ GM2 = (\{q_3, q_4\}, \{a, b\} \cup \mathcal{R}_0, s = q_3, F = \{q_4\} \]

\[ \Delta_{G2} = (\{q_3, b^*a(a \cup b)^* \cup b^*q_4]) \]

4. (1pt) Write the regular expression \( r \), such that \( L(M) = r \) and COMPARE with your answer to S1

\[ L(M_G) = b^*a(a \cup b)^* \cup b^* = b^* \cup b^*a(a \cup b)^* = L(M) \]